Estimating Equipment Costs by Scaling

It is often necessary to estimate the cost of a piece of equipment when no cost data are available for the particular size of operational capacity involved. Good results can be obtained by using the logarithmic relationship known as the *six-tenths-factor* rule, if the new piece of equipment is similar to one of another capacity for which cost data are available. According to this rule, if the cost of a given unit at one capacity is known, the cost of a similar unit with $X$ times the capacity of the first is approximately $(X)^{0.6}$ times the cost of the initial unit.

\[
\text{Cost of equip. } a = \text{cost of equip. } b \left( \frac{\text{capac. equip. } a}{\text{capac. equip. } b} \right)^{0.6} \tag{1}
\]

The preceding equation indicates that a log-log plot of capacity versus equipment cost for a given type of equipment should be a straight line with a slope equal to 0.6. Figure 6-5 presents a plot of this sort for shell-and-tube heat exchangers. However, the application of the 0.6 rule of thumb for most purchased equipment is an oversimplification of a valuable cost concept since the actual values of the cost capacity factor vary from less than 0.2 to greater than 1.0 as shown in Table 5. Because of this, the 0.6 factor should only be used in the absence of other information. In general, the cost-capacity concept should not be used beyond a tenfold range of capacity, and care must be taken to make certain the two pieces of equipment are similar with regard to type of construction, materials of construction, temperature and pressure operating range, and other pertinent variables.
**TABLE 5**  
Typical exponents for equipment cost vs. capacity

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Size range</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blender, double cone rotary, <strong>C.S.</strong></td>
<td>SO-250 ft³</td>
<td>0.49</td>
</tr>
<tr>
<td>Blower, centrifugal</td>
<td>$10^3-10^4 \text{ ft}^3/\text{min}$</td>
<td>0.59</td>
</tr>
<tr>
<td>Centrifuge, solid bowl, <strong>C.S.</strong></td>
<td>10-10² hp drive</td>
<td>0.67</td>
</tr>
<tr>
<td>Crystallizer, vacuum batch, <strong>C.S.</strong></td>
<td>500-7000 ft³</td>
<td>0.37</td>
</tr>
<tr>
<td>Compressor, reciprocating, air cooled, two-stage, 150 psi discharge</td>
<td>$10^4-400 \text{ ft}^3/\text{min}$</td>
<td>0.69</td>
</tr>
<tr>
<td>Compressor, rotary, single-stage, sliding vane, 150 psi discharge</td>
<td>$10^2-10^3 \text{ ft}^3/\text{min}$</td>
<td>0.79</td>
</tr>
<tr>
<td>Dryer, drum, single vacuum</td>
<td>10-10² ft²</td>
<td>0.76</td>
</tr>
<tr>
<td>Dryer, drum, single atmospheric</td>
<td>10-10² ft²</td>
<td>0.40</td>
</tr>
<tr>
<td>Evaporator (installed), horizontal <strong>tank</strong></td>
<td>$10^2-10^4 \text{ ft}^2$</td>
<td>0.54</td>
</tr>
<tr>
<td>Fan, centrifugal</td>
<td>$10^3-10^4 \text{ ft}^3/\text{min}$</td>
<td>0.44</td>
</tr>
<tr>
<td>Fan, centrifugal</td>
<td>$2 \times 10^4-7 \times 10^4 \text{ ft}^3/\text{min}$</td>
<td>1.17</td>
</tr>
<tr>
<td>Heat exchanger, shell and tube, floating head, <strong>C.S.</strong></td>
<td>100-400 ft²</td>
<td>0.60</td>
</tr>
<tr>
<td>Heat exchanger, shell and tube, <strong>fixed sheet. C.S.</strong></td>
<td>100-400 ft²</td>
<td>0.44</td>
</tr>
<tr>
<td>Kettle, cast iron, jacketed</td>
<td>250-800 gal</td>
<td>0.27</td>
</tr>
<tr>
<td>Kettle, glass lined, jacketed</td>
<td>200-800 gal</td>
<td>0.31</td>
</tr>
<tr>
<td>Motor, squirrel cage, induction, 440 volts, explosion proof</td>
<td>5-20 hp</td>
<td>0.69</td>
</tr>
<tr>
<td>Motor, squirrel cage, induction, 440 volts, explosion proof</td>
<td>20-200 hp</td>
<td>0.99</td>
</tr>
<tr>
<td>Pump, reciprocating, horizontal cast iron (includes motor)</td>
<td>2-100 gpm</td>
<td>0.34</td>
</tr>
<tr>
<td>Pump, centrifugal, horizontal, cast steel (includes motor)</td>
<td>$10^4-10^5 \text{ gpm} \times \text{ psi}$</td>
<td>0.33</td>
</tr>
<tr>
<td>Reactor, glass lined, jacketed (without drive)</td>
<td>50-600 gal</td>
<td>0.54</td>
</tr>
<tr>
<td>Reactor, <strong>S.S.</strong>, 300 psi</td>
<td>$10^2-10^3 \text{ gal}$</td>
<td>0.56</td>
</tr>
<tr>
<td>Separator, centrifugal, <strong>C.S.</strong></td>
<td>50-250 ft³</td>
<td>0.49</td>
</tr>
<tr>
<td>Tank, flat head, <strong>C.S.</strong></td>
<td>$10^2-10^4 \text{ gal}$</td>
<td>0.57</td>
</tr>
<tr>
<td>Tank, <strong>C.S.</strong>, glass lined</td>
<td>$10^2-10^3 \text{ gal}$</td>
<td>0.49</td>
</tr>
<tr>
<td>Tower, <strong>C.S.</strong></td>
<td>$10^3-2 \times 10^6 \text{ lb}$</td>
<td>0.62</td>
</tr>
<tr>
<td>Tray, bubble cup, <strong>C.S.</strong></td>
<td>3-10 ft diameter</td>
<td>1.20</td>
</tr>
<tr>
<td>Tray, sieve, <strong>C.S.</strong></td>
<td>3-10 ft diameter</td>
<td>0.86</td>
</tr>
</tbody>
</table>

**Example 2**  Estimating cost of equipment using scaling factors and cost index.  
The purchased cost of a **50-gal** glass-lined, jacketed reactor (without drive) was $8350 in 1981. Estimate the purchased cost of a similar **300-gal**, glass-lined, jacketed reactor (without drive) in 1986. Use the annual average Marshall and Swift equipment-cost index (all industry) to update the purchase cost of the reactor.

**Solution.**  Marshall and Swift equipment-cost index (all industry)

(From Table 3) For 1981  721
(From Table 3) For 1986  798
From Table 5, the equipment vs. capacity exponent is given as 0.54:

\[
\text{In 1986, cost of reactor} = (\$8350) \left( \frac{798}{721} \right) \left( \frac{300}{50} \right)^{0.54}
\]

\[= \$24,300\]

Purchased-equipment costs for vessels, tanks, and process- and materials-handling equipment can often be estimated on the basis of weight. The fact that a wide variety of types of equipment have about the same cost per unit weight is quite useful, particularly when other cost data are not available. Generally, the cost data generated by this method are sufficiently reliable to permit order-of-magnitude estimates.

**Purchased-Equipment Installation**

The installation of equipment involves costs for labor, foundations, supports, platforms, construction expenses, and other factors directly related to the erection of purchased equipment. Table 6 presents the general range of installation cost as a percentage of the purchased-equipment cost for various types of equipment.

Installation labor cost as a function of equipment size shows wide variations when scaled from previous installation estimates. Table 7 shows exponents varying from 0.0 to 1.56 for a few selected pieces of equipment.

<table>
<thead>
<tr>
<th>Type of equipment</th>
<th>Installation cost, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centrifugal separators</td>
<td>20-60</td>
</tr>
<tr>
<td>Compressors</td>
<td>30-60</td>
</tr>
<tr>
<td>Dryers</td>
<td>25-60</td>
</tr>
<tr>
<td>Evaporators</td>
<td>25-90</td>
</tr>
<tr>
<td>Filters</td>
<td>65-80</td>
</tr>
<tr>
<td>Heat exchangers</td>
<td>30-60</td>
</tr>
<tr>
<td>Mechanical crystallizers</td>
<td>30-60</td>
</tr>
<tr>
<td>Metal tanks</td>
<td>30-60</td>
</tr>
<tr>
<td>Mixers</td>
<td>20-40</td>
</tr>
<tr>
<td>Pumps</td>
<td>25-60</td>
</tr>
<tr>
<td>Towers</td>
<td>60-90</td>
</tr>
<tr>
<td>Vacuum crystallizers</td>
<td>40-70</td>
</tr>
<tr>
<td>Wood tanks</td>
<td>30-60</td>
</tr>
</tbody>
</table>

TABLE 7
Typical exponents for equipment installation labor vs. size

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Size range</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conduit, aluminum</td>
<td>0.5–2-in. diam.</td>
<td>0.49</td>
</tr>
<tr>
<td>Conduit, aluminum</td>
<td>2–4-in. diam.</td>
<td>1.11</td>
</tr>
<tr>
<td>Motor, squirrel cage, induction, 440 volts</td>
<td>1–10 hp</td>
<td>0.19</td>
</tr>
<tr>
<td>Motor, squirrel cage, induction, 440 volts</td>
<td>10–50 hp</td>
<td>0.50</td>
</tr>
<tr>
<td>Pump, centrifugal, horizontal</td>
<td>0.5–1.5 hp</td>
<td>0.63</td>
</tr>
<tr>
<td>Pump, centrifugal, horizontal</td>
<td>1.5–40 hp</td>
<td>0.09</td>
</tr>
<tr>
<td>Tower, c.s.</td>
<td>Constant diam.</td>
<td>0.88</td>
</tr>
<tr>
<td>Tower, c.s.</td>
<td>Constant height</td>
<td>1.56</td>
</tr>
<tr>
<td>Transformer, single phase, dry</td>
<td>9–225 kva</td>
<td>0.58</td>
</tr>
<tr>
<td>Transformer, single phase, oil, class A</td>
<td>15–225 kva</td>
<td>0.34</td>
</tr>
<tr>
<td>Tubular heat exchanger</td>
<td>Any size</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Tubular heat exchangers appear to have zero exponents, implying that direct labor cost is independent of size. This reflects the fact that such equipment is set with cranes and hoists, which, when adequately sized for the task, recognize no appreciable difference in size of weight of the equipment. The higher labor exponent for installing carbon-steel towers indicates the increasing complexity of tower internals (trays, downcomers, etc.) as tower diameter increases.

Analyses of the total installed costs of equipment in a number of typical chemical plants indicate that the cost of the purchased equipment varies from 65 to 80 percent of the installed cost depending upon the complexity of the equipment and the type of plant in which the equipment is installed. Installation costs for equipment, therefore, are estimated to vary from 25 to 55 percent of the purchased-equipment cost.

Insulation Costs

When very high or very low temperatures are involved, insulation factors can become important, and it may be necessary to estimate insulation costs with a great deal of care. Expenses for equipment insulation and piping insulation are often included under the respective headings of equipment-installation costs and piping costs.

The total cost for the labor and materials required for insulating equipment and piping in ordinary chemical plants is approximately 8 to 9 percent of the purchased-equipment cost. This is equivalent to approximately 2 percent of the total capital investment.

Instrumentation and Controls

Instrument costs, installation-labor costs, and expenses for auxiliary equipment and materials constitute the major portion of the capital investment required for
instrumentation. This part of the capital investment is sometimes combined with the general equipment groups. Total instrumentation cost depends on the amount of control required and may amount to 6 to 30 percent of the purchased cost for all equipment. Computers are commonly used with controls and have the effect of increasing the cost associated with controls.

For the normal solid-fluid chemical processing plant, a value of 13 percent of the purchased equipment is normally used to estimate the total instrumentation cost. This cost represents approximately 3 percent of the total capital investment. Depending on the complexity of the instruments and the service, additional charges for installation and accessories may amount to 50 to 70 percent of the purchased cost, with the installation charges being approximately equal to the cost for accessories.

Piping

The cost for piping covers labor, valves, fittings, pipe, supports, and other items involved in the complete erection of all piping used directly in the process. This includes raw-material, intermediate-product, finished-product, steam, water, air, sewer, and other process piping. Since process-plant piping can run as high as 80 percent of purchased-equipment cost or 20 percent of tied-capital investment, it is understandable that accuracy of the entire estimate can be seriously affected by the improper application of estimation techniques to this one component.

Piping estimation methods involve either some degree of piping take-off from detailed drawings and flow sheets or using a factor technique when neither detailed drawings nor flow sheets are available. Factoring by percent of purchased-equipment cost and percent of fixed-capital investment is based strictly on experience gained from piping costs for similar previously installed chemical-process plants. Table 8 presents a rough estimate of the piping costs for various types of chemical processes. Additional information for estimating

<table>
<thead>
<tr>
<th>Type of process plant</th>
<th>Percent of purchased-equipment</th>
<th>Percent of fixed-capital investment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material</td>
<td>Labor</td>
</tr>
<tr>
<td>Solid †</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Solid-fluid ‡</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td>Fluid §</td>
<td>36</td>
<td>30</td>
</tr>
</tbody>
</table>

† A coal briquetting plant would be a typical solid-processing plant.
‡ A shale oil plant with crushing, grinding, retorting, and extraction would be a typical solid-fluid processing plant.
§ A distillation unit would be a typical fluid-processing plant.
piping costs is presented in Chap. 14. Labor for installation is estimated as approximately 40 to 50 percent of the total installed cost of piping. Material and labor for pipe insulation is estimated to vary from 15 to 25 percent of the total installed cost of the piping and is influenced greatly by the extremes in temperature which are encountered in the process streams.

### Electrical Installations

The cost for electrical installations consists primarily of installation labor and materials for power and lighting, with building-service lighting usually included under the heading of building-and-services costs. In ordinary chemical plants, electrical-installations cost amounts to 10 to 15 percent of the value of all purchased equipment. However, this may range to as high as 40 percent of purchased-equipment cost for a specific process plant. There appears to be little relationship between percent of total cost and percent of equipment cost, but there is a better relationship to fixed-capital investment. Thus, the electrical installation cost is generally estimated between 3 and 10 percent of the fixed-capital investment.

The electrical installation consists of four major components, namely, power wiring, lighting, transformation and service, and instrument and control wiring. Table 9 shows these component costs as ratios of the total electrical cost.

### Buildings Including Services

The cost for buildings including services consists of expenses for labor, materials, and supplies involved in the erection of all buildings connected with the plant. Costs for plumbing, heating, lighting, ventilation, and similar building services are included. The cost of buildings, including services for different types of process plants, is shown in Tables 10 and 11 as a percentage of purchased-equipment cost and tied-capital investment.

---

### Table 9

Component electrical costs as percent of total electrical cost

<table>
<thead>
<tr>
<th>Component</th>
<th>Range, %</th>
<th>Typical value, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power wiring</td>
<td>25-50</td>
<td>40</td>
</tr>
<tr>
<td>Lighting</td>
<td>1-25</td>
<td>12</td>
</tr>
<tr>
<td>Transformation and service</td>
<td>9-65</td>
<td>40</td>
</tr>
<tr>
<td>Instrument control wiring</td>
<td>3-8</td>
<td>5</td>
</tr>
</tbody>
</table>

The lower range is generally applicable to grass-roots single-product plants; the higher percentages apply to complex chemical plants and expansions to major chemical plants.
TABLE 10
Cost of buildings including services based on purchased-equipment cost

<table>
<thead>
<tr>
<th>Type of process plant†</th>
<th>Percentage of purchased-equipment cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New plant at new site (Grass roots)</td>
</tr>
<tr>
<td>Solid</td>
<td>68</td>
</tr>
<tr>
<td>Solid-fluid</td>
<td>47</td>
</tr>
<tr>
<td>Fluid</td>
<td>45</td>
</tr>
</tbody>
</table>

† See Table 8 for definition of types of process plants.
‡ The lower figure is applicable to petroleum refining and related industries.

TABLE 11
Cost of buildings and services as a percentage of fixed-capital investment for various types of process plants

<table>
<thead>
<tr>
<th>Type of process plant†</th>
<th>New plant at new site</th>
<th>New unit at existing site</th>
<th>Expansion at an existing site</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>18</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Solid-fluid</td>
<td>12</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Fluid</td>
<td>10</td>
<td>2–4†</td>
<td>2</td>
</tr>
</tbody>
</table>

† See Table 8 for definition of types of process plants.
‡ The lower figure is applicable to petroleum refining and related industries.

Yard Improvements

Costs for fencing, grading, roads, sidewalks, railroad sidings, landscaping, and similar items constitute the portion of the capital investment included in yard improvements. Yard-improvements cost for chemical plants approximates 10 to 20 percent of the purchased-equipment cost. This is equivalent to approximately 2 to 5 percent of the fixed-capital investment. Table 12 shows the range in variation for various components of yard improvements in terms of the fixed-capital investment.

Service Facilities

Utilities for supplying steam, water, power, compressed air, and fuel are part of the service facilities of an industrial plant. Waste disposal, fire protection, and miscellaneous service items, such as shop, first aid, and cafeteria equipment and facilities, require capital investments which are included under the general heading of service-facilities cost.

The total cost for service facilities in chemical plants generally ranges from 30 to 80 percent of the purchased-equipment cost with 55 percent representing
TABLE 12
Typical variation in percent of fixed-capital investment for yard improvements

<table>
<thead>
<tr>
<th>Yard improvement</th>
<th>Range, %</th>
<th>Typical value, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site clearing</td>
<td>0.4-1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Roads and walks</td>
<td>0.2-1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Railroads</td>
<td>0.3-0.9</td>
<td>0.6</td>
</tr>
<tr>
<td>Fences</td>
<td>0.1-0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Yard and fence lighting</td>
<td>0.1-0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Parking areas</td>
<td>0.1-0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Landscaping</td>
<td>0.1-0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Other improvements</td>
<td>0.2-0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

An average for a normal solid-fluid processing plant. For a single-product, small, continuous-process plant, the cost is likely to be in the lower part of the range. For a large, new, multiprocess plant at a new location, the costs are apt to be near the upper limit of the range. The cost of service facilities, in terms of capital investment, generally ranges from 8 to 20 percent with 13 percent considered as an average value. Table 13 lists the typical variations in percentages of fixed-capital investment that can be encountered for various components of service facilities. Except for entirely new facilities, it is unlikely that all service facilities will be required in all process plants. This accounts to a large degree for the wide variation range assigned to each component in Table 13. The range also reflects the degree to which utilities which depend on heat balance are used in the process. Service facilities largely are functions of plant physical size and will be present to some degree in most plants. However, not always will there be a need for each service-facility component. The omission of these utilities would tend to increase the relative percentages of the other service facilities actually used in the plant. Recognition of this fact, coupled with a careful appraisal as to the extent that service facilities are used in the plant, should result in selecting from Table 13 a reasonable cost ratio applicable to a specific process design.

Land

The cost for land and the accompanying surveys and fees depends on the location of the property and may vary by a cost factor per acre as high as thirty to fifty between a rural district and a highly industrialized area. As a rough average, land costs for industrial plants amount to 4 to 8 percent of the purchased-equipment cost or 1 to 2 percent of the total capital investment. Because the value of land usually does not decrease with time, this cost should not be included in the fixed-capital investment when estimating certain annual operating costs, such as depreciation.
### TABLE 13
**Typical** variation in percent of fixed-capital investment for service facilities

<table>
<thead>
<tr>
<th>Service facilities</th>
<th>Range, %</th>
<th>Typical value, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam generation</td>
<td>2.6–6.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Steam distribution</td>
<td>0.2–2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Water supply, cooling, and pumping</td>
<td>0.4–3.7</td>
<td>1.8</td>
</tr>
<tr>
<td>Water treatment</td>
<td>0.5–2.1</td>
<td>1.3</td>
</tr>
<tr>
<td>Water distribution</td>
<td>0.1–2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Electric substation</td>
<td>0.9–2.6</td>
<td>1.3</td>
</tr>
<tr>
<td>Electric distribution</td>
<td>0.4–2.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Gas supply and distribution</td>
<td>0.2–0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Air compression and distribution</td>
<td>0.2–3.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Refrigeration including distribution</td>
<td>1.0–3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Process waste disposal</td>
<td>0.6–2.4</td>
<td>1.5</td>
</tr>
<tr>
<td>Sanitary waste disposal</td>
<td>0.2–0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Communications</td>
<td>0.1–0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Raw-material storage</td>
<td>0.3–3.2</td>
<td>0.5</td>
</tr>
<tr>
<td>Finished-product storage</td>
<td>0.7–2.4</td>
<td>1.5</td>
</tr>
<tr>
<td>Fire-protection system</td>
<td>0.3–1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Safety installations</td>
<td>0.2–0.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Engineering and Supervision**

The costs for construction design and engineering, drafting, purchasing, accounting, construction and cost engineering, travel, reproductions, communications, and home office expense including overhead constitute the capital investment for engineering and supervision. This cost, since it cannot be directly charged to equipment, materials, or labor, is normally considered an indirect cost in fixed-capital investment and is approximately 30 percent of the purchased-equipment cost or 8 percent of the total direct costs of the process plant. Typical percentage variations of tied-capital investment for various components of engineering and supervision are given in Table 14.

**Construction Expense**

Another expense which is included under indirect plant cost is the item of construction or field expense and includes temporary construction and operation, construction tools and rentals, home office personnel located at the construction site, construction payroll, travel and living, taxes and insurance, and other construction overhead. This expense item is occasionally included under equipment installation, or more often under engineering, supervision, and construction. If construction or field expenses are to be estimated separately, then Table 15 will be useful in establishing the variation in percent of fixed-capital investment for this indirect cost. For ordinary chemical-process...
TABLE 14
Typical variation in percent of fixed-capital investment for engineering and services

<table>
<thead>
<tr>
<th>Component</th>
<th>Range, %</th>
<th>Typical value, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering</td>
<td>1.5-6.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Drafting</td>
<td>2.0-12.0</td>
<td>4.8</td>
</tr>
<tr>
<td>Purchasing</td>
<td>0.2-0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Accounting, construction, and cost engineering</td>
<td>0.2-1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Travel and living</td>
<td>0.1-1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Reproductions and communications</td>
<td>0.2-0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Total engineering and supervision</td>
<td>4.0-21.0</td>
<td>8.1</td>
</tr>
</tbody>
</table>

plants the construction expenses average roughly 10 percent of the total direct costs for the plant.

Contractor’s Fee

The contractor’s fee varies for different situations, but it can be estimated to be about 2 to 8 percent of the direct plant cost or 1.5 to 6 percent of the fixed-capital investment.

Contingencies

A contingency factor is usually included in an estimate of capital investment to compensate for unpredictable events, such as storms, floods, strikes, price

TABLE 15
Typical variation in percent of fixed-capital investment for construction expenses

<table>
<thead>
<tr>
<th>Component</th>
<th>Range, %</th>
<th>Typical value, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary construction and operations</td>
<td>1.0-3.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Construction tools and rental</td>
<td>1.0-3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Home office personnel in field</td>
<td>0.2-2.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Field payroll</td>
<td>0.4-4.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Travel and living</td>
<td>0.1-0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>Taxes and insurance</td>
<td>1.0-2.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Startup materials and labor</td>
<td>0.2-1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>overhead</td>
<td>0.3-0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Total construction expenses</td>
<td>4.2-16.6</td>
<td>7.0</td>
</tr>
</tbody>
</table>
changes, small design changes, errors in estimation, and other unforeseen expenses, which previous estimates have statistically shown to be of a recurring nature. This factor may or may not include allowance for escalation. Contingency factors ranging from 5 to 15 percent of the direct and indirect plant costs are commonly used, with 8 percent being considered a fair average value.

**Startup Expense**

After plant construction has been completed, there are quite frequently changes that have to be made before the plant can operate at maximum design conditions. These changes involve expenditures for materials and equipment and result in loss of income while the plant is shut down or is operating at only partial capacity. Capital for these startup changes should be part of any capital appropriation because they are essential to the success of the venture. These expenses may be as high as 12 percent of the fixed-capital investment. In general, however, an allowance of 8 to 10 percent of the fixed-capital investment for this item is satisfactory.

Startup expense is not necessarily included as part of the required investment; so it is not presented as a component in the summarizing Table 26 for capital investment at the end of this chapter. In the overall cost analysis, startup expense may be represented as a one-time-only expenditure in the first year of the plant operation or as part of the total capital investment depending on the company policies.

**Methods for estimating capital investment**

Various methods can be employed for estimating capital investment. The choice of any one method depends upon the amount of detailed information available and the accuracy desired. Seven methods are outlined in this chapter, with each method requiring progressively less detailed information and less preparation time. Consequently, the degree of accuracy decreases with each succeeding method. A maximum accuracy within approximately ±5 percent of the actual capital investment can be obtained with method A.

**METHOD A DETAILED-ITEM ESTIMATE.** A detailed-item estimate requires careful determination of each individual item shown in Table 1. Equipment and material needs are determined from completed drawings and specifications and are priced either from current cost data or preferably from firm delivered quotations. Estimates of installation costs are determined from accurate labor rates, efficiencies, and employee-hour calculations. Accurate estimates of engineering, drafting, field supervision employee-hours, and field-expenses must be detailed in the same manner. Complete site surveys and soil data must be available to minimize errors in site development and construction cost estimates. In fact, in this type of estimate, an attempt is made to firm up as much of the estimate as possible by obtaining quotations from vendors and suppliers. Because of the extensive data necessary and the large amounts of engineering
time required to prepare such a detailed-item estimate, this type of estimate is almost exclusively only prepared by contractors bidding on lump-sum work from finished drawings and specifications.

**METHOD B UNIT-COST ESTIMATE.** The unit-cost method results in good estimating accuracies for fixed-capital investment provided accurate records have been kept of previous cost experience. This method, which is frequently used for preparing definitive and preliminary estimates, also requires detailed estimates of purchased price obtained either from quotations or index-corrected cost records and published data. Equipment installation labor is evaluated as a fraction of the delivered-equipment cost. Costs for concrete, steel, pipe, **electricals**, instrumentation, insulation, etc., are obtained by take-offs from the drawings and applying unit costs to the material and labor needs. A unit cost is also applied to engineering employee-hours, number of drawings, and specifications. A factor for construction expense, contractor’s fee, and contingency is estimated from previously completed projects and is used to complete this type of estimate. A cost equation summarizing this method can be given as:

\[
C_n = \left[ \sum (E + E_L) + \sum (f_x M_x + f_y M_L) + \sum f_e H_e + \sum f_d d_n \right] (f_F)
\]

where

- \( C_n \) = new capital investment
- \( E \) = purchased-equipment cost
- \( E_L \) = purchased-equipment labor cost
- \( f_x \) = specific material unit cost, e.g., \( f_p \) = unit cost of pipe
- \( M_x \) = specific material quantity in compatible units
- \( f_y \) = specific material labor unit cost per employee-hour
- \( M_L \) = labor employee-hours for specific material
- \( f_e \) = unit cost-for engineering
- \( H_e \) = engmeermg employee-hours
- \( f_d \) = unit cost per drawing or specification
- \( d_n \) = number of drawings or specifications
- \( f_F \) = construction or field expense factor always greater than 1

Approximate corrections to the base equipment cost of complete, main-plant items for specific materials of construction or extremes of operating pressure and temperature can be applied in the form of factors as shown in Table 16.

**METHOD C PERCENTAGE OF DELIVERED-EQUIPMENT COST.** This method for estimating the fixed or total-capital investment requires determination of the delivered-equipment cost. The other items included in the total direct plant cost are then estimated as percentages of the delivered-equipment cost. The additional components of the capital investment are based on average percentages of the total direct plant cost, total direct and indirect plant costs, or total capital.

---

TABLE 16
Correction factors for operating pressure, operating temperature, and material of construction to apply for fixed-capital investment of major plant items†‡

<table>
<thead>
<tr>
<th>Operating pressure, psia (atm)</th>
<th>Correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08 (0.005)</td>
<td>1.3</td>
</tr>
<tr>
<td>0.2 (0.014)</td>
<td>1.2</td>
</tr>
<tr>
<td>0.7 (0.048)</td>
<td>1.1</td>
</tr>
<tr>
<td>3000 (204)</td>
<td>1.0 (base)</td>
</tr>
<tr>
<td>6000 (408)</td>
<td>1.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating temperature, °C</th>
<th>Correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>-80</td>
<td>1.3</td>
</tr>
<tr>
<td>0</td>
<td>1.00 (base)</td>
</tr>
<tr>
<td>100</td>
<td>1.05</td>
</tr>
<tr>
<td>600</td>
<td>1.1</td>
</tr>
<tr>
<td>5,000</td>
<td>1.2</td>
</tr>
<tr>
<td>10,000</td>
<td>1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material of construction</th>
<th>Correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon steel-mild</td>
<td>1.0 (base)</td>
</tr>
<tr>
<td>Bronze</td>
<td>1.05</td>
</tr>
<tr>
<td>Carbon/molybdenum steel</td>
<td>1.065</td>
</tr>
<tr>
<td>Aluminum</td>
<td>1.075</td>
</tr>
<tr>
<td>Cast steel</td>
<td>1.11</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>1.28 to 1.5</td>
</tr>
<tr>
<td>Worthite alloy</td>
<td>1.41</td>
</tr>
<tr>
<td>Hastelloy C alloy</td>
<td>1.54</td>
</tr>
<tr>
<td>Monel alloy</td>
<td>1.65</td>
</tr>
<tr>
<td>Nickel/inconel alloy</td>
<td>1.71</td>
</tr>
<tr>
<td>Titanium</td>
<td>2.0</td>
</tr>
</tbody>
</table>


‡ It should be noted that these factors are to be used only for complete, main-plant items and serve to correct from the base case to the indicated conditions based on pressure or temperature extremes that may be involved or special materials of construction that may be required. For the case of small or single pieces of equipment which are completely dedicated to the extreme conditions, the factors given in this table may be far too low and factors or methods given in other parts of this book must be used.
investment. This is summarized in the following cost equation:

\[ C_n = \left( \sum E + \sum (f_1 E + f_2 E + f_3 E + \ldots) \right) (f_I) \]  

(3)

where \( f_1, f_2 \ldots \) = multiplying factors for piping, electrical, instrumentation, etc.

\( f_I = \) indirect cost factor always greater than 1.

The percentages used in making an estimation of this type should be determined on the basis of the type of process involved, design complexity, required materials of construction, location of the plant, past experience, and other items dependent on the particular unit under consideration. Average values of the various percentages have been determined for typical chemical plants, and these values are presented in Table 17.

Estimating by percentage of delivered-equipment cost is commonly used for preliminary and study estimates. It yields most accurate results when applied to projects similar in configuration to recently constructed plants. For comparable plants of different capacity, this method has sometimes been reported to yield definitive estimate accuracies.

**Example 3 Estimation of fixed-capital investment by percentage of delivered-equipment cost.** Prepare a study estimate of the fixed-capital investment for the process plant described in Example 1 if the delivered-equipment cost is $100,000.

**Solution.** Use the ratio factors outlined in Table 17 with modifications for instrumentation and outdoor operation.

<table>
<thead>
<tr>
<th>Components</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchased equipment (delivered), ( E )</td>
<td>$100,000</td>
</tr>
<tr>
<td>Purchased equipment installation, 39% ( E )</td>
<td>39,000</td>
</tr>
<tr>
<td>Instrumentation (installed), 28% ( E )</td>
<td>28,000</td>
</tr>
<tr>
<td>Piping (installed), 31% ( E )</td>
<td>31,000</td>
</tr>
<tr>
<td>Electrical (installed), 10% ( E )</td>
<td>10,000</td>
</tr>
<tr>
<td>Buildings (including services), 22% ( E )</td>
<td>22,000</td>
</tr>
<tr>
<td>Yard improvements, 10% ( E )</td>
<td>10,000</td>
</tr>
<tr>
<td>Service facilities (installed), 55% ( E )</td>
<td>55,000</td>
</tr>
<tr>
<td>Land, 6% ( E )</td>
<td>6,000</td>
</tr>
</tbody>
</table>

\[
\text{Total direct plant cost } D = 301,000
\]

\[
\text{Engineering and supervision, 32\% } E = 32,000
\]

\[
\text{Construction expenses, 34\% } E = 34,000
\]

\[
\text{Total direct and indirect cost } (D + I) = 367,000
\]

\[
\text{Contractor's fee, 5\% } (D + I) = 18,000
\]

\[
\text{Contingency, 10\% } (D + I) = 37,000
\]

\[
\text{Fixed-capital investment} = 422,000
\]

**METHOD D “LANG” FACTORS FOR APPROXIMATION OF CAPITAL INVESTMENT.** This technique, proposed originally by Lang† and used quite frequently to obtain order-of-magnitude cost estimates, recognizes that the cost of a

TABLE 17
Ratio factors for estimating capital-investment items based on delivered-equipment cost

Values presented are applicable for major process plant additions to an existing site where the necessary land is available through purchase or present ownership. The values are based on fixed-capital investments ranging from under $1 million to over $20 million.

<table>
<thead>
<tr>
<th>Item</th>
<th>Percent of delivered-equipment cost for</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solid-processing plant ‡</td>
</tr>
<tr>
<td></td>
<td>Solid-fluid-processing plant ‡</td>
</tr>
<tr>
<td></td>
<td>Fluid-processing plant ‡</td>
</tr>
<tr>
<td>Purchased equipment-delivered (including fabricated equipment and process machinery) §</td>
<td>100</td>
</tr>
<tr>
<td>Purchased-equipment installation</td>
<td>45</td>
</tr>
<tr>
<td>Instrumentation and controls (installed)</td>
<td>9</td>
</tr>
<tr>
<td>Piping (installed)</td>
<td>16</td>
</tr>
<tr>
<td>Electrical (installed)</td>
<td>10</td>
</tr>
<tr>
<td>Buildings (including services)</td>
<td>25</td>
</tr>
<tr>
<td>Yard improvements</td>
<td>13</td>
</tr>
<tr>
<td>Service facilities (installed)</td>
<td>40</td>
</tr>
<tr>
<td>Land (if purchase is required)</td>
<td>6</td>
</tr>
<tr>
<td><strong>Total direct plant cost</strong></td>
<td><strong>264</strong></td>
</tr>
<tr>
<td>Indirect costs</td>
<td></td>
</tr>
<tr>
<td>Engineering and supervision</td>
<td>33</td>
</tr>
<tr>
<td>Construction expenses</td>
<td>39</td>
</tr>
<tr>
<td><strong>Total direct and indirect plant costs</strong></td>
<td><strong>336</strong></td>
</tr>
<tr>
<td>Contractor’s fee (about 5% of direct and indirect plant costs)</td>
<td>17</td>
</tr>
<tr>
<td>Contingency (about 10% of direct and indirect plant costs)</td>
<td>34</td>
</tr>
<tr>
<td><strong>Fixed-capital investment</strong></td>
<td><strong>387</strong></td>
</tr>
<tr>
<td>Working capital (about 15% of total capital investment)</td>
<td>68</td>
</tr>
<tr>
<td><strong>Total capital investment</strong></td>
<td><strong>455</strong></td>
</tr>
</tbody>
</table>

† Because of the extra expense involved in supplying service facilities, storage facilities, loading terminals, transportation facilities, and other necessary utilities at a completely undeveloped site, the fixed-capital investment for a new plant located at an undeveloped site may be as much as 100 percent greater than for an equivalent plant constructed as an addition to an existing plant.

‡ See Table 8 for definition of types of process plants.

§ Includes pumps and compressors.
TABLE 18
Lang multiplication factors for estimation of fixed-capital investment or total capital investment

<table>
<thead>
<tr>
<th>Type of plant</th>
<th>Factor for Fixed-capital investment</th>
<th>Factor for Total capital investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid-processing plant</td>
<td>3.9</td>
<td>4.6</td>
</tr>
<tr>
<td>Solid-fluid-processing plant</td>
<td>4.1</td>
<td>4.9</td>
</tr>
<tr>
<td>Fluid-processing plant</td>
<td>4.8</td>
<td>5.7</td>
</tr>
</tbody>
</table>

process plant may be obtained by multiplying the basic equipment cost by some factor to approximate the capital investment. These factors vary depending upon the type of process plant being considered. The percentages given in Table 17 are rough approximations which hold for the types of process plants indicated. These values, therefore, may be combined to give Lang multiplication factors that can be used for estimating the total direct plant cost, the fixed-capital investment, or the total capital investment. Factors for estimating the fixed-capital investment or the total capital investment are given in Table 18. It should be noted that these factors include costs for land and contractor’s fees.

Greater accuracy of capital investment estimates can be achieved in this method by using not one but a number of factors. One approach is to use different factors for different types of equipment. Another approach is to use separate factors for erection of equipment, foundations, utilities, piping, etc., or even to break up each item of cost into material and labor factors. With this approach, each factor has a range of values and the chemical engineer must rely on past experience to decide, in each case, whether to use a high, average, or low figure.

Since tables are not convenient for computer calculations it is better to combine the separate factors into an equation similar to the one proposed by Hirsch and Glazier†

\[ C_n = f_1\left[ E(1 + f_F + f_p + f_m) + E_i + A \right] \]  

where the three installation-cost factors are, in turn, defined by the following three equations:

\[
\log f_F = 0.635 - 0.154 \log 0.001E - 0.992; + 0.506 \frac{f_v}{E}
\]  

(5)

\[
\log f_p = -0.266 - 0.014 \log 0.001E - 0.156; + 0.556 \frac{p}{E}
\]  

(6)

\[
\log f_m = 0.344 + 0.033 \log 0.001 E + 1.194;
\]  

(7)

and the various parameters are defined accordingly:

\[
E = \text{purchased-equipment on an f.o.b. basis}
\]

\[
f_F = \text{indirect cost factor always greater than 1 (normally taken as 1.4)}
\]

\[
f_p = \text{cost factor for field labor}
\]

\[
f_m = \text{cost factor for piping materials}
\]

\[
f_m = \text{cost factor for miscellaneous items, including the materials cost for insulation, instruments, foundations, structural steel, building, wiring, painting, and the cost of freight and field supervision}
\]

\[
E_i = \text{cost of equipment already installed at site}
\]

\[
A = \text{incremental cost of corrosion-resistant alloy materials}
\]

\[
e = \text{total heat exchanger cost (less incremental cost of alloy)}
\]

\[
f_v = \text{total cost of field-fabricated vessels (less incremental cost of alloy)}
\]

\[
p = \text{total pump plus driver cost (less incremental cost of alloy)}
\]

\[
t = \text{total cost of tower shells (less incremental cost of alloy)}
\]

Note that Eq. (4) is designed to handle both purchased equipment on an f.o.b. basis and completely installed equipment.

**METHOD E POWER FACTOR APPLIED TO PLANT-CAPACITY RATIO.** This method for study or order-of-magnitude estimates relates the fixed-capital investment of a new process plant to the fixed-capital investment of similar previously constructed plants by an exponential power ratio. That is, for certain similar process plant configurations, the fixed-capital investment of the new facility is equal to the fixed-capital investment of the constructed facility \(C\) multiplied by the ratio \(R\), defined as the capacity of the new facility divided by the capacity of the old, raised to a power \(x\). This power has been found to average between 0.6 and 0.7 for many process facilities. Table 19 gives the capacity power factor \((x)\) for various kinds of processing plants.

\[
C_n = C(R)^x
\]  

(8)

A closer approximation for this relationship which involves the direct and indirect plant costs has been proposed as

\[
C_n = f[D(R)^x + I]
\]  

(9)
### TABLE 19
Capital-cost data for processing plants *(1990)*

<table>
<thead>
<tr>
<th>Product of process</th>
<th>Process remarks</th>
<th>Typical plant size, 1000 tons / yr</th>
<th>Fixed-capital investment, million $</th>
<th>$ of fixed-capital investment per annual ton of product</th>
<th>Power factor <em>(x)‡</em> for plant-capacity ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetic acid</td>
<td>CH₃OH and CO-catalytic</td>
<td>10</td>
<td>6</td>
<td>650</td>
<td>0.68</td>
</tr>
<tr>
<td>Acetone</td>
<td>Propylene-copper chloride catalyst</td>
<td>100</td>
<td>32</td>
<td>320</td>
<td>0.45</td>
</tr>
<tr>
<td>Ammonia</td>
<td>Steam reforming</td>
<td>100</td>
<td>24</td>
<td>240</td>
<td>0.53</td>
</tr>
<tr>
<td>Ammonium nitrate</td>
<td>Ammonia and nitric acid</td>
<td>100</td>
<td>5</td>
<td>50</td>
<td>0.65</td>
</tr>
<tr>
<td>Butanol</td>
<td>Propylene, CO, and H₂O—catalytic</td>
<td>50</td>
<td>40</td>
<td>800</td>
<td>0.40</td>
</tr>
<tr>
<td>Chlorine</td>
<td>Electrolysis of NaCl</td>
<td>50</td>
<td>28</td>
<td>550</td>
<td>0.45</td>
</tr>
<tr>
<td>Ethylene</td>
<td>Refinery gases</td>
<td>50</td>
<td>13</td>
<td>260</td>
<td>0.83</td>
</tr>
<tr>
<td>Ethylene oxide</td>
<td>Ethylene-catalytic</td>
<td>50</td>
<td>50</td>
<td>1000</td>
<td>0.78</td>
</tr>
<tr>
<td>Formaldehyde <em>(37%)</em></td>
<td>Methanol-catalytic</td>
<td>10</td>
<td>16</td>
<td>1600</td>
<td>0.55</td>
</tr>
<tr>
<td>Glycol</td>
<td>Ethylene and chlorine</td>
<td>5</td>
<td>15</td>
<td>2900</td>
<td>0.75</td>
</tr>
<tr>
<td>Hydrofluoric acid</td>
<td>Hydrogen fluoride and H₂O</td>
<td>10</td>
<td>8</td>
<td>800</td>
<td>0.68</td>
</tr>
<tr>
<td>Methanol</td>
<td>CO₂, natural gas, and steam</td>
<td>60</td>
<td>13</td>
<td>200</td>
<td>0.60</td>
</tr>
<tr>
<td>Nitric acid <em>(high strength)</em></td>
<td>Ammonia-catalytic</td>
<td>100</td>
<td>6</td>
<td>65</td>
<td>0.60</td>
</tr>
<tr>
<td>Phosphoric acid</td>
<td>Calcium phosphate and H₂SO₄</td>
<td>5</td>
<td>3</td>
<td>650</td>
<td>0.60</td>
</tr>
<tr>
<td>Polyethylene <em>(high density)</em></td>
<td>Ethylene-catalytic</td>
<td>5</td>
<td>16</td>
<td>3200</td>
<td>0.65</td>
</tr>
<tr>
<td>Propylene</td>
<td>Refinery gases</td>
<td>10</td>
<td>3</td>
<td>320</td>
<td>0.70</td>
</tr>
<tr>
<td>Sulfuric acid</td>
<td>Sulfur-catalytic</td>
<td>100</td>
<td>3</td>
<td>32</td>
<td>0.65</td>
</tr>
<tr>
<td>Urea</td>
<td>Ammonia and CO₂</td>
<td>60</td>
<td>8</td>
<td>130</td>
<td>0.70</td>
</tr>
</tbody>
</table>
### TABLE 19
Capital-cost data for processing plants (1990) (Continued)

<table>
<thead>
<tr>
<th>Product or process</th>
<th>Process remarks</th>
<th>Typical plant size, 1000 bbl / day</th>
<th>Fixed-capital investment, million $</th>
<th>$ of fixed-capital investment per bbl / day</th>
<th>Power factor (x)‡ for plant-capacity ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alkylation (H₂SO₄)</td>
<td>Catalytic</td>
<td>10</td>
<td>19</td>
<td>1900</td>
<td>0.60</td>
</tr>
<tr>
<td>Coking (delayed)</td>
<td>Thermal</td>
<td>10</td>
<td>26</td>
<td>2600</td>
<td>0.38</td>
</tr>
<tr>
<td>Coking (fluid)</td>
<td>Thermal</td>
<td>10</td>
<td>16</td>
<td>1600</td>
<td>0.42</td>
</tr>
<tr>
<td>Cracking (fluid)</td>
<td>Catalytic</td>
<td>10</td>
<td>16</td>
<td>1600</td>
<td>0.70</td>
</tr>
<tr>
<td>Cracking</td>
<td>Thermal</td>
<td>10</td>
<td>5</td>
<td>500</td>
<td>0.70</td>
</tr>
<tr>
<td>Distillation (atm.)</td>
<td>65% vaporized</td>
<td>100</td>
<td>32</td>
<td>320</td>
<td>0.90</td>
</tr>
<tr>
<td>Distillation (vac.)</td>
<td>65% vaporized</td>
<td>100</td>
<td>19</td>
<td>200</td>
<td>0.70</td>
</tr>
<tr>
<td>Hydrotreating</td>
<td>Catalytic</td>
<td>10</td>
<td>3</td>
<td>320</td>
<td>0.65</td>
</tr>
<tr>
<td>Reforming</td>
<td>Catalytic</td>
<td>10</td>
<td>29</td>
<td>2900</td>
<td>0.60</td>
</tr>
<tr>
<td>Polymerization</td>
<td>Catalytic</td>
<td>10</td>
<td>5</td>
<td>500</td>
<td>0.58</td>
</tr>
</tbody>
</table>


‡ These power factors apply within roughly a three-fold ratio extending either way from the plant size as given.
### TABLE 20

Relative labor rate and productivity indexes in the chemical and allied products industries for the United States (1989)†

<table>
<thead>
<tr>
<th>Geographical area</th>
<th>Relative labor rate</th>
<th>Relative productivity factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>New England</td>
<td>1.14</td>
<td>0.95</td>
</tr>
<tr>
<td>Middle Atlantic</td>
<td>1.06</td>
<td>0.96</td>
</tr>
<tr>
<td>South Atlantic</td>
<td>0.84</td>
<td>0.91</td>
</tr>
<tr>
<td>Midwest</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>Gulf</td>
<td>0.95</td>
<td>1.22</td>
</tr>
<tr>
<td>Southwest</td>
<td>0.88</td>
<td>1.04</td>
</tr>
<tr>
<td>Mountain</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td>Pacific Coast</td>
<td>1.22</td>
<td>0.89</td>
</tr>
</tbody>
</table>


where $f$ is a lumped cost-index factor relative to the original installation cost. $D$ is the direct cost and $Z$ is the total indirect cost for the previously installed facility of a similar unit on an equivalent site. The value of $x$ approaches unity when the capacity of a process facility is increased by adding identical process units instead of increasing the size of the process equipment. The lumped cost-index factor $f$ is the product of a geographical labor cost index, the corresponding area labor productivity index, and a material and equipment cost index. Table 20 presents the relative median labor rate and productivity factor for various geographical areas in the United States.

**Example 4** Estimating relative costs of construction labor as a function of geographical area. If a given chemical process plant is erected near Dallas (Southwest area) with a construction labor cost of $100,000 what would be the construction labor cost of an identical plant if it were to be erected at the same time near Los Angeles (Pacific Coast Area) for the time when the factors given in Table 20 apply?

**Solution**

Relative median labor rate-Southwest 0.88 from Table 20  
Relative median labor rate-Pacific Coast 1.22 from Table 20

\[
\text{Relative labor rate ratio} = \frac{1.22}{0.88} = 1.3864
\]
COST ESTIMATION 189

Relative productivity factor—Southwest 1.04 from Table 20
Relative productivity factor—Pacific Coast 0.89 from Table 20

Relative productivity factor ratio = \(\frac{0.89}{1.04}\) = 0.8558

Construction labor cost of Southwest to Pacific Coast = \((1.3864)/(0.8558)\) = 1.620
Construction labor cost at Los Angeles = \((1.620)(\$100,000)\) = $162,000

To determine the fixed-capital investment required for a new similar single-process plant at a new location with a different capacity and with the same number of process units, the following relationship has given good results:

\[
C_n = R^{x}[f_EE + f_MM + f_Lf_FFEE + f_WM] \frac{C}{C - I} \tag{10}
\]

where

- \(f_E\) = current equipment cost index relative to cost of the purchased equipment
- \(f_M\) = current material cost index relative to cost of material
- \(M\) = material cost
- \(f_L\) = current labor cost index in new location relative to \(E_L\) and \(M_L\) at old location
- \(f_E\) = labor efficiency index in new location relative to \(E_L\) and \(M_L\) at old location
- \(E_L\) = purchased-equipment labor cost
- \(M_L\) = labor employee-hours for specific material
- \(f_W\) = specific material labor cost per employee-hour
- \(C\) = original capital investment

In those situations where estimates of fixed-capital investment are desired for a similar plant at a new location and with a different capacity, but with multiples of the original process units, Eq. (11) often gives results with somewhat better than study-estimate accuracy.

\[
C_n = \left[R^x f_E E + R^x f_M M + R^x f_L f_E E L + f_WM L \right] \frac{C}{C - I} \tag{11}
\]

More accurate estimates by this method are obtained by subdividing the process plant into various process units, such as crude distillation units, reformers, alkylation units, etc., and applying the best available data from similar previously installed process units separately to each subdivision. Table 19 lists some typical process unit capacity-cost data and exponents useful for making this type of estimate.

**Example 5**  Estimation of fixed-capital investment with power factor applied to plant-capacity ratio. If the process plant, described in Example 1, was erected in the Dallas area for a fixed-capital investment of $436,000 in 1975, determine what the estimated fixed-capital investment would have been in 1980 for a similar process plant located near Los Angeles with twice the process capacity but with an equal number of process units? Use the power-factor method to evaluate the new fixed-capital investment and assume the factors given in Table 20 apply.
Solution. If Eq. (8) is used with a 0.6 power factor and the Marshall and Swift all-industry index (Table 3), the fixed-capital investment is
\[ C_n = C_{fE}(R)^x \]
\[ C_n = (436,000) \left( \frac{660}{444} \right) (2)^{0.6} = 982,000 \]

If Eq. (8) is used with a 0.7 power factor and the Marshall and Swift all-industry index (Table 3), the fixed-capital investment is
\[ C_n = (436,000) \left( \frac{660}{444} \right) (2)^{0.7} = 1,053,000 \]

If Eq. (9) is used with a 0.6 power factor, the Marshall and Swift all-industry index (Table 3), and the relative labor and productivity indexes (Table 20), the fixed-capital investment is
\[ C_n = f [D(R)^x + I] \]
where \( f = f_E f_L e_L \), and \( D \) and \( Z \) are obtained from Example 1,
\[ C_n = \left( \frac{660}{444} \right) \left( \frac{1.22}{0.88} \right) \left( \frac{1.04}{0.88} \right) \left[ (308,000)(2)^{0.6} + 128,000 \right] \]
\[ C_n = (1.486)(1.620)(467,000 + 128,000) \]
\[ C_n = 1,432,000 \]

If Eq. (9) is used with a 0.7 power factor, the Marshall and Swift all-industry index (Table 3), and the relative labor and productivity indexes (Table 20), the fixed-capital investment is
\[ C_n = 1,513,000 \]

Results obtained using this procedure have shown high correlation with fixed-capital investment estimates that have been obtained with more detailed techniques. Properly used, these factoring methods can yield quick fixed-capital investment requirements with accuracies sufficient for most economic-evaluation purposes.

**METHOD F INVESTMENT COST PER UNIT OF CAPACITY.** Many data have been published giving the fixed-capital investment required for various processes per unit of annual production capacity such as those shown in Table 19. Although these values depend to some extent on the capacity of the individual plants, it is possible to determine the unit investment costs which apply for average conditions. An order-of-magnitude estimate of the fixed-capital investment for a given process can then be obtained by multiplying the appropriate investment cost per unit of capacity by the annual production capacity of the proposed plant. The necessary correction for change of costs with time can be made with the use of cost indexes.

**METHOD G TURNOVER RATIOS.** A rapid evaluation method suitable for order-of-magnitude estimates is known as the “turnover ratio” method. Turnover ratio is defined as the ratio of gross annual sales to the fixed-capital investment,
\[ \text{Turnover ratio} = \frac{\text{gross annual sales}}{\text{fixed-capital investment}} \] (12)
where the product of the annual production rate and the average selling price of the commodities is the gross annual sales figures. The reciprocal of the turnover ratio is sometimes defined as the capital ratio or the investment ratio.† Turnover ratios of up to 5 are common for some business establishments and some are as low as 0.2. For the chemical industry, as a very rough rule of thumb, the ratio can be approximated as 1.

**ORGANIZATION FOR PRESENTING CAPITAL INVESTMENT ESTIMATES BY COMPARTMENTALIZATION**

The methods for estimating capital investment presented in the preceding sections represent the fundamental approaches that can be used. However, the direct application of these methods can often be accomplished with considerable improvement by considering the fixed-capital investment requirement by parts. With this approach, each identified part is treated as a separate unit to obtain the total investment cost directly related to it. Various forms of compartmentalization for this type of treatment have been proposed. Included in these are (1) the modular estimate,†† (2) the unit-operations estimate,§ (3) the functional-unit estimate,¶ and (4) the average-unit-cost estimate.†††

The same principle of breakdown into individual components is used for each of the four approaches. For the modular estimate, the basis is to consider individual modules in the total system with each consisting of a group of similar items. For example, all heat exchangers might be included in one module, all furnaces in another, all vertical process vessels in another, etc. The total cost estimate is considered under six general groupings including chemical processing, solids handling, site development, industrial buildings, offsite facilities, and

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†When the term investment ratio is used, the investment is usually considered to be the total capital investment which includes working capital as well as other capitalized costs.


project indirects. As an example of an equipment cost module for heat exchangers, the module would include the basic delivered cost of the piece of equipment with factors similar to Lang factors being presented for supplemental items needed to get the equipment ready for use such as piping, insulation, paint, labor, auxiliaries, indirect costs, and contingencies.

In presenting the basic data for the module factors, the three critical variables are size or capacity of the equipment, materials of construction, and operating pressure with temperature often being given as a fourth critical variable. It is convenient to establish the base cost of all equipment as that constructed of carbon steel and operated at atmospheric pressure. Factors, such as are presented in Table 16, are then used to change the estimated costs of the equipment to account for variation in the preceding critical variables. Once the equipment cost for the module is determined, various factors are applied to obtain the final fixed-capital investment estimate for the item completely installed and ready for operation. Figure 6-6 shows two typical module approaches with Fig. 6-6a representing a module that applies to a “normal” chemical process where the overall Lang factor for application to the f.o.b. cost of the original equipment is 3.482 and Fig. 6-6b representing a “normal” module for a piece of mechanical equipment where the Lang factor has been determined to be 2.456.

The modules referred to in the preceding can be based on combinations of equipment that involve similar types of operations requiring related types of auxiliaries. An example would be a distillation operation requiring the distillation column with the necessary auxiliaries of reboiler, condenser, pumps, holdup tanks, and structural supports. This type of compartmentalization for estimating purposes can be considered as resulting in a so-called unit-operations estimate. Similarly, the functional-unit estimate is based on the grouping of equipment by function such as distillation or filtration and including the fundamental pieces of equipment as the initial basis with factors applied to give the final estimate of the capital investment.

The average-unit-cost method puts special emphasis on the three variables of size of equipment, materials of construction, and operating pressure as well as on the type of process involved. In its simplest form, all of these variables and the types of process can be accounted for by one number so that a given factor to convert the process equipment cost to total fixed-capital investment can apply for each “average unit cost.” The latter is defined as the total cost of the process equipment divided by the number of equipment items in that particular process. As the “average unit cost” increases, the size of the factor for converting equipment cost to total fixed-capital investment decreases with a range of factor values applicable for each “average unit cost” depending on the particular type of process, operating conditions, and materials of construction.

ESTIMATION OF TOTAL PRODUCT COST

Methods for estimating the total capital investment required for a given plant are presented in the first part of this chapter. Determination of the necessary
Direct material labor total cost \( E + M )

\[
\begin{align*}
E &= \text{F.O.B. equipment} \\
M &= \text{Labor factor} \\
M_L &= \text{Material factor} \\
M_C &= \text{Field installation factor}
\end{align*}
\]

\[
\begin{align*}
162.2 + 32.0 &= 220.2 \\
E + M &= 0.36 \\
\text{Indirect cost factor} &= 1.34
\end{align*}
\]

\[
\begin{align*}
127.0 + 34.3 &= 161.3 \\
E + M &= 0.27 \\
\text{Indirect cost factor} &= 1.27
\end{align*}
\]

*Field installation

Total bore module cost

Contingency and fee (18%)

Total module cost

(a) "Normal" module for a chemical process unit with resultant Long factor of 3.482

(b) "Normal" module for a mechanical equipment unit with resultant Long factor of 2.456

**FIGURE 6-6**

Example of a “normal” module as applied for estimating capital investment for a chemical process and a mechanical equipment unit. [Adapted from K. M. Guthrie, Capital Cost Estimating, *Chem. Eng.*, 76(6):114 (March 24, 1969).]

153
Raw materials
Operating labor
Operating supervision
Steam
Electricity
Fuel
Refrigeration
Water
Maintenance and repairs
Operating supplies
Laboratory charges
Royalties (if not on lump-sum basis)
Catalysts and solvents
Depreciation
Taxes (property)
Insurance
Rent
Medical
Safety and protection
General plant overhead
Payroll overhead
Packaging
Restaurant
Recreation
Salvage
Control laboratories
Plant superintendence
Storage facilities
Executive salaries
Clerical wages
Engineering and legal costs
Office maintenance
Communications
Sales offices
Salesmen expenses
Shipping
Advertising
Technical sales service
Research and development
Financing (interest)
(often considered a fixed charge)
Gross-earnings expense

FIGURE 6-7
Costs involved in total product cost for a typical chemical process plant.
capital investment is only one part of a complete cost estimate. Another equally important part is the estimation of costs for operating the plant and selling the products. These costs can be grouped under the general heading of total product cost. The latter, in turn, is generally divided into the categories of manufacturing costs and general expenses. Manufacturing costs are also known as operating or production costs. Further subdivision of the manufacturing costs is somewhat dependent upon the interpretation of direct and indirect costs.

Accuracy is as important in estimating total product cost as it is in estimating capital investment costs. The largest sources of error in total-product-cost estimation are overlooking elements of cost. A tabular form is very useful for estimating total product cost and constitutes a valuable checklist to preclude omissions. Figure 6-7 provides a suggested checklist which is typical of the costs involved in chemical processing operations.

Total product costs are commonly calculated on one of three bases: namely, daily basis, unit-of-product basis, or annual basis. The annual cost basis is probably the best choice for estimation of total cost because (1) the effect of seasonal variations is smoothed out, (2) plant on-stream time or equipment-operating factor is considered, (3) it permits more-rapid calculation of operating costs at less than full capacity, and (4) it provides a convenient way of considering infrequently occurring but large expenses such as annual turnaround costs in a refinery.

The best source of information for use in total-product-cost estimates is data from similar or identical projects. Most companies have extensive records of their operations, so that quick, reliable estimates of manufacturing costs and general expenses can be obtained from existing records. Adjustments for increased costs as a result of inflation must be made, and differences in plant site and geographical location must be considered.

Methods for estimating total product cost in the absence of specific information are discussed in the following paragraphs. The various cost elements are presented in the order shown in Fig. 6-7.

Manufacturing Costs

All expenses directly connected with the manufacturing operation or the physical equipment of a process plant itself are included in the manufacturing costs. These expenses, as considered here, are divided into three classifications as follows: (1) direct production costs, (2) fixed charges, and (3) plant-overhead costs.

Direct production costs include expenses directly associated with the manufacturing operation. This type of cost involves expenditures for raw materials (including transportation, unloading, etc.,); direct operating labor; supervisory and clerical labor directly connected with the manufacturing operation; plant maintenance and repairs; operating supplies; power; utilities; royalties; and catalysts.
It should be recognized that some of the variable costs listed here as part of the direct production costs have an element of fixed cost in them. For instance, maintenance and repair decreases, but not directly, with production level because a maintenance and repair cost still occurs when the process plant is shut down.

Fixed charges are expenses which remain practically constant from year to year and do not vary widely with changes in production rate. Depreciation, property taxes, insurance, and rent require expenditures that can be classified as fixed charges.

Plant-overhead costs are for hospital and medical services; general plant maintenance and overhead; safety services; payroll overhead including pensions, vacation allowances, social security, and life insurance; packaging, restaurant and recreation facilities, salvage services, control laboratories, property protection, plant superintendence, warehouse and storage facilities, and special employee benefits. These costs are similar to the basic fixed charges in that they do not vary widely with changes in production rate.

General Expenses

In addition to the manufacturing costs, other general expenses are involved in any company's operations. These general expenses may be classified as (1) administrative expenses, (2) distribution and marketing expenses, (3) research and development expenses, (4) financing expenses, and (5) gross-earnings expenses.

Administrative expenses include costs for executive and clerical wages, office supplies, engineering and legal expenses, upkeep on office buildings, and general communications.

Distribution and marketing expenses are costs incurred in the process of selling and distributing the various products. These costs include expenditures for materials handling, containers, shipping, sales offices, salesmen, technical sales service, and advertising.

Research and development expenses are incurred by any progressive concern which wishes to remain in a competitive industrial position. These costs are for salaries, wages, special equipment, research facilities, and consultant fees related to developing new ideas or improved processes.

Financing expenses include the extra costs involved in procuring the money necessary for the capital investment. Financing expense is usually limited to interest on borrowed money, and this expense is sometimes listed as a fixed charge.

Gross-earnings expenses are based on income-tax laws. These expenses are a direct function of the gross earnings made by all the various interests held by the particular company. Because these costs depend on the company-wide picture, they are often not included in predesign or preliminary cost-estimation figures for a single plant, and the probable returns are reported as the gross earnings obtainable with the given plant design. However, when considering net
profits, the expenses due to income taxes are extremely important, and this cost must be included as a special type of general expense.

**DIRECT PRODUCTION COSTS**

**Raw Materials**

In the chemical industry, one of the major costs in a production operation is for the raw materials involved in the process. The amount of the raw materials which must be supplied per unit of time or per unit of product can be determined from process material balances. In many cases, certain materials act only as an agent of production and may be recoverable to some extent. Therefore, the cost should be based on the amount of raw materials actually consumed as determined from the overall material balances.

Direct price quotations from prospective suppliers are preferable to published market prices. For preliminary cost analyses, market prices are often used for estimating raw-material costs. These values are published regularly in journals such as the *Chemical Marketing Reporter* (formerly the *Oil, Paint, and Drug Reporter*).

Freight or transportation charges should be included in the raw-material costs, and these charges should be based on the form in which the raw materials are to be purchased for use in the final plant. Although bulk shipments are cheaper than smaller-container shipments, they require greater storage facilities and inventory. Consequently, the demands to be met in the final plant should be considered when deciding on the cost of raw materials.

The ratio of the cost of raw materials to total plant cost obviously will vary considerably for different types of plants. In chemical plants, raw-material costs are usually in the range of 10 to 50 percent of the total product cost.

**Operating Labor**

In general, operating labor may be divided into skilled and unskilled labor. Hourly wage rates for operating labor in different industries at various locations can be obtained from the U.S. Bureau of Labor *Monthly Labor Review*. For chemical processes, operating labor usually amounts to about 15 percent of the total product cost.

In preliminary costs analyses, the quantity of operating labor can often be estimated either from company experience with similar processes or from published information on similar processes. Because the relationship between labor requirements and production rate is not always a linear one, a 0.2 to 0.25 power of the capacity ratio when plant capacities are scaled up or down is often used.

If a flow sheet and drawings of the process are available, the operating labor may be estimated from an analysis of the work to be done. Consideration
### TABLE 21
Typical labor requirements for process equipment

<table>
<thead>
<tr>
<th>Type of equipment</th>
<th>Workers/unit/shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dryer, rotary</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>Dryer, spray</td>
<td>1</td>
</tr>
<tr>
<td>Dryer, tray</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>Centrifugal separator</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>Crystallizer, mechanical</td>
<td>( \frac{6}{4} )</td>
</tr>
<tr>
<td>Filter, vacuum</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>Evaporator</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>Reactor, batch</td>
<td>1</td>
</tr>
<tr>
<td>Reactor, continuous</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>Steam plant (100,000 lb/h)</td>
<td>3</td>
</tr>
</tbody>
</table>

**FIGURE 6-8**
Operating labor requirements for chemical process industries.
COST ESTIMATION

must be given to such items as the type and arrangement of equipment, multiplicity of units, amount of instrumentation and control for the process, and company policy in establishing labor requirements. Table 21 indicates some typical labor requirements for various types of process equipment.

Another method of estimating labor requirements as a function of plant capacity is based on adding up the various principal processing steps on the flow

### Table 22
Operating labor, fuel, steam, power, and water requirements for various processes†

<table>
<thead>
<tr>
<th>Capacity thousand ton/yr</th>
<th>Operating labor and supervision workhours/ton</th>
<th>Maintenance labor and supervision workhours/ton</th>
<th>Power and utilities, per ton/yr or bbl/day capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Fuel MM Btu/h lb/h kWh gph</td>
</tr>
<tr>
<td>Chemical plants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acetone 100</td>
<td>0.518</td>
<td>0.315</td>
<td>1.73 310 5.18</td>
</tr>
<tr>
<td>Acetic acid 10</td>
<td>1.483</td>
<td>0.984</td>
<td>4.88 140 0.148</td>
</tr>
<tr>
<td>Butadiene 100</td>
<td>0.345</td>
<td>0.285</td>
<td>0.012 130 0.73</td>
</tr>
<tr>
<td>Ethylene oxide 100</td>
<td>0.232</td>
<td>0.104</td>
<td>34.6 200 0.029</td>
</tr>
<tr>
<td>Formaldehyde 100</td>
<td>0.259</td>
<td>0.328</td>
<td>2.62 160 0.186</td>
</tr>
<tr>
<td>Hydrogen peroxide 100</td>
<td>0.288</td>
<td>0.352</td>
<td>0.81 710 0.001</td>
</tr>
<tr>
<td>Isoprene 100</td>
<td>0.230</td>
<td>0.325</td>
<td>0.18 40 0.03</td>
</tr>
<tr>
<td>Phosphoric acid 10</td>
<td>1.85</td>
<td>0.442</td>
<td>0.23 450 0.0004</td>
</tr>
<tr>
<td>Polyethylene 100</td>
<td>0.259</td>
<td>0.295</td>
<td>0.33 135 0.0002</td>
</tr>
<tr>
<td>Urea 100</td>
<td>0.238</td>
<td>0.215</td>
<td>1.34 275 0.27</td>
</tr>
<tr>
<td>Vinyl acetate 100</td>
<td>0.432</td>
<td>0.528</td>
<td></td>
</tr>
<tr>
<td>Refinery units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thousand bbl/day</td>
<td>Workhours/ bbl</td>
<td>Workhours/ bbl</td>
<td></td>
</tr>
<tr>
<td>Alkylation 10</td>
<td>0.007</td>
<td>0.0895</td>
<td>10.83 0.07 1.48</td>
</tr>
<tr>
<td>Coking (delayed) 10</td>
<td>0.0096</td>
<td>0.0058</td>
<td>0.012 1.85 0.07</td>
</tr>
<tr>
<td>Coking (fluid) 10</td>
<td>0.0122</td>
<td>0.0115</td>
<td>(4.73) 0.02 0.33</td>
</tr>
<tr>
<td>Cracking (fluid) 10</td>
<td>0.0096</td>
<td>0.0025</td>
<td>(2.55) 0.06 0.64</td>
</tr>
<tr>
<td>Cracking (thermal) 10</td>
<td>0.0048</td>
<td>0.0042</td>
<td>0.004 0.25 0.03</td>
</tr>
<tr>
<td>Distillation (atm) 10</td>
<td>0.0024</td>
<td>0.0154</td>
<td>0.003 0.95 0.04</td>
</tr>
<tr>
<td>Distillation (MC) 10</td>
<td>0.0048</td>
<td>0.0028</td>
<td>0.006 0.92 0.01</td>
</tr>
<tr>
<td>Hydrotreating 10</td>
<td>0.0048</td>
<td>0.0078</td>
<td>0.002 1.38 0.23</td>
</tr>
<tr>
<td>Reforming, catalytic 10</td>
<td>0.0024</td>
<td>0.0158</td>
<td>4.85 0.07 0.43</td>
</tr>
</tbody>
</table>

‡ Includes two coke cutters (1 shift/day).
§ Net steam generated.
### TABLE 23
Cost tabulation for selected utilities and labor†‡
1989 costs based on U.S. Gulf Coast location

<table>
<thead>
<tr>
<th>Steam costs</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exhaust, $/1000 lb</td>
<td>1.10</td>
</tr>
<tr>
<td>Pressure of 100 psig, $/1000 lb</td>
<td>2.40</td>
</tr>
<tr>
<td>Pressure of 500 psig, $/1000 lb</td>
<td>3.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fuel costs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas at well head including gathering-system costs:</td>
<td></td>
</tr>
<tr>
<td>Existing contracts, $/million Btu</td>
<td>2.40</td>
</tr>
<tr>
<td>New contracts, $/million Btu</td>
<td>3.00</td>
</tr>
<tr>
<td>Fuel oil in $/million Btu with 6.25 million Btu/bbl</td>
<td>3.00</td>
</tr>
<tr>
<td>Gas transmission costs in $/100 miles</td>
<td>7.30</td>
</tr>
<tr>
<td>Plant fuel gas in $/million Btu</td>
<td>3.20</td>
</tr>
<tr>
<td>Purchased power for midcontinent USA in $/kWh</td>
<td>7.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water costs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Process water (treated) in $/1000 gal</td>
<td>80</td>
</tr>
<tr>
<td>Cooling water in $/1000 gal (tower or river)</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labor rates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Supervisor, $/h</td>
<td>28.00</td>
</tr>
<tr>
<td>Operators, $/h</td>
<td>21.00</td>
</tr>
<tr>
<td>Helpers, $/h</td>
<td>17.40</td>
</tr>
<tr>
<td>Chemists, $/h</td>
<td>20.00</td>
</tr>
<tr>
<td>Labor burden as % of direct labors</td>
<td>25</td>
</tr>
<tr>
<td>Plant general overhead as % of total labor + burden</td>
<td>40</td>
</tr>
</tbody>
</table>

‡ See Appendix B for a more detailed listing of utility and related costs.
§ Labor burden refers to costs the company must pay associated with and above the base labor rate, such as for Social Security, insurance, and other benefits.
¶ In this method, a process step is defined as any unit operation, unit process, or combination thereof, which takes place in one or more units of integrated equipment on a repetitive cycle or continuously, e.g., reaction, distillation, evaporation, drying, filtration, etc. Once the plant capacity is fixed, the number of employee-hours per ton of product per step is obtained from Fig. 6-8 and multiplied by the number of process steps to give the total employee-hours per ton of production. Variations in labor requirements from highly automated processing steps to batch operations are provided by selection of the appropriate curve on Fig. 6-8.

### Table 24

Engineering News-Record labor indexes to permit estimation of prevailing wage rates by location

(See table 23 for values of labor rates as $/h)

<table>
<thead>
<tr>
<th>Location</th>
<th>ENB</th>
<th>Skilled</th>
<th>Labor Index (December values)</th>
<th>(Based on 1967 = 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta</td>
<td>256</td>
<td>304</td>
<td>330</td>
<td>312</td>
</tr>
<tr>
<td>Baltimore</td>
<td>281</td>
<td>304</td>
<td>333</td>
<td>337</td>
</tr>
<tr>
<td>Birmingham</td>
<td>289</td>
<td>309</td>
<td>320</td>
<td>332</td>
</tr>
<tr>
<td>Boston</td>
<td>265</td>
<td>2%</td>
<td>353</td>
<td>378</td>
</tr>
<tr>
<td>Chicago</td>
<td>289</td>
<td>314</td>
<td>349</td>
<td>355</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>314</td>
<td>342</td>
<td>359</td>
<td>378</td>
</tr>
<tr>
<td>Cleveland</td>
<td>294</td>
<td>315</td>
<td>349</td>
<td>382</td>
</tr>
<tr>
<td>Dallas</td>
<td>302</td>
<td>352</td>
<td>386</td>
<td>409</td>
</tr>
<tr>
<td>Denver</td>
<td>281</td>
<td>324</td>
<td>366</td>
<td>406</td>
</tr>
<tr>
<td>Detroit</td>
<td>314</td>
<td>350</td>
<td>356</td>
<td>369</td>
</tr>
<tr>
<td>Kansas City</td>
<td>307</td>
<td>340</td>
<td>372</td>
<td>394</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>336</td>
<td>375</td>
<td>375</td>
<td>452</td>
</tr>
<tr>
<td>Minneapolis</td>
<td>276</td>
<td>314</td>
<td>351</td>
<td>388</td>
</tr>
<tr>
<td>New Orleans</td>
<td>297</td>
<td>325</td>
<td>350</td>
<td>376</td>
</tr>
<tr>
<td>New York</td>
<td>250</td>
<td>274</td>
<td>303</td>
<td>334</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>267</td>
<td>307</td>
<td>324</td>
<td>354</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>282</td>
<td>304</td>
<td>324</td>
<td>342</td>
</tr>
<tr>
<td>St Louis</td>
<td>262</td>
<td>297</td>
<td>306</td>
<td>318</td>
</tr>
<tr>
<td>San Francisco</td>
<td>307</td>
<td>330</td>
<td>381</td>
<td>400</td>
</tr>
<tr>
<td>Seattle</td>
<td>327</td>
<td>363</td>
<td>386</td>
<td>387</td>
</tr>
</tbody>
</table>

† Published in Engineering News Record monthly in the second issue of the month with summaries in the third issue of the March and December issues.

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Example 6 Estimation of labor requirements. Consider a highly automated processing plant having a capacity of 100 tons/day of product and requiring principal processing steps of heat transfer, reaction, and distillation. What are the average operating labor requirements for an annual operation of 300 days?

**Solution.** The process plant is considered to require three process steps. From Fig. 6-8, for a capacity of 100 tons product/day, the highly automated process plant requires 33 employee-hours/day/processing step. Thus, for 300 days annual operation, operating labor required = \(3(33 \times 300)\) = 29,700 employee-hours/year.

Because of new technological developments including computerized controls and long-distance control arrangements, the practice of relating employee-hour requirements directly to production quantities for a given product can give inaccurate results unless very recent data are used. As a general
rule of thumb, the labor requirements for a fluids-processing plant, such as an ethylene oxide plant or others as shown in Table 22, would be in the low range of $\frac{1}{3}$ to 2 employee-hours per ton of product; for a solid-fluids plant, such as a polyethylene plant, the labor requirement would be in the intermediate range of 2 to 4 employee-hours per ton of product; for plants primarily engaged in solids processing such as a coal briquetting plant, the large amount of materials handling would make the labor requirements considerably higher than for other types of plants with a range of 4 to 8 employee-hours per ton of product being reasonable. The data shown in Fig. 6-8 and Table 22, where plant capacity and specific type of process are taken into account, are much more accurate than the preceding rule of thumb if the added necessary information is available.

In determining costs for labor, account must be taken of the type of worker required, the geographical location of the plant, the prevailing wage rates, and worker productivity. Table 20 presents data that can be used as a guide for relative median labor rates and productivity factors for workers in various geographical areas of the United States. Tables 23 and 24 provide data on labor rates in dollars per hour for the U.S. Gulf Coast region and average labor indexes to permit estimation of prevailing wage rates.

**Direct Supervisory and Clerical Labor**

A certain amount of direct supervisory and clerical labor is always required for a manufacturing operation. The necessary amount of this type of labor is closely related to the total amount of operating labor, complexity of the operation, and product quality standards. The cost for direct supervisory and clerical labor averages about 15 percent of the cost for operating labor. For reduced capacities, supervision usually remains fixed at the 50-percent-capacity rate.

**Utilities**

The cost for utilities, such as steam, electricity, process and cooling water, compressed air, natural gas, and fuel oil, varies widely depending on the amount of consumption, plant location, and source. For example, costs for a few selected utilities in the U.S. Gulf Coast region are given in Table 23. A more detailed list of average rates for various utilities is presented in Appendix B. The required utilities can sometimes be estimated in preliminary cost analyses from available information about similar operations as shown in Table 22. If such information is unavailable, the utilities must be estimated from a preliminary design. The utility may be purchased at predetermined rates from an outside source, or the service may be available from within the company. If the company supplied its own service and this is utilized for just one process, the entire cost of the service installation is usually charged to the manufacturing process. If the service is utilized for the production of several different products,

the service cost is apportioned among the different products at a rate based on the amount of individual consumption.

Steam requirements include the amount consumed in the manufacturing process plus that necessary for auxiliary needs. An allowance for radiation and line losses must also be made.

Electrical power must be supplied for lighting, motors, and various process-equipment demands. These direct-power requirements should be increased by a factor of 1.1 to 1.25 to allow for line losses and contingencies. As a rough approximation, utility costs for ordinary chemical processes amount to 10 to 20 percent of the total product cost.

M aintenance and R epairs

A considerable amount of expense is necessary for maintenance and repairs if a plant is to be kept in efficient operating condition. These expenses include the cost for labor, materials, and supervision.

Annual costs for equipment maintenance and repairs may range from as low as 2 percent of the equipment cost if service demands are light to 20 percent for cases in which there are severe operating demands. Charges of this type for buildings average 3 to 4 percent of the building cost. In the process industries, the total plant cost per year for maintenance and repairs is roughly equal to an average of 6 percent of the fixed-capital investment. Table 25 provides a guide for estimation of maintenance and repair costs as a function of process conditions.

For operating rates less than plant capacity, the maintenance and repair cost is generally estimated as 85 percent of that at 100 percent capacity for a 75 percent operating rate, and 75 percent of that at 100 percent capacity for a 50 percent operating rate.

<table>
<thead>
<tr>
<th>Type of operation</th>
<th>Maintenance cost as percentage of fixed-capital investment (on annual basis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wages</td>
</tr>
<tr>
<td>Simple chemical processes</td>
<td>1-3</td>
</tr>
<tr>
<td>Average processes with normal operating conditions</td>
<td>2-4</td>
</tr>
<tr>
<td>Complicated processes, severe corrosion operating conditions, or extensive instrumentation</td>
<td>3-5</td>
</tr>
</tbody>
</table>

TABLE 25
Estimation of costs for maintenance and repairs
Operating Supplies

In any manufacturing operation, many miscellaneous supplies are needed to keep the process functioning efficiently. Items such as charts, lubricants, test chemicals, custodial supplies, and similar supplies cannot be considered as raw materials or maintenance and repair materials, and are classified as operating supplies. The annual cost for this type of supplies is about 15 percent of the total cost for maintenance and repairs.

Laboratory Charges

The cost of laboratory tests for control of operations and for product-quality control is covered in this manufacturing cost. This expense is generally calculated by estimating the employee-hours involved and multiplying this by the appropriate rate. For quick estimates, this cost may be taken as 10 to 20 percent of the operating labor.

Patents and Royalties

Many manufacturing processes are covered by patents, and it may be necessary to pay a set amount for patent rights or a royalty based on the amount of material produced. Even though the company involved in the operation obtained the original patent, a certain amount of the total expense involved in the development and procurement of the patent rights should be borne by the plant as an operating expense. In cases of this type, these costs are usually amortized over the legally protected life of the patent. Although a rough approximation of patent and royalty costs for patented processes is 0 to 6 percent of the total product cost, the engineer must use judgement because royalties vary with such factors as the type of product and the industry.

Catalysts and Solvents

Costs for catalysts and solvents can be significant and depend upon the specific manufacturing processes chosen.

FIXEDCHARGES

Certain expenses are always present in an industrial plant whether or not the manufacturing process is in operation. Costs that are invariant with the amount of production are designated as fixed costs or fixed charges. These include costs for depreciation, local property taxes, insurance, and rent. Expenses of this type are a direct function of the capital investment. As a rough approximation, these charges amount to about 10 to 20 percent of the total product cost.
Depreciation

Equipment, buildings, and other material objects comprising a manufacturing plant require an initial investment which must be written off as a manufacturing expense. In order to write off this cost, a decrease in value is assumed to occur throughout the usual life of the material possessions. This decrease in value is designated as depreciation.

Since depreciation rates are very important in determining the amount of income tax, the Internal Revenue Service has established allowable depreciation rates based on the probable useful life of various types of equipment and other fixed items involved in manufacturing operations. While several alternative methods may be used for determining the rate of depreciation, a straight-line method is usually assumed for engineering projects. In applying this method, a useful-life period and a salvage value at the end of the useful life are assumed, with due consideration being given to possibilities of obsolescence and economic changes. The difference between initial cost and the salvage value divided by the total years of useful life gives the annual cost due to depreciation.

The annual depreciation rate for machinery and equipment ordinarily is about 10 percent of the tied-capital investment, while buildings are usually depreciated at an annual rate of about 3 percent of the initial cost.

Local Taxes

The magnitude of local property taxes depends on the particular locality of the plant and the regional laws. Annual property taxes for plants in highly populated areas are ordinarily in the range of 2 to 4 percent of the fixed-capital investment. In less populated areas, local property taxes are about 1 to 2 percent of the tied-capital investment.

Insurance

Insurance rates depend on the type of process being carried out in the manufacturing operation and on the extent of available protection facilities. On an annual basis, these rates amount to about 1 percent of the fixed-capital investment.

Rent

Annual costs for rented land and buildings amount to about 8 to 12 percent of the value of the rented property.

PLANT OVERHEAD COSTS

The costs considered in the preceding sections are directly related with the production operation. In addition, however, many other expenses are always
involved if the complete plant is to function as an efficient unit. The expenditures required for routine plant services are included in *plant-overhead* costs. Nonmanufacturing machinery, equipment, and buildings are necessary for many of the general plant services, and the fixed charges and direct costs for these items are part of the plant-overhead costs.

Expenses connected with the following comprise the bulk of the charges for plant overhead:

- Hospital and medical services
- General engineering
- Safety services
- Cafeteria and recreation facilities
- General plant maintenance and overhead
- Payroll overhead including employee benefits
- Control laboratories
- Packaging
- Plant protection
- Janitor and similar services
- Employment offices
- Distribution of utilities
- Shops
- Lighting
- Interplant communications and transportation
- Warehouses
- Shipping and receiving facilities

These charges are closely related to the costs for all labor 'directly connected with the production operation. The plant-overhead cost for chemical plants is about 50 to 70 percent of the total expense for operating labor, supervision, and maintenance.

**ADMINISTRATIVE COSTS**

The expenses connected with top-management or administrative activities cannot be charged directly to manufacturing costs; however, it is necessary to include the *administrative costs* if the economic analysis is to be complete. Salaries and wages for administrators, secretaries, accountants, stenographers, typists, and similar workers are part of the administrative expenses, along with costs for office supplies and equipment, outside communications, administrative buildings, and other overhead items related with administrative activities. These costs may vary markedly from plant to plant and depend somewhat on whether the plant under consideration is a new one or an addition to an old plant. In the
absence of more-accurate cost figures from company records, or for a quick estimate, the administrative costs may be approximated as 20 to 30 percent of the operating labor.

**DISTRIBUTION AND MARKETING COSTS**

From a practical viewpoint, no manufacturing operation can be considered a success until the products have been sold or put to some profitable use. It is necessary, therefore, to consider the expenses involved in selling the products. Included in this category are salaries, wages, supplies, and other expenses for sales offices; salaries, commissions, and traveling expenses for salesmen; shipping expenses; cost of containers; advertising expenses; and technical sales service.

*Distribution and marketing costs* vary widely for different types of plants depending on the particular material being produced, other products sold by the company, plant location, and company policies. These costs for most chemical plants are in the range of 2 to 20 percent of the total product cost. The higher figure usually applies to a new product or to one sold in small quantities to a large number of customers. The lower figure applies to large-volume products, such as bulk chemicals.

**RESEARCH AND DEVELOPMENT COSTS**

New methods and products are constantly being developed in the chemical industries. These accomplishments are brought about by emphasis on research and development. *Research and development costs* include salaries and wages for all personnel directly connected with this type of work, fixed and operating expenses for all machinery and equipment involved, costs for materials and supplies, direct overhead expenses, and miscellaneous costs. In the chemical industry, these costs amount to about 2 to 5 percent of every sales dollar.

**FINANCING**

**Interest**

*Interest* is considered to be the compensation paid for the use of borrowed capital. A fixed rate of interest is established at the time the capital is borrowed; therefore, interest is a definite cost if it is necessary to borrow the capital used to make the investment for a plant. Although the interest on borrowed capital is a fixed charge, there are many persons who claim that interest should not be considered as a manufacturing cost. It is preferable to separate interest from the other fixed charges and list it as a separate expense under the general heading of management or financing cost. Annual interest rates amount to 5 to 10 percent of the total value of the borrowed capital.

When the capital investment is supplied directly from the existing funds of a company, it is a debatable point whether interest should be charged as a cost.
For income-tax calculations, interest on owned money cannot be charged as a cost. In design calculations, however, interest can be included as a cost unless there is assurance that the total capital investment will be supplied from the company’s funds and the company policies permit exclusion of interest as a cost.

GROSS-EARNINGS COSTS

The total income minus the total production cost gives the gross earnings made by the particular production operation, which can then be treated mathematically by any of several methods to measure the profitability of the proposed venture or project. These methods will be discussed later in Chaps. 7 and 10.

Because of income-tax demands, the final net profit is often much less than the gross earnings. Income-tax rates are based on the gross earnings received from all the company interests. Consequently, the magnitude of these costs varies widely from one company to another.

On an annual basis, the corporate income-tax laws for the United States in 1979 required payment of a 17, 20, 30, and 40 percent normal tax on the 1st, 2nd, 3rd, and 4th $25,000, respectively, of the annual gross earnings of a corporation plus 46 percent of all annual gross earnings above $100,000. In addition, if other levies, such as state income taxes, were included, the overall tax rate could have been even higher. By 1988, the corporate income-tax laws had been changed to 15 percent on the first $50,000 of annual gross earnings, 25 percent on annual gross earnings of $50,000 to $75,000, and 34 percent on annual gross earnings above $75,000 plus a special graduated-tax phase-out of 5 percent on the gross earnings from $100,000 to $335,000. Tax rates vary from year to year depending on Federal and state regulations as is shown in the following example where 1988 Federal-tax rates are considered.

**Example 7 Break-even point, gross earnings, and net profit for a process plant.**

The annual direct production costs for a plant operating at 70 percent capacity are $280,000 while the sum of the annual fixed charges, overhead costs, and general expenses is $200,000. What is the break-even point in units of production per year if total annual sales are $560,000 and the product sells at $40 per unit? What were the annual gross earnings and net profit for this plant at 100 percent capacity in 1988 when corporate income taxes required a 15 percent tax on the first $50,000 of annual gross earnings, 25 percent on annual gross earnings of $50,000 to $75,000, 34 percent on annual gross earnings above $75,000, and 5 percent on gross earnings from $100,000 to $335,000?

**Solution.** The break-even point (Fig. 6-3) occurs when the total annual product cost equals the total annual sales. The total annual product cost is the sum of the fixed costs (including fixed charges, overhead, and general expenses) and the direct production costs for \( n \) units per year. The total annual sales is the product of the number of units and the selling price per unit. Thus

\[
\text{Direct production cost/unit} = \frac{280,000}{(560,000/40)} = \$20/\text{unit}
\]
and the number of units needed for a break-even point is given by
\[200,000 + 20n = 40n\]
\[n = \frac{200,000}{20} = 10,000 \text{ units/year} \]

This is \([(10,000)/(14,000/0.7)] \times 100 = 50\% \) of the present plant operating capacity.

Gross annual earnings = total annual sales $\rightarrow$ total annual product cost

\[= \frac{14,000}{0.7} \text{units (MO/unit)} \]

\[-\left[200,000 + \frac{14,000}{0.7} \text{ units ($20/unit)} \right] \]

\[= 800,000 \rightarrow 600,000 \]

\[= $200,000 \]

Net annual earnings = gross annual earnings $\rightarrow$ income taxes

\[= 200,000 - [(0.15)(50,000) + (0.25)(25,000) + (0.34)(200,000 - 75,000) + (0.05)(200,000 - 100,000)] \]

\[= 200,000 - 61,250 \]

\[= $138,750 \]

CONTINGENCIES

Unforeseen events, such as strikes, storms, floods, price variations, and other contingencies, may have an effect on the costs for a manufacturing operation. When the chemical engineer predicts total costs, it is advisable to take these factors into account. This can be accomplished by including a contingency factor equivalent to 1 to 5 percent of the total product cost.

SUMMARY

This chapter has outlined the economic considerations which are necessary when a chemical engineer prepares estimates of capital investment cost or total product cost for a new venture or project. Methods for obtaining predesign cost estimates have purposely been emphasized because the latter are extremely important for determining the feasibility of a proposed investment and to compare alternative designs. It should be remembered, however, that predesign estimates are often based partially on approximate percentages or factors that are applicable to a particular plant or process under consideration. Tables 26 and 27 summarize the predesign estimates for capital investment costs and total product costs, respectively. The percentages indicated in both tables give the ranges encountered in typical chemical plants. Because of the wide variations in different types of plants, the factors presented should be used only when more accurate data are not available.
TABLE 26
Estimation of capital investment cost (showing individual components)
The percentages indicated in the following summary of the various costs constituting the capital investment are approximations applicable to ordinary chemical processing plants. It should be realized that the values given can vary depending on many factors, such as plant location, type of process, complexity of instrumentation, etc.

I. Direct costs = material and labor involved in actual installation of complete facility
(70–85% of fixed-capital investment)
   A. Equipment + installation + instrumentation + piping + electrical + insulation + painting
      (50–60% of fixed-capital investment)
      1. Purchased equipment (15–40% of fixed-capital investment)
      2. Installation, including insulation and painting (25–55% of purchased-equipment cost)
      3. Instrumentation and controls, installed (6–30% of purchased-equipment cost)
      4. Piping, installed (10–80% of purchased-equipment cost)
      5. Electrical, installed (10–40% of purchased-equipment cost)
   B. Buildings, process and auxiliary (10–70% of purchased-equipment cost)
   C. Service facilities and yard improvements (40–100% of purchased-equipment cost)
   D. Land (1–2% of fixed-capital investment or 4–8% of purchased-equipment cost)

II. Indirect costs = expenses which are not directly involved with material and labor of actual installation of complete facility (15–30% of fixed-capital investment)
   A. Engineering and supervision (5–30% of direct costs)
   B. Construction expense and contractor's fee (6–30% of direct costs)
   C. Contingency (5–15% of fixed-capital investment)

III. Fixed-capital investment = direct costs + indirect costs

IV. Working capital (10–20% of total capital investment)

V. Total capital investment = fixed-capital investment + working capital

TABLE 27
Estimation of total product cost (showing individual components)
The percentages indicated in the following summary of the various costs involved in the complete operation of manufacturing plants are approximations applicable to ordinary chemical processing plants. It should be realized that the values given can vary depending on many factors, such as plant location, type of process, and company policies.
Percentages are expressed on an annual basis.

I. Manufacturing cost = direct production costs + fixed charges + plant overhead costs
   A. Direct production costs (about 60% of total product cost)
      1. Raw materials (10–50% of total product cost)
      2. Operating labor (10–20% of total product cost)
      3. Direct supervisory and clerical labor (10–25% of operating labor)
      4. Utilities (10–20% of total product cost)
      5. Maintenance and repairs (2–10% of fixed-capital investment)
      6. Operating supplies (10–20% of cost for maintenance and repairs, or 0.5–1% of fixed-capital investment)
      7. Laboratory charges (10–20% of operating labor)
      8. Patents and royalties (0–6% of total product cost)
   B. Fixed charges (10–20% of total product cost)
      1. Depreciation (depends on life period, salvage value, and method of calculation—about
         10% of fixed-capital investment for machinery and equipment and 2–3% of building
         value for buildings)
      2. Local taxes (1–4% of fixed-capital investment)
      3. Insurance (0.4–1% of fixed-capital investment)
      4. Rent (8–12% of value of rented land and buildings)
TABLE 27
Estimation of total product cost (showing individual components)  (Continued)

C. Plant-overhead costs (SO-70% of cost for operating labor, supervision, and maintenance, or 5-15% of total product cost); includes costs for the following: general plant upkeep and overhead, payroll overhead, packaging, medical services, safety and protection, restaurants, recreation, salvage, laboratories, and storage facilities.

II. General expenses = administrative costs + distribution and selling costs + research and development costs

A. Administrative costs (about 15% of costs for operating labor, supervision, and maintenance, or 2-6% of total product cost); includes costs for executive salaries, clerical wages, legal fees, office supplies, and communications

B. Distribution and selling costs (2-20% of total product cost); includes costs for sales offices, salesmen, shipping, and advertising

C. Research and development costs (2-5% of every sales dollar or about 5% of total product cost)

D. Financing (interest)? (0-10% of total capital investment)

III. Total product cost = manufacturing cost + general expenses

IV. Gross-earnings cost (gross earnings = total income - total product cost; amount of gross-earnings cost depends on amount of gross earnings for entire company and income-tax regulations; a general range for gross-earnings cost is 30-40% of gross earnings)

† Interest on borrowed money is often considered as a fixed charge.
‡ If desired, a contingency factor can be included by increasing the total product cost by 1-5%.

NOMENCLATURE FOR CHAPTER 6

A = incremental cost of corrosion-resistant alloy materials
A, = nonmanufacturing fixed-capital investment
\( c_o \) = costs for operations \( \text{(not including depreciation)} \)
\( \bar{C} \) = original capital investment
\( C_n \) = new capital investment
\( d \) = depreciation charge
\( d_n \) = number of drawings and specifications
\( D \) = total direct cost of plant
\( e \) = total heat exchanger cost (less incremental cost of alloy)
\( e_L \) = labor efficiency index in new location relative to cost of \( E_L \) and \( M_L' \)
\( E \) = purchased-equipment cost (installation cost not included) on f.o.b. basis
\( E_i \) = installed-equipment cost (purchased and installation cost included)
\( E_i' \) = purchased-equipment labor cost (base)
\( f \) = lumped cost index relative to original installation cost
\( f_1, f_2 \) = multiplying factors for piping, electrical, instrumentation, etc.
\( f_d \) = unit cost per drawing and specification
\( f_e \) = unit cost for engineering
\( f_e' \) = current equipment cost index relative to cost of \( E \)
\( f_F \) = construction or field-labor expense factor always greater than 1
\( f_i \) = indirect cost factor always greater than 1
\[ f_L = \text{current labor cost index in new location relative to cost of } E_L \text{ and } M_L' \]

\[ f_M = \text{current material cost index relative to cost of } M \]

\[ f_m = \text{cost factor for miscellaneous items} \]

\[ f_p = \text{cost factor for piping materials} \]

\[ f_v = \text{total cost of field-fabricated vessels (less incremental cost of alloy)} \]

\[ f_s = \text{specific material unit cost, e.g., } f_p = \text{unit cost of pipe} \]

\[ f_L = \text{specific material labor unit cost per employee-hour} \]

\[ H_e = \text{engineering employee-hours} \]

\[ I = \text{total indirect cost of plant} \]

\[ M = \text{material cost} \]

\[ M_L = \text{labor employee-hours for specific material} \]

\[ M_r = \text{direct labor cost for equipment installation and material handling} \]

\[ M_s = \text{specific material quantity in compatible units} \]

\[ P = \text{total pump plus driver cost (less incremental cost of alloy)} \]

\[ R = \text{ratio of new to original capacity} \]

\[ s_i = \text{total income from sales} \]

\[ I = \text{total cost of tower shells (less incremental cost of alloy)} \]

\[ T = \text{total capital investment} \]

\[ V = \text{manufacturing fixed-capital investment} \]

\[ W = \text{working-capital investment} \]

\[ \chi = \text{exponential power for cost-capacity relationships} \]

**PROBLEMS**

1. The purchased cost of a shell-and-tube heat exchanger (floating head and carbon-steel tubes) with 100 ft\(^2\) of heating surface was $3000 in 1980. What will be the purchased cost of a similar heat exchanger with 200 ft\(^2\) of heating surface in 1980 if the purchased-cost-capacity exponent is 0.60 for surface area ranging from 100 to 400 ft\(^2\)? If the purchased-cost-capacity exponent for this type of exchanger is 0.81 for surface areas ranging from 400 to 2000 ft\(^2\), what will be the purchased cost of a heat exchanger with 1000 ft\(^2\) of heating surface in 1985?

2. Plot the 1985 purchased cost of the shell-and-tube heat exchanger outlined in the previous problem as a function of the surface area from 100 to 2000 ft\(^2\). Note that the purchased-cost-capacity exponent is not constant over the range of surface area requested.

3. The purchased and installation costs of some pieces of equipment are given as a function of weight rather than capacity. An example of this is the installed costs of large tanks. The 1980 cost for an installed aluminum tank weighing 100,000 lb was $390,000. For a size range from 200,000 to 1,000,000 lb, the installed cost-weight exponent for aluminum tanks is 0.93. If an aluminum tank weighing 700,000 lb is required, what is the present capital investment needed?

4. What weight of installed stainless-steel tank could have been obtained for the same capital investment as in the previous problem? The 1980 cost for an installed 304 stainless-steel tank weighing 300,000 lb was $670,000. The installed cost-weight exponent for stainless tanks is 0.88 for a size range from 300,000 to 700,000 lb.

5. The purchased cost of a 1400-gal stainless-steel tank in 1980 was $7500. The tank is cylindrical with flat top and bottom, and the diameter is 6 ft. If the entire outer
surface of the tank is to be covered with 2 in. thickness of magnesia block, estimate the present total cost for the installed and insulated tank. The Jan. 1, 1980 cost for the 2-in. magnesia block was $2.20 per ft$^2$ while the labor for installing the insulation was $5.00 per ft$^2$.

6. A one-story warehouse 120 by 60 ft is to be added to an existing plant. An asphalt-pavement service area 60 by 30 ft will be added adjacent to the warehouse. It will also be necessary to put in 500 lin ft of railroad siding to service the warehouse. Utility service lines are already available at the warehouse site. The proposed warehouse has a concrete floor and steel frame, walls, and roof. No heat is necessary, but lighting and sprinklers must be installed. Estimate the total cost of the proposed addition. Consult App. B for necessary cost data.

7. The purchased cost of equipment for a solid-processing plant is $500,000. The plant is to be constructed as an addition to an existing plant. Estimate the total capital investment and the tied-capital investment for the plant. What percentage and amount of the fixed-capital investment is due to wst for land and contractor’s fee?

8. The purchased-equipment cost for a plant which produces pentaerythritol (solid-fuel-processing plant) is $300,000. The plant is to be an addition to an existing formaldehyde plant. The major part of the building cost will be for indoor construction, and the contractor’s fee will be 7 percent of the direct plant cost. All other costs are close to the average values found for typical chemical plants. On the basis of this information, estimate the following:
   (a) The total direct plant cost.
   (b) The fixed-capital investment.
   (c) The total capital investment.

9. Estimate by the turnover-ratio method the fixed-capital investment required for a proposed sulfuric acid plant (battery limit) which has a capacity of 140,000 tons of 100 percent sulfuric acid per year (contact-catalytic process) using the data from Table 19 for 1990 with sulfuric acid cost at $72 per ton. The plant may be considered as operating full time. Repeat using the cost-capacity-exponent method with data from Table 19.

10. The total capital investment for a chemical plant is $1 million, and the working capital is $100,000. If the plant can produce an average of 8000 kg of final product per day during a 365-day year, what selling price in dollars per kilogram of product would be necessary to give a turnover ratio of 1.0?

11. A process plant was constructed in the Philadelphia area (Middle Atlantic) at a labor cost of $200,000 in 1980. What would the average costs for the same plant to be in the Miami, Florida area (South Atlantic) if it were constructed in late 1988? Assume, for simplicity, that the relative labor rate and relative productivity factor remain essentially constant.

12. A company has been selling a soap containing 30 percent by weight water at a price of $10 per 100 lb f.o.b. (i.e., freight on board, which means the laundry pays the freight charges). The company offers an equally effective soap containing only 5 percent water. The water content is of no importance to the laundry, and it is willing to accept the soap containing 5 percent water if the delivered costs are equivalent. If the freight rate is 70 cents per 100 lb, how much should the company charge the laundry per 100 lb f.o.b. for the soap containing 5 percent water?

13. The total capital investment for a conventional chemical plant is $1,500,000, and the plant produces 3 million kg of product annually. The selling price of the product is
$0.82/kg. Working capital amounts to 15 percent of the total capital investment. The investment is from company funds, and no interest is charged. Raw-materials costs for the product are $0.09/kg, labor $0.08/kg, utilities $0.05/kg, and packaging $0.008/kg. Distribution costs are 5 percent of the total product cost. Estimate the following:

(a) Manufacturing cost per kilogram of product.
(b) Total product cost per year.
(c) Profit per kilogram of product before taxes.
(d) Profit per kilogram of product after taxes (use current rate).

14. Estimate the manufacturing cost per 100 lb of product under the following conditions:

- Fixed-capital investment = $2 million
- Annual production output = 10 million lb of product
- Raw materials cost = $0.12/lb of product
- Utilities
  - 100 psig steam = 50 lb/lb of product
  - Purchased electrical power = 0.4 kWh/lb of product
  - Filtered and softened water = 10 gal/lb of product
- Operating labor = 20 men per shift at $12.00 per employee-hour
- Plant operates three hundred 24-h days per year
- Corrosive liquids are involved
- Shipments are in bulk carload lots
- A large amount of direct supervision is required
- There are no patent, royalty, interest, or rent charges
- Plant-overhead costs amount to 50 percent of the cost for operating labor, supervision, and maintenance

15. A company has direct production costs equal to 50 percent of total annual sales and fixed charges, overhead, and general expenses equal to $200,000. If management proposes to increase present annual sales of $800,000 by 30 percent with a 20 percent increase in fixed charges, overhead, and general expenses, what annual sales dollar is required to provide the same gross earnings as the present plant operation? What would be the net profit if the expanded plant were operated at full capacity with an income tax on gross earnings fixed at 34 percent? What would be the net profit for the enlarged plant if total annual sales remained the same as at present? What would be the net profit for the enlarged plant if the total annual sales actually decreased to $700,000?

16. A process plant making 2000 tons per year of a product selling for $0.80 per lb has annual direct production costs of $2 million at 100 percent capacity and other fixed costs of $700,000. What is the fixed cost per pound at the break-even point? If the selling price of the product is increased by 10 percent, what is the dollar increase in net profit at full capacity if the income tax rate is 34 percent of gross earnings?

17. A rough rule of thumb for the chemical industry is that $1 of annual sales requires $1 of fixed-capital investment. In a chemical processing plant where this rule applies, the total capital investment is $2,500,000 and the working capital is 20 percent of the total capital investment. The annual total product cost amounts to $1,500,000. If the national and regional income-tax rates on gross earnings total 36 percent, determine the following:

(a) Percent of total capital investment returned annually as gross earnings.
(b) Percent of total capital investment returned annually as net profit.
18. The total capital investment for a proposed chemical plant which will produce $1,500,000 worth of goods per year is estimated to be $1 million. It will be necessary to do a considerable amount of research and development work on the project before the final plant can be constructed, and management wishes to estimate the permissible research and development costs. It has been decided that the net profits from the plant should be sufficient to pay off the total capital investment plus all research and development costs in 7 years. A return after taxes of at least 12 percent of sales must be obtained, and 34 percent of the research and development cost is tax-free (i.e., income-tax rate for the company is 34 percent of the gross earnings). Under these conditions, what is the total amount the company can afford to pay for research and development?

19. A chemical processing unit has a capacity for producing 1 million kg of a product per year. After the unit has been put into operation, it is found that only 500,000 kg of the product can be disposed of per year. An analysis of the existing situation shows that all fixed and other invariant charges, which must be paid whether or not the unit is operating, amount to 35 percent of the total product cost when operating at full capacity. Raw-material costs and other production costs that are directly proportional to the quantity of production (i.e., constant per kilogram of product at any production rate) amount to 40 percent of the total product cost at full capacity. The remaining 25 percent of the total product cost is for variable overhead and miscellaneous expenses, and the analysis indicates that these costs are directly proportional to the production rate during operation raised to the 1.5 power. What will be the percent change in total cost per kilogram of product if the unit is switched from the 1-million-kg-per-year rate to a time and rate schedule which will produce 500,000 kg of product per year at the least total cost? All costs referred to above are on a per-kilogram basis.

20. Estimate the total operating cost per day for labor, power, steam, and water in a plant producing 100 tons of acetone per day from the data given in Table 22 and using utility costs from Table 23. Consider all water as treated city water. The steam pressure may be assumed to be 100 psig. Labor costs average $20 per employee-hour. Electricity must be purchased. Plant operates 365 days per year.
A considerable amount of confusion exists among engineers over the role of interest in determining costs for a manufacturing operation. The confusion is caused by the attempt to apply the classical economist’s definition of interest. According to the classical definition, interest is the money returned to the owners of capital for use of their capital. This would mean that any profit obtained through the uses of capital could be considered as interest. Modern economists seldom adhere to the classical definition. Instead, they prefer to substitute the term return on capital or return on investment for the classical interest.

Engineers define interest as the compensation paid for the use of borrowed capital. This definition permits distinction between profit and interest. The rate at which interest will be paid is usually fixed at the time the capital is borrowed, and a guarantee is made to return the capital at some set time in the future or on an agreed-upon pay-off schedule.

TYPES OF INTEREST

Simple Interest

In economic terminology, the amount of capital on which interest is paid is designated as the principal, and rate of interest is defined as the amount of interest earned by a unit of principal in a unit of time. The time unit is usually taken as one year. For example, if $100 were the compensation demanded for
giving someone the use of $1000 for a period of one year, the principal would be $1000, and the rate of interest would be \( \frac{100}{1000} = 0.1 \) or 10 percent/year.

The simplest form of interest requires compensation payment at a constant interest rate based only on the original principal. Thus, if $1000 were loaned for a total time of 4 years at a constant interest rate of 10 percent/year, the simple interest earned would be

\[
\$1000 \times 0.1 \times 4 = \$400
\]

If \( P \) represents the principal, \( n \) the number of time units or interest periods, and \( i \) the interest rate based on the length of one interest period, the amount of simple interest \( Z \) during \( n \) interest periods is

\[ Z = Pin \]  

(1)

The principal must be repaid eventually; therefore, the entire amount \( S \) of principal plus simple interest due after \( n \) interest periods is

\[ S = P + Z = P(1 + in) \]  

(2)

Ordinary and Exact Simple Interest

The time unit used to determine the number of interest periods is usually 1 year, and the interest rate is expressed on a yearly basis. When an interest period of less than 1 year is involved, the ordinary way to determine simple interest is to assume the year consists of twelve 30-day months, or 360 days. The exact method accounts for the fact that there are 365 days in a normal year. Thus, if the interest rate is expressed on the regular yearly basis and \( d \) represents the number of days in an interest period, the following relationships apply:

\[
\text{Ordinary simple interest} = Pi \frac{d}{360} 
\]

(3)

\[
\text{Exact simple interest} = Pi \frac{d}{365} 
\]

(4)

Ordinary interest is commonly accepted in business practices unless there is a particular reason to use the exact value.

Compound Interest

In the payment of simple interest, it makes no difference whether the interest is paid at the end of each time unit or after any number of time units. The same total amount of money is paid during a given length of time, no matter which method is used. Under these conditions, there is no incentive to pay the interest until the end of the total loan period.

Interest, like all negotiable capital, has a time value. If the interest were paid at the end of each time unit, the receiver could put this money to use for earning additional returns. Compound interest takes this factor into account by
stipulating that interest is due regularly at the end of each interest period. If payment is not made, the amount due is added to the principal, and interest is charged on this converted principal during the following time unit. Thus, an initial loan of $1000 at an annual interest rate of 10 percent would require payment of $100 as interest at the end of the first year. If this payment were not made, the interest for the second year would be \((1000 + 100)(0.10) = 110\), and the total \textbf{compound amount} due after 2 years would be

\[1000 + 100 + 110 = 1210\]

The compound amount due after any discrete number of interest periods can be determined as follows:

<table>
<thead>
<tr>
<th>Period</th>
<th>Principal at start of period (P)</th>
<th>Interest earned during period ((i = \text{interest rate based on length of one period}))</th>
<th>Compound amount (S) at end of period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(P)</td>
<td>(Pi)</td>
<td>(P + Pi = P(1 + i))</td>
</tr>
<tr>
<td>2</td>
<td>(P(1 + i))</td>
<td>(P(1 + i)(i))</td>
<td>(P(1 + i) + P(1 + i)(i) = P(1 + i)^2)</td>
</tr>
<tr>
<td>3</td>
<td>(P(1 + i)^2)</td>
<td>(P(1 + i)^2(i))</td>
<td>(P(1 + i)^2 + P(1 + i)^2(i) = P(1 + i)^3)</td>
</tr>
<tr>
<td>(n)</td>
<td>(P(1 + i)^{n-1})</td>
<td>(P(1 + i)^{n-1}(i))</td>
<td>(P(1 + i)^n)</td>
</tr>
</tbody>
</table>

Therefore, the total amount of principal plus compounded interest due after \(n\) interest periods and designated as \(S\) is

\[S = P(1 + i)^n\]  \(5\)

The term \((1 + i)^n\) is commonly referred to as the \textit{discrete single-payment compound-amount factor}. Values for this factor at various interest rates and numbers of interest periods are given in Table 1.

Figure 7-1 shows a comparison among the total amounts due at different times for the cases where simple interest, discrete compound interest, and continuous interest are used.

**NOMINAL AND EFFECTIVE INTEREST RATES**

In common industrial practice, the length of the discrete interest period is assumed to be 1 year and the fixed interest rate \(i\) is based on 1 year. However, there are cases where other time units are employed. Even though the actual interest period is not 1 year, the interest rate is often expressed on an annual basis. Consider an example in which the interest rate is 3 percent per period

\(\dagger\)For the analogous equation for continuous interest compounding, see Eq. (12).
### TABLE 1
Discrete compound-interest factor \((1 + i)^n\) at various values of \(i\) and \(n\)

<table>
<thead>
<tr>
<th>Number of interest periods, (n)</th>
<th>Value of ((1 + i)^n) at indicated percent interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1.0100, 1.0200, 1.0300, 1.0400, 1.0500, 1.0600, 1.0709, 1.0800, 1.1000, 1.1209, 1.1409, 1.1600, 1.1800, 1.2000</td>
</tr>
<tr>
<td>2%</td>
<td>1.0201, 1.0404, 1.0609, 1.0816, 1.1025, 1.1236, 1.1449, 1.1664, 1.2010, 1.2344, 1.2696, 1.3056, 1.3466, 1.3917, 1.4400</td>
</tr>
<tr>
<td>3%</td>
<td>1.0303, 1.0612, 1.0927, 1.1249, 1.1576, 1.1910, 1.2250, 1.2597, 1.3131, 1.4040, 1.4815, 1.5609, 1.6434, 1.7280, 1.8173</td>
</tr>
<tr>
<td>4%</td>
<td>1.0406, 1.0824, 1.1256, 1.1699, 1.2155, 1.2625, 1.3108, 1.3605, 1.4641, 1.5735, 1.6880, 1.8100, 1.9338, 2.0736, 2.2533</td>
</tr>
<tr>
<td>5%</td>
<td>1.0510, 1.1041, 1.1593, 1.2167, 1.2763, 1.3382, 1.4026, 1.4693, 1.6105, 1.7623, 1.9264, 2.1005, 2.2871, 2.4883, 2.7206</td>
</tr>
<tr>
<td>6%</td>
<td>1.0615, 1.1262, 1.1940, 1.2635, 1.3401, 1.4185, 1.5007, 1.5859, 1.7716, 1.9670, 2.1818, 2.4161, 2.6696, 2.9428, 3.2364</td>
</tr>
<tr>
<td>7%</td>
<td>1.0721, 1.1487, 1.2289, 1.3159, 1.4171, 1.5306, 1.6658, 1.8506, 2.1436, 2.5967, 3.1620, 3.8448, 4.6788, 5.6946, 6.8651</td>
</tr>
<tr>
<td>8%</td>
<td>1.0829, 1.1717, 1.2668, 1.3868, 1.5175, 1.6838, 1.8152, 2.0563, 2.4136, 2.9567, 3.6348, 4.4716, 5.5015, 6.6851, 8.0599</td>
</tr>
<tr>
<td>9%</td>
<td>1.0937, 1.1961, 1.3048, 1.4233, 1.5513, 1.7098, 1.9000, 2.3579, 2.7721, 3.2620, 4.0038, 4.9314, 6.0303, 7.3109, 8.7651</td>
</tr>
<tr>
<td>10%</td>
<td>1.1046, 1.2190, 1.3439, 1.4802, 1.6289, 1.7908, 1.9672, 2.1589, 2.5931, 3.1058, 3.7072, 4.4114, 5.3315, 6.4197, 7.6917</td>
</tr>
<tr>
<td>11%</td>
<td>1.1157, 1.2434, 1.3842, 1.5395, 1.7103, 1.8983, 2.1049, 2.3316, 2.5825, 3.0819, 3.6620, 4.3219, 5.1762, 6.2439, 7.5014</td>
</tr>
<tr>
<td>12%</td>
<td>1.1268, 1.2662, 1.4258, 1.6010, 1.7959, 2.0122, 2.2522, 2.5182, 3.0084, 3.6900, 4.5870, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>13%</td>
<td>1.1381, 1.3195, 1.5126, 1.7137, 1.9260, 2.1609, 2.4258, 2.7216, 3.1423, 3.7695, 4.5870, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>14%</td>
<td>1.1495, 1.3395, 1.5519, 1.7717, 1.9979, 2.2408, 2.5290, 2.8447, 3.2715, 3.8448, 4.6716, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>15%</td>
<td>1.1610, 1.3549, 1.5580, 1.7809, 2.0136, 2.2769, 2.5506, 2.8447, 3.2715, 3.8448, 4.6716, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>16%</td>
<td>1.1726, 1.3728, 1.5947, 1.8370, 2.0920, 2.3825, 2.6928, 2.9522, 3.2715, 3.8448, 4.6716, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>17%</td>
<td>1.1843, 1.4002, 1.5947, 1.8149, 2.0568, 2.3158, 2.5890, 2.8374, 3.1554, 3.7305, 4.6009, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>18%</td>
<td>1.1961, 1.4282, 1.6072, 1.8258, 2.0686, 2.3379, 2.5980, 2.8447, 3.1554, 3.7305, 4.6009, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>19%</td>
<td>1.2081, 1.4568, 1.6495, 1.8525, 2.0686, 2.3379, 2.5980, 2.8447, 3.1554, 3.7305, 4.6009, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
<tr>
<td>20%</td>
<td>1.2202, 1.4859, 1.6861, 1.8870, 2.0829, 2.3404, 2.6040, 2.8750, 3.1722, 3.7832, 4.6109, 5.6960, 7.0287, 8.5916, 10.4073</td>
</tr>
</tbody>
</table>

**Percent interest = \((i)(100)\).**

**Notes:**
- The table provides discrete compound-interest factors at various values of interest rate \(i\) and number of periods \(n\).
- The interest rates range from 1% to 20%.
- The number of periods ranges from 1 to 20.

**Example Usage:** To find the compound-interest factor for a 20% interest rate over 5 periods, locate the row for 20% and the column for 5 periods. The value is 1.3208. Thus, \((1 + 0.20)^5 = 1.3208\).

**Example Calculation:** If you want to calculate the future value of an investment after 5 years with an annual interest rate of 5%, you would use the factor in the 5% row and 5 column. The future value is the present value multiplied by \((1 + 0.05)^5\).
and the interest is compounded at half-year periods. A rate of this type would be referred to as “6 percent compounded semiannually.” Interest rates stated in this form are known as nominal interest rates. The actual annual return on the principal would not be exactly 6 percent but would be somewhat larger because of the compounding effect at the end of the semiannual period.

It is desirable to express the exact interest rate based on the original principal and the convenient time unit of 1 year. A rate of this type is known as the effective interest rate. In common engineering practice, it is usually preferable to deal with effective interest rates rather than with nominal interest rates. The only time that nominal and effective interest rates are equal is when the interest is compounded annually.

Nominal interest rates should always include a qualifying statement indicating the compounding period. For example, using the common annual basis, $100 invested at a nominal interest rate of 20 percent compounded annually would amount to $120.00 after 1 year; if compounded semiannually, the amount would be $121.00; and, if compounded continuously, the amount would be $122.14. The corresponding effective interest rates are 20.00 percent, 21.00 percent, and 22.14 percent, respectively.

If nominal interest rates are quoted, it is possible to determine the effective interest rate by proceeding from Eq. (5).

\[ S = P(1 + i)^n \]  

(5)

In this equation, \( S \) represents the total amount of principal plus interest due after \( n \) periods at the periodic interest rate \( i \). Let \( r \) be the nominal interest rate under conditions where there are \( m \) conversions or interest periods per year.
Then the interest rate based on the length of one interest period is \( r/m \), and the amount \( S \) after 1 year is

\[
S_{\text{after 1 year}} = P \left(1 + \frac{r}{m}\right)^m \tag{6}
\]

Designating the effective interest rate as \( i_{\text{eff}} \), the amount \( S \) after 1 year can be expressed in an alternate form as

\[
S_{\text{after 1 year}} = P(1 + i_{\text{eff}}) \tag{7}
\]

By equating Eqs. (6) and (7), the following equation can be obtained for the effective interest rate in terms of the nominal interest rate and the number of periods per year:

\[
\text{Effective annual interest rate} = i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 \tag{8}
\]

Similarly, by definition,

\[
\text{Nominal annual interest rate} = m \left(\frac{r}{m}\right) = r \tag{9}
\]

**Example 1** Applications of different types of interest. It is desired to borrow $1000 to meet a financial obligation. This money can be borrowed from a loan agency at a monthly interest rate of 2 percent. Determine the following:

(a) The total amount of principal plus simple interest due after 2 years if no intermediate payments are made.

(b) The total amount of principal plus compounded interest due after 2 years if no intermediate payments are made.

(c) The nominal interest rate when the interest is compounded monthly.

(d) The effective interest rate when the interest is compounded monthly.

**Solution**

(a) Length of one interest period = 1 month

Number of interest periods in 2 years = 24

For simple interest, the total amount due after \( n \) periods at periodic interest rate \( i \) is

\[
S = P(1 + in) \tag{2}
\]

\( P = \text{initial principal} = \$1000 \)
\( i = 0.02 \) on a monthly basis
\( n = 24 \) interest periods in 2 years

\[
S = \$1000(1 + 0.02 \times 24) = \$1480
\]

(b) For compound interest, the total amount due after \( n \) periods at periodic interest rate \( i \) is

\[
S = P(1 + i)^n \tag{5''}
\]

\[
S = \$1000(1 + 0.02)^{24} = \$1608
\]

(c) Nominal interest rate = \( 2 \times 12 = 24\% \) per year compounded monthly
Number of interest periods per year = \( m = 12 \)

Nominal interest rate = \( r = 0.24 \)

Effective interest rate = \( \left(1 + \frac{r}{m}\right)^m \approx 1 \) \( (8) \)

Effective interest rate = \( \left(1 + \frac{0.24}{12}\right)^{12} \approx 1 = 0.268 = 26.8\% \)

**CONTINUOUS INTEREST**

The preceding discussion of types of interest has considered only the common form of interest in which the payments are charged at periodic and discrete intervals, where the intervals represent a finite length of time with interest accumulating in a discrete amount at the end of each interest period. Although in practice the basic time interval for interest accumulation is usually taken as one year, shorter time periods can be used as, for example, one month, one day, one hour, or one second. The extreme case, of course, is when the time interval becomes infinitesimally small so that the interest is compounded continuously.

The concept of continuous interest is that the cost or income due to interest flows regularly, and this is just as reasonable an assumption for most cases as the concept of interest accumulating only at discrete intervals. The reason why continuous interest has not been used widely is that most industrial and financial practices are based on methods which executives and the public are used to and can understand. Because normal interest comprehension is based on the discrete-interval approach, little attention has been paid to the concept of continuous interest even though this may represent a more realistic and idealized situation.

**The Basic Equations for Continuous Interest Compounding**

Equations \( (6), (7), \) and \( (8) \) represent the basic expressions from which continuous-interest relationships can be developed. The symbol \( r \) represents the nominal interest rate with \( m \) interest periods per year. If the interest is compounded continuously, \( m \) approaches infinity, and Eq. \( (6) \) can be written as

\[
S_{\text{after } n \text{ years}} = P \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{mn} = P \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{(m/r)(mn)}
\]

The fundamental definition for the base of the natural system of logarithms \( e = 2.71828 \) is

\[
\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{m/r} = e = 2.71828\ldots . \]

Thus, with continuous interest compounding at a nominal annual interest rate of $r$, the amount $S$ an initial principal $P$ will compound to in $n$ years is†

$$ S = Pe^{rn} $$

(12)

Similarly, from Eq. (8), the effective annual interest rate $i_{\text{eff}}$, which is the conventional interest rate that most executives comprehend, is expressed in terms of the nominal interest rate $r$ compounded continuously as

$$ i_{\text{eff}} = e^r - 1 $$

(13)

$$ r = \ln(i_{\text{eff}} + 1) $$

(14)

Therefore,

$$ e^{rn} = (1 + i_{\text{eff}})^n $$

(15)

and

$$ S = Pe^{rn} = P(1 + i_{\text{eff}})^n $$

(16)

As is illustrated in the following example, a conventional interest rate (i.e., effective annual interest rate) of 22.14 percent is equivalent to a 20.00 percent nominal interest rate compounded continuously. Note, also, that a nominal interest rate compounded daily gives results very close to those obtained with continuous compounding.

†The same result can be obtained from calculus by noting that, for the case of continuous compounding, the differential change of $S$ with time must equal the nominal continuous interest rate times $S$, or $dS/dn = rS$. This expression can be integrated as follows to give Eq. (12):

$$ \int_P^S \frac{dS}{S} = r \int_0^n dn $$

$$ \ln \frac{S}{P} = rn \text{ or } S = Pe^{rn} $$

(12)

‡A generalized way to express both Eq. (12) and Eq. (5), with direct relationship to the other interest equations in this chapter, is as follows:

Future worth = present worth x compound interest factor

$$ S = PC $$

or

Future worth x discount factor = present worth

$$ SF = P $$

Discount factor = $F = \frac{1}{\text{compound interest factor}} = \frac{1}{C}$

Although the various factors for different forms of interest expressions are derived in terms of interest rate in this chapter, the overall concept of interest evaluations is simplified by the use of the less-complicated nomenclature where designated factors are applied. Thus, expressing both Eqs. (12) and (5) as $SF = P$ would mean that $F$ is $e^{-rn}$ for the continuous interest case of Eq. (12) and $(1 + i)^{-n}$ for the discrete interest case of Eq. (5). See Table 4 for further information on this subject.
Example 2 Calculations with continuous interest compounding. For the case of a nominal annual interest rate of 20.00 percent, determine:

(a) The total amount to which one dollar of initial principal would accumulate after one 365-day year with daily compounding.

(b) The total amount to which one dollar of initial principal would accumulate after one year with continuous compounding.

(c) The effective annual interest rate if compounding is continuous.

**Solution**

(a) Using Eq. (6), $P = 1.0$, $r = 0.20$, $m = 365$,

\[ S_{\text{after 1 year}} = P \left(1 + \frac{r}{m}\right)^m = (1.0) \left(1 + \frac{0.20}{365}\right)^{365} = 1.2213 \]

(b) Using Eq. (12),

\[ S = Pe^{rn} = (1.0)(e^{0.20 \times 1}) = 1.2214 \]

(c) Using Eq. (13),

\[ i_{\text{eff}} = e^r - 1 = 1.2214 - 1 = 0.2214 \text{ or } 22.14\% \]

Tabulated values of $i_{\text{eff}}$ and the corresponding $r$ with continuous interest compounding are shown in Table 2.

<table>
<thead>
<tr>
<th>Effective annual rate of return, %</th>
<th>Nominal continuous rate of return, %</th>
<th>Effective annual rate of return, %</th>
<th>Nominal continuous rate of return, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99504</td>
<td>35</td>
<td>30.010</td>
</tr>
<tr>
<td>2</td>
<td>1.9803</td>
<td>40</td>
<td>33.647</td>
</tr>
<tr>
<td>3</td>
<td>2.9559</td>
<td>45</td>
<td>37.156</td>
</tr>
<tr>
<td>4</td>
<td>3.9221</td>
<td>50</td>
<td>40.547</td>
</tr>
<tr>
<td>5</td>
<td>4.8790</td>
<td>60</td>
<td>47.000</td>
</tr>
<tr>
<td>6</td>
<td>5.8269</td>
<td>70</td>
<td>53.063</td>
</tr>
<tr>
<td>7</td>
<td>6.7659</td>
<td>80</td>
<td>58.779</td>
</tr>
<tr>
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<td>90</td>
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<td>8.6178</td>
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<tr>
<td>10</td>
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<tr>
<td>15</td>
<td>13.976</td>
<td>120</td>
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<tr>
<td>20</td>
<td>18.232</td>
<td>130</td>
<td>83.291</td>
</tr>
<tr>
<td>25</td>
<td>22.314</td>
<td>140</td>
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</tr>
<tr>
<td>30</td>
<td>26.236</td>
<td>150</td>
<td>91.629</td>
</tr>
</tbody>
</table>
Example 3 Use of digital computer to give tabulated values of amount accumulated with continuous interest compounding. Present the digital computer program and the tabulated printout to six significant figures giving the amount to which an initial principal of $100 will accumulate year by year from 1 to 20 years with continuous interest compounding based on a nominal interest rate of 20 percent.

Solution. The equation to be solved on the digital computer is

$$ S = Pe^{rn} $$

where S will be evaluated to six significant figures for

$$ n = 1, 2, 3, \ldots, 20 $$

$$ P = 100 $$

$$ r = 0.20 $$

The Fortran IV program and the computer print-out follow:

```
$IBJOB MAP
$IBFTC DECK 1
1    DO 1 N = 1, 20
2      AN = N
3    S = 100.*EXP(.20*AN)
4      WRITE(6,2)N,S
5     END
6
$ENTRY
```

Printout

<table>
<thead>
<tr>
<th>N</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>122.140</td>
</tr>
<tr>
<td>2</td>
<td>149.182</td>
</tr>
<tr>
<td>3</td>
<td>182.212</td>
</tr>
<tr>
<td>4</td>
<td>222.554</td>
</tr>
<tr>
<td>5</td>
<td>271.828</td>
</tr>
<tr>
<td>6</td>
<td>332.012</td>
</tr>
<tr>
<td>7</td>
<td>405.520</td>
</tr>
<tr>
<td>8</td>
<td>495.303</td>
</tr>
<tr>
<td>9</td>
<td>604.965</td>
</tr>
<tr>
<td>10</td>
<td>738.906</td>
</tr>
</tbody>
</table>

(Note: The preceding illustrates the method used to prepare tabulated results of factors and emphasizes the simplicity of the procedure with a digital computer. This set of results represents a standard exponential function available in tabulated form in standard mathematical tables. See Prob. 8 at the end of this chapter for a requested computer solution for a more complicated continuous-interest case.)

PRESENT WORTH AND DISCOUNT

It is often necessary to determine the amount of money which must be available at the present time in order to have a certain amount accumulated at some definite time in the future. Because the element of time is involved, interest must be taken into consideration. The present worth (or present value) of a
future amount is the present principal which must be deposited at a given interest rate to yield the desired amount at some future date.

In Eq. (5), \( S \) represents the amount available after \( n \) interest periods if the initial principal is \( P \) and the discrete compound-interest rate is \( i \). Therefore, the present worth can be determined by merely rearranging Eq. (5).

\[
\text{Present worth} = P = S \frac{1}{(1 + i)^n}
\]

The factor \( 1/(1 + i)^n \) is commonly referred to as the \textit{discrete single-payment present-worth factor}.

Similarly, for the case of continuous interest compounding, Eq. (12) gives

\[
\text{Present worth} = P = S \frac{1}{e^{in}}
\]

Some types of capital are in the form of bonds having an indicated value at a future date. In business terminology, the difference between the indicated future value and the present worth (or present value) is known as the \textit{discount}.

Example 4 Determination of present worth and discount. A bond has a maturity value of $1000 and is paying discrete compound interest at an effective annual rate of 3 percent. Determine the following at a time four years before the bond reaches maturity value:

(a) Present worth.
(b) Discount.
(c) Discrete compound rate of effective interest which will be received by a purchaser if the bond were obtained for $700.
(d) Repeat part (a) for the case where the nominal bond interest is 3 percent compounded continuously.

Solution

(a) By Eq. (17), present worth = \( S/(1 + i)^n = \$1000/(1 + 0.03)^4 = \$888 \)

(b) Discount = future value - present worth = \$1000 - \$888 = \$112

(c) Principal = \$700 = \( S/(1 + i)^n = \$1000/(1 + i)^4 \)

\[ i = \left( \frac{1000}{700} \right)^{1/4} - 1 = 0.0935 \text{ or } 9.35\%
\]

(d) By Eq. (18), present worth = \( S/e^{in} = \$1000/e^{(0.03\times4)} = \$869 \)

\textbf{ANNUITIES}

An \textit{annuity} is a series of equal payments occurring at equal time intervals. Payments of this type can be used to pay off a debt, accumulate a desired

\*In the analyses presented in this chapter, effects of inflation or deflation on future worth are not considered. See Chap. 11 (Optimum Design and Design Strategy) for information on the strategy for dealing with inflation or deflation in design economic evaluations.
amount of capital, or receive a lump sum of capital that is due in periodic installments as in some life-insurance plans. Engineers often encounter annuities in depreciation calculations, where the decrease in value of equipment with time is accounted for by an annuity plan.

The common type of annuity involves payments which occur at the end of each interest period. This is known as an ordinary annuity. Interest is paid on all accumulated amounts, and the interest is compounded each payment period. An annuity term is the time from the beginning of the first payment period to the end of the last payment period. The amount of an annuity is the sum of all the payments plus interest if allowed to accumulate at a definite rate of interest from the time of initial payment to the end of the annuity term.

Relation between Amount of Ordinary Annuity and the Periodic Payments

Let \( R \) represent the uniform periodic payment made during \( n \) discrete periods in an ordinary annuity. The interest rate based on the payment period is \( i \), and \( S \) is the amount of the annuity. The first payment of \( R \) is made at the end of the first period and will bear interest for \( n - 1 \) periods. Thus, at the end of the annuity term, this first payment will have accumulated to an amount of \( R(1 + i)^{n-1} \). The second payment of \( R \) is made at the end of the second period and will bear interest for \( n - 2 \) periods giving an accumulated amount of \( R(1 + i)^{n-2} \). Similarly, each periodic payment will give an additional accumulated amount until the last payment of \( R \) is made at the end of the annuity term.

By definition, the amount of the annuity is the sum of all the accumulated amounts from each payment; therefore,

\[
S = R(1 + i)^{n-1} + R(1 + i)^{n-2} + R(1 + i)^{n-3} + \cdots + R(1 + i) + R \quad (19)
\]

To simplify Eq. (19), multiply each side by \( 1 + i \) and subtract Eq. (19) from the result. This gives

\[
Si = R(1 + i)^n - R \quad (20)
\]

or

\[
S = R \left( \frac{(1 + i)^n - 1}{i} \right) \quad (21)
\]

The term \( \frac{(1 + i)^n - 1}{i} \) is commonly designated as the discrete uniform-series compound-amount factor or the series compound-amount factor.

Continuous Cash Flow and Interest Compounding

The expression for the case of continuous cash flow and interest compounding, equivalent to Eq. (21) for discrete cash flow and interest compounding, is developed as follows:

As before, let \( r \) represent the nominal interest rate with \( m \) conversions or interest periods per year so that \( i = \frac{r}{m} \) and the total number of interest
periods in $n$ years is $mn$. With $m$ annuity payments per year, let $\bar{R}$ represent the total of all ordinary annuity payments occurring regularly and uniformly throughout the year so that $\bar{R}/m$ is the uniform annuity payment at the end of each period. Under these conditions, Eq. (21) becomes

$$S = \frac{\bar{R}}{m} \left[ \frac{1 + (r/m)}{r/m} \right]^{(m/r)(m'n)} - 1$$

(22)

For the case of continuous cash flow and interest compounding, $m$ approaches infinity, and Eq. (22), by use of Eq. (11), becomes

$$S = \bar{R} \left( \frac{e^{rn} - 1}{r} \right)$$

(23)

Present Worth of an Annuity

The present worth of an annuity is defined as the principal which would have to be invested at the present time at compound interest rate $i$ to yield a total amount at the end of the annuity term equal to the amount of the annuity. Let $P$ represent the present worth of an ordinary annuity. Combining Eq. (5) with Eq. (21) gives, for the case of discrete interest compounding,

$$P = \frac{R(1 + i)^n - 1}{i(1 + i)^n}$$

(24)

The expression $[(1 + i)^n - 1]/[i(1 + i)^n]$ is referred to as the discrete uniform-series present-worth factor or the series present-worth factor, while the reciprocal $[i(1 + i)^n]/[(1 + i)^n - 1]$ is often called the capital-recovery factor.

For the case of continuous cash flow and interest compounding, combination of Eqs. (12) and (23) gives the following equation which is analogous to Eq. (24):

$$P = \bar{R} \frac{e^{rn} - 1}{re^{rn}}$$

(25)

Example 5 Application of annuities in determining amount of depreciation with discrete interest compounding. A piece of equipment has an initial installed value of $12,000. It is estimated that its useful life period will be 10 years and its scrap
value at the end of the useful life will be $2000. The depreciation will be charged as a cost by making equal charges each year, the first payment being made at the end of the first year. The depreciation fund will be accumulated at an annual interest rate of 6 percent. At the end of the life period, enough money must have been accumulated to account for the decrease in equipment value. Determine the yearly cost due to depreciation under these conditions.

(Note: This method for determining depreciation is based on an ordinary annuity and is known as the sinking-fund method.)

**Solution.** This problem is a typical case of an ordinary annuity. Over a period of 10 years, equal payments must be made each year at an interest rate of 6 percent. After 10 years, the amount of the annuity must be equal to the total amount of depreciation.

Amount of annuity = S  
Total amount of depreciation = $12,000 - $2000 = $10,000 = S  
Equal payments per year = R = yearly cost due to depreciation  
Number of payments = n = 10  
Annual interest rate = i = 0.06.

From Eq. (21),

\[ R = \frac{S}{(1 + \frac{i}{n})^n - 1} = \frac{10,000}{(1.06)^{10} - 1} = \$759/\text{year} \]

Yearly cost due to depreciation = $759

**Example 6** Application of annuities in determining amount of depreciation with continuous cash flow and interest compounding. Repeat Example 5 with continuous cash flow and nominal annual interest of 6 percent compounded continuously.

Solution. This problem is solved in exactly the same manner as Example 5, except the appropriate Eq. (23) for the continuous-interest case is used in place of the discrete-interest equation.

Amount of annuity = S  
Total amount of depreciation = $12,000 - $2000 = S  
Equal payments per year based on continuous cash flow and interest compounding = \( \bar{R} \) = yearly cost due to depreciation  
Number of years = n = 10  
Nominal interest rate with continuous compounding = r = 0.06

From Eq. (23),

\[ \bar{R} = \frac{r}{e^{rn} - 1} = \frac{10,000}{e^{(0.06)(10)} - 1} = \$730/\text{year} \]

Yearly cost due to depreciation = $730
Special Types of Annuities

One special form of an annuity requires that payments be made at the beginning of each period instead of at the end of each period. This is known as an annuity due. An annuity in which the first payment is due after a definite number of years is called a deferred annuity. Determination of the periodic payments, amount of annuity, or present value for these two types of annuities can be accomplished by methods analogous to those used in the case of ordinary annuities.

PERPETUITIES AND CAPITALIZED COSTS

A perpetuity is an annuity in which the periodic payments continue indefinitely. This type of annuity is of particular interest to engineers, for in some cases they may desire to determine a total cost for a piece of equipment or other asset under conditions which permit the asset to be replaced perpetually without considering inflation or deflation.

Consider the example in which the original cost of a certain piece of equipment is $12,000. The useful-life period is 10 years, and the scrap value at the end of the useful life is $2000. The engineer reasons that this piece of equipment, or its replacement, will be in use for an indefinitely long period of time, and it will be necessary to supply $10,000 every 10 years in order to replace the equipment. He therefore wishes to provide a fund of sufficient size so that it will earn enough interest to pay for the periodic replacement. If the discrete annual interest rate is 6 percent, this fund would need to be $12,650. At 6 percent interest compounded annually, the fund would amount to $(12,650)(1 + 0.06)^n = 22,650 after 10 years. Thus, at the end of 10 years, the equipment can be replaced for $10,000 and $12,650 will remain in the fund. This cycle could now be repeated indefinitely. If the equipment is to perpetuate itself, the theoretical amount of total capital necessary at the start would be $12,000 for the equipment plus $12,650 for the replacement fund. The total capital determined in this manner is called the capitalized cost. Engineers use capitalized costs principally for comparing alternative choices.\footnote{For further discussion of capitalized costs used in engineering, see Chap. 10 and F. C. Jelen and M. S. Cole. Methods for Economic Analysis, Part I, Hydrocarbon Proc., 53(7):133 (1974); Part II, Hydrocarbon Proc., 53(9):227 (1974).}

In a perpetuity, such as in the preceding example, the amount required for the replacement must be earned as compounded interest over a given length of time. Let $P$ be the amount of present principal (i.e., the present worth) which can accumulate to an amount of $S$ during $n$ interest periods at periodic interest rate $i$. Then, by Eq. (5),

$$S = P(1 + i)^n$$  \hspace{1cm} (5)
If perpetuation is to occur, the amount $S$ accumulated after $n$ periods minus the cost for the replacement must equal the present worth $P$. Therefore, letting $C_R$ represent the replacement cost,

$$P = S - C_R,$$  \hspace{1cm} (26)

Combining Eqs. (5) and (26),

$$P = \frac{C_R}{(1 + i)^n - 1}$$  \hspace{1cm} (27)

The capitalized cost is defined as the original cost of the equipment plus the present value of the renewable perpetuity. Designating $K$ as the capitalized cost and $C_V$ as the original cost of the equipment,$^1$

$$K = C_V + \frac{C_R}{(1 + i)^n - 1}$$  \hspace{1cm} (28)

**Example 7** Determination of capitalized cost. A new piece of completely installed equipment costs $12,000 and will have a scrap value of $2000 at the end of its useful life. If the useful-life period is 10 years and the interest is compounded at 6 percent per year, what is the capitalized cost of the equipment?

Solution. The cost for replacement of the equipment at the end of its useful life (assuming costs unchanged) = $12,000 - $2000 = $10,000.

By Eq. (28)

$$\text{Capitalized cost} = C_V + \frac{C_R}{(1 + i)^n - 1}$$

where $C_V = $12,000

$C_R = $10,000

$i = 0.06$

$n = 10$

Capitalized cost = $12,000 + \frac{$10,000}{(1 + 0.06)^{10} - 1}$

= $12,000 + $12,650 = $24,650

**Example 8** Comparison of alternative investments using capitalized costs. A reactor, which will contain corrosive liquids, has been designed. If the reactor is made of mild steel, the initial installed cost will be $5000, and the useful-life period will be 3 years. Since stainless steel is highly resistant to the corrosive action of the liquids, stainless steel, as the material of construction, has been proposed as an alternative to mild steel. The stainless-steel reactor would have an initial installed cost of $15,000. The scrap value at the end of the useful life would be

\[\text{For the continuous-interest-compounding expression equivalent to the discrete-interest-compounding case given in Eq. (28), see Prob. 13 at the end of the chapter.}\]
zero for either type of reactor, and both could be replaced at a cost equal to the original price. On the basis of equal capitalized costs for both types of reactors, what should be the useful-life period for the stainless-steel reactor if money is worth 6 percent compounded annually?

**Solution.** By Eq. (28), the capitalized cost for the mild-steel reactor is

\[
K = C_V + \frac{C_R}{(1 + i)^n} = \frac{5000}{(1 + 0.06)^n - 1}
\]

\[
K = \frac{5000}{31,180} = 162,000
\]

Therefore, the capitalized cost for the stainless-steel reactor must also be $31,180.

For the stainless-steel reactor,

\[
31,180 = C_V + \frac{C_R}{(1 + i)^n} = \frac{15,000}{(1 + 0.06)^n - 1}
\]

Solving algebraically for \( n \),

\[
11.3 \text{ years}
\]

Thus, the useful-life period of the stainless-steel reactor should be 11.3 years for the two types of reactors to have equal capitalized costs. If the stainless-steel reactor would have a useful life of more than 11.3 years, it would be the recommended choice, while the mild-steel reactor would be recommended if the useful life using stainless steel were less than 11.3 years.

**RELATIONSHIPS FOR CONTINUOUS CASH FLOW AND CONTINUOUS INTEREST OF IMPORTANCE FOR PROFITABILITY ANALYSES**

The fundamental relationships dealing with continuous interest compounding can be divided into two general categories: (1) those that involve instantaneous or lump-sum payments, such as a required initial investment or a future payment that must be made at a given time, and (2) those that involve continuous payments or continuous cash flow, such as construction costs distributed evenly over a construction period or regular income that flows constantly into an overall operation. Equation (12) is a typical example of a lump-sum formula, while Eqs. (23) and (25) are typical of continuous-cash-flow formulas.

The symbols \( S, P, \) and \( R \) represent discrete lump-sum payments as future worth, present principal (or present worth), and end-of-period (or end-of-year) payments, respectively. A bar above the symbol, such as \( \bar{S}, \bar{P}, \) or \( \bar{R} \), means that the payments are made **continuously** throughout the time period under consid-
For example, consider the case where construction of a plant requires a continuous flow of cash to the project for one year, with the plant ready for operation at the end of the year of construction. The symbol $\overline{P}$ represents the total amount of cash put into the project on the basis of one year with a continuous flow of cash. At the end of the year, the compound amount of this $\overline{P}$ is

$$S_{\text{at end of one individual year}} = \overline{P} e^r - 1 \overline{r} = P_{\text{at startup}}$$  \hspace{1cm} (29)$$

If the plant is ready for operation after one year of construction time and the startup of the plant is designated as zero time, the future worth of the plant construction cost after $n$ years with continuous interest compounding is

$$S_{\text{after } n \text{ years in operation}} = (P_{\text{at startup}}) e^{rn} = \overline{P} e^r - 1 \overline{r} e^{rn}$$  \hspace{1cm} (30)$$

For profitability analyses, certain discounting or compounding factors based on continuous interest compounding are of sufficient importance that tables have been prepared which give values of the factors for various interest rates and time periods. Table 3 gives examples of tabulated factors for the following cases:

(a) Discount factors to give present worths for cash flows which occur in an instant at a point in time after the reference point. These factors are used to convert one dollar of money, which must be available in an instant after time $n$ (such as scrap value, working capital, or land value), to the present worth of this one dollar with continuous interest compounding. The appropriate equation for calculating the factor, therefore, is based on Eq. (12), and

$$\text{Factor} = 1.0 \frac{1}{e^{rn}} = F_a$$  \hspace{1cm} (31)$$

For example, if the nominal continuous interest rate is 20 percent and the

\footnote{† It should be noted that $\overline{S}$, $\overline{P}$ and $\overline{R}$ represent sums accumulated by continuous payment over an indicated time period without any interest accumulation. $\overline{R}$ represents a periodic accumulation normally based on one year, while $\overline{S}$ and $\overline{P}$ represent accumulations during a given period of time. Thus, $\overline{S}$ and $\overline{P}$ are interchangeable depending on the basic form of equation being used, and $\overline{S}$, $\overline{P}$, and $\overline{R}$ are interchangeable if the time period under consideration is limited to one year or one basic interest period.}

\footnote{‡ See Table 4 for a summary of the significance and meaning of the factors presented in Table 3. Extended values of the factors for parts (a) to (d) of Table 3 are given in Tables 5 to 8.}

\footnote{§ For illustrations of the applications of continuous interest compounding and continuous cash flow to cases of profitability evaluation, see Examples 2 and 3 in Chap. 10.}
TABLE 3
Discount and compounding factors for continuous interest and cash flows?

<table>
<thead>
<tr>
<th>r as percent</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>80%</th>
<th>100%</th>
</tr>
</thead>
</table>

Discount factors to give present worths for cash flows which

(a) Occur in an instant at a point in time after the reference point

\[ n = 1 \]

<table>
<thead>
<tr>
<th>P.W.</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
<th>( n = 4 )</th>
<th>( n = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.0 \left( \frac{1}{e^{r n}} \right) = F_a )</td>
<td>( 0.990 )</td>
<td>( 0.951 )</td>
<td>( 0.905 )</td>
<td>( 0.861 )</td>
<td>( 0.819 )</td>
</tr>
<tr>
<td>( 0.973 )</td>
<td>( 0.933 )</td>
<td>( 0.887 )</td>
<td>( 0.840 )</td>
<td>( 0.792 )</td>
<td>( 0.741 )</td>
</tr>
<tr>
<td>( 0.961 )</td>
<td>( 0.912 )</td>
<td>( 0.861 )</td>
<td>( 0.810 )</td>
<td>( 0.759 )</td>
<td>( 0.710 )</td>
</tr>
<tr>
<td>( 0.951 )</td>
<td>( 0.901 )</td>
<td>( 0.852 )</td>
<td>( 0.802 )</td>
<td>( 0.751 )</td>
<td>( 0.702 )</td>
</tr>
</tbody>
</table>

(b) Occur uniformly over one-year periods after the reference point

\[ n = 1 \]

<table>
<thead>
<tr>
<th>P.W.</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 3 )</th>
<th>( n = 4 )</th>
<th>( n = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.0 \left( \frac{e^{r n} - 1}{r} \right) e^{-r n} = F_b )</td>
<td>( 0.995 )</td>
<td>( 0.975 )</td>
<td>( 0.952 )</td>
<td>( 0.929 )</td>
<td>( 0.906 )</td>
</tr>
<tr>
<td>( 0.985 )</td>
<td>( 0.965 )</td>
<td>( 0.943 )</td>
<td>( 0.921 )</td>
<td>( 0.900 )</td>
<td>( 0.880 )</td>
</tr>
<tr>
<td>( 0.975 )</td>
<td>( 0.955 )</td>
<td>( 0.935 )</td>
<td>( 0.915 )</td>
<td>( 0.895 )</td>
<td>( 0.876 )</td>
</tr>
<tr>
<td>( 0.966 )</td>
<td>( 0.946 )</td>
<td>( 0.925 )</td>
<td>( 0.905 )</td>
<td>( 0.885 )</td>
<td>( 0.866 )</td>
</tr>
<tr>
<td>( 0.956 )</td>
<td>( 0.936 )</td>
<td>( 0.915 )</td>
<td>( 0.895 )</td>
<td>( 0.875 )</td>
<td>( 0.855 )</td>
</tr>
</tbody>
</table>

(c) Occur uniformly over a period of years-For period of years = \( T = 5 \) years

\[ n = 5 \]

<table>
<thead>
<tr>
<th>P.W.</th>
<th>( n = 5 )</th>
<th>( n = 10 )</th>
<th>( n = 15 )</th>
<th>( n = 20 )</th>
<th>( n = 25 )</th>
<th>( n = 30 )</th>
<th>( n = 40 )</th>
<th>( n = 50 )</th>
<th>( n = 60 )</th>
<th>( n = 80 )</th>
<th>( n = 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.0 \left( \frac{e^{r T} - 1}{r} \right) e^{-r n} = F_c )</td>
<td>( 0.975 )</td>
<td>( 0.885 )</td>
<td>( 0.797 )</td>
<td>( 0.709 )</td>
<td>( 0.621 )</td>
<td>( 0.533 )</td>
<td>( 0.445 )</td>
<td>( 0.357 )</td>
<td>( 0.270 )</td>
<td>( 0.182 )</td>
<td>( 0.095 )</td>
</tr>
<tr>
<td>( 0.963 )</td>
<td>( 0.873 )</td>
<td>( 0.784 )</td>
<td>( 0.695 )</td>
<td>( 0.606 )</td>
<td>( 0.517 )</td>
<td>( 0.428 )</td>
<td>( 0.339 )</td>
<td>( 0.250 )</td>
<td>( 0.161 )</td>
<td>( 0.072 )</td>
<td></td>
</tr>
</tbody>
</table>
| \( 0.951 \) | \( 0.861 \) | \( 0.773 \) | \( 0.684 \) | \( 0.595 \) | \( 0.506 \) | \( 0.417 \) | \( 0.328 \) | \( 0.239 \) | \( 0.150 \) | \( 0.061 \) | (Continued)
<table>
<thead>
<tr>
<th>( r ) as percent</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>80%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d) Decline to zero at a constant rate over a period of years starting with the reference point</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st 5 years</td>
<td>0.983</td>
<td>0.922</td>
<td>0.852</td>
<td>0.791</td>
<td>0.736</td>
<td>0.687</td>
<td>0.643</td>
<td>0.606</td>
<td>0.568</td>
<td>0.506</td>
<td>0.456</td>
<td>0.377</td>
</tr>
<tr>
<td>1st 10 years</td>
<td>0.968</td>
<td>0.852</td>
<td>0.736</td>
<td>0.643</td>
<td>0.568</td>
<td>0.506</td>
<td>0.456</td>
<td>0.377</td>
<td>0.320</td>
<td>0.278</td>
<td>0.219</td>
<td>0.180</td>
</tr>
<tr>
<td>1st 15 years</td>
<td>0.952</td>
<td>0.791</td>
<td>0.643</td>
<td>0.536</td>
<td>0.456</td>
<td>0.394</td>
<td>0.347</td>
<td>0.278</td>
<td>0.231</td>
<td>0.198</td>
<td>0.153</td>
<td>0.124</td>
</tr>
<tr>
<td>1st 20 years</td>
<td>0.936</td>
<td>0.736</td>
<td>0.568</td>
<td>0.456</td>
<td>0.377</td>
<td>0.320</td>
<td>0.278</td>
<td>0.219</td>
<td>0.180</td>
<td>0.153</td>
<td>0.117</td>
<td>0.095</td>
</tr>
<tr>
<td>1st 25 years</td>
<td>0.922</td>
<td>0.687</td>
<td>0.506</td>
<td>0.394</td>
<td>0.320</td>
<td>0.269</td>
<td>0.231</td>
<td>0.180</td>
<td>0.147</td>
<td>0.124</td>
<td>0.095</td>
<td>0.077</td>
</tr>
</tbody>
</table>

Compounding factors to give future worths for cash flows which

(e) Occur in an instant at a point in time before the reference point

\[
\frac{2}{r n r} \left[1 - (1 - e^{-r n r})/r n r\right] = F_d
\]

<table>
<thead>
<tr>
<th></th>
<th>1/2 year before</th>
<th>1 year before</th>
<th>1 1/2 years before</th>
<th>2 years before</th>
<th>3 years before</th>
</tr>
</thead>
<tbody>
<tr>
<td>F.W.</td>
<td>1.005</td>
<td>1.010</td>
<td>1.015</td>
<td>1.020</td>
<td>1.030</td>
</tr>
<tr>
<td>1.0(e^m) = C_t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(f) Occur uniformly before the reference point

\[
\frac{1}{T} \left(e^r T - 1\right) = C_f
\]

<table>
<thead>
<tr>
<th></th>
<th>1/2 year before</th>
<th>1 year before</th>
<th>1 1/2 years before</th>
<th>2 years before</th>
<th>3 years before</th>
</tr>
</thead>
<tbody>
<tr>
<td>F.W.</td>
<td>1.002</td>
<td>1.005</td>
<td>1.008</td>
<td>1.010</td>
<td>1.015</td>
</tr>
<tr>
<td>1.0(e^T - 1)/r = C_f</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( r \) = nominal interest compounded continuously, percent/100; \( n \) = number of years; \( T \) and \( n_T \) = number of years in a time period.

See Table 4 for significance and meaning of compounding factors. Extended values of factors for parts (a), (b), and (d) are given in Tables 5, 6, and 8, and Table 7 gives extended values for part (c) with \( n = T \).
TABLE 4
Summary of significance and meaning of discount and compounding factors presented in Tables 3, 5, 6, 7, and 8†

As indicated in the footnote for Eq. (12), the common interest expressions can be written in simplified form by using discount-factor and compound-interest-factor notation. Following is a summary showing the significance and meaning of the compounding factors presented in Table 3. Derivations of the factors are presented in the text.

For part (a) in Table 3 and for Table 5

\[ F_a = \text{Discount factor to give present worth for cash flows which occur in an instant at a point in time after the reference point.} \]

\[ P = F_a S \quad F_a = e^{-rn} \]

For part (b) in Table 3 and for Table 6

\[ F_b = \text{Discount factor to give present worth for cash flows which occur uniformly over one-year periods after the reference point. } (S \text{ is the total cash flow for the } n \text{th year.)} \]

\[ P = F_b S_{(n\text{th year})} \quad F_b = \left( \frac{e^r - 1}{r} \right) e^{-rn} \]

For part (c) in Table 3 and for Table 7 with \( n = T \)

\[ F_c = \text{Discount factor to give present worth for cash flows which occur uniformly over a period of years } T. \ (S \text{ is the total cash flow for the } T\text{-year period.)} \]

\[ P = F_c S_{(T\text{-year period})} \quad F_c = \frac{1}{T} \left( \frac{e^{rT} - 1}{r} \right) e^{-rn} \]

**Note:** For the case when the period of years \( T \) is based on the period immediately after the reference point, \( n = T \), and \( F_c = (1 - e^{-rT})/rT \). This is the factor presented in Table 7.

For part (d) in Table 3 and for Table 8

\[ F_d = \text{Discount factor to give present worth for cash flows which decline to zero at a constant rate over a period of years } nT \text{ starting with the reference point. } (S \text{ is the total cash flow for the } nT\text{-year period.)} \]

\[ P = F_d S_{(\text{declining to zero at constant rate in } nT \text{ years})} \quad F_d = \frac{2}{rnT} \left( 1 - \frac{1 - e^{-rnT}}{rnT} \right) \]

For part (e) in Table 3

\[ C_s = \text{Compounding factor to give future worth for cash flows which occur in an instant at a point in time before the reference point.} \]

\[ s = C_s P \quad C_s = e^{rn} = \frac{1}{F_a} \]

**Note:** Table 6 gives reciprocal values of \( C_s \).

(Continued)
TABLE 4
Summary of significance and meaning of discount and compounding factors presented in Tables 3, 5, 6, 7, and 8† (Continued)

For part (f) in Table 3

\[ C_f = \text{Compounding factor to give future worth for cash flows which occur uniformly over a period of years } T \text{ before the reference point.} \]

\[ S = C_f \bar{P} \quad c_r = \frac{e^{rT} - 1}{rT} \]

For \( n = T \), \( C_f = \frac{F_e}{F_a} = F_eC_e \).

**Note:** On basis of above relationship, \( C_f \) can be generated from \( F_a \) and \( F_e \) values given in Tables 5 and 7.

† \( r = \text{nominal interest compounded continuously, percent}/100; n = \text{number of years; } T \text{ and } nT = \text{number of years in a time period.} \)

time period is 5 years, the appropriate factor, as shown in Table 3, is

\[ \text{Factor} = \frac{1}{e^{(0.2)(5)}} = \frac{1}{2.7183} = 0.368 \]

(b) Discount factors to give present worths for cash flows which occur uniformly over one-year periods after the reference point. For this situation, the factor would convert one dollar of money, as the total yearly amount flowing continuously and uniformly during the year (such as cash receipts for one year), to the present worth of this one dollar at zero time with continuous interest compounding. Thus, \( \bar{R} \) (or \( \bar{s} \)) for the year in question is 1.0, and the appropriate equation for calculating the factor, based on Eqs. (23) and (12), is

\[ \text{Factor} = 1 \cdot 0 \cdot e^{-r} = F_b \]  

As an example, if \( r \) represents 20 percent and \( n \) is the fifth year, the appropriate factor, as shown in Table 3, is

\[ \text{Factor} = \frac{e^{(0.2)}}{0.2} = \frac{1}{2.7183} = 0.368 \]

(c) Discount factors to give present worths for cash flows which occur uniformly over a period of years. For this situation, a total amount of one dollar over a given time period is used as the basis. The cash flows continuously and uniformly during the entire period, and the factor converts the total of one dollar put in over the given time period to the present worth at zero time. This condition would be applicable to a case where cash receipts are steady over a given period of time, such as for five years. Designating \( T \) as the time
period involved, the total amount put in each year is $\$1/T$, and the factor, based on Eqs. (23) and (12), is

\[
\text{Factor} = \frac{1}{T} e^{rT} - \frac{1}{r} e^{-rn} = F_c
\]  

(33)

For example, if the time period involved is the second five years (i.e., the 6th through the 10th years) and $r$ represents 20 percent, the appropriate factor, as shown in Table 3, is

\[
\text{Factor} = \frac{1}{5} \left( \frac{e^{0.2(5)} - 1}{0.2} \right) \left( \frac{1}{e^{0.2(10)}} \right) = \frac{1}{5} \left( \frac{2.7183 - 1}{0.2} \right) \frac{1}{7.3891} = 0.232
\]

(d) **Discount factors to give present worths for cash flows declining to zero at a constant rate over a period of years starting with the reference point.** For this case, the assumption is made that the continuous cash flow declines linearly with time from the initial flow at time zero to zero flow at time $n_T$. A situation similar to this exists when the sum-of-the-years-digits method is used for calculating depreciation in that depreciation allowances decline linearly with time from a set value in the first year to zero at the end of the life.†‡ For the case of continuous cash flow declining to zero at a constant rate over a time period of $n_T$, the linear equation for $\bar{R}$ is

\[
\bar{R} = a - gn
\]  

(34)

where $g$ = the constant declining rate or the gradient  
$\bar{R}$ = instantaneous value of the cash flow  
a = a constant

The discount factor is based on a total amount of one dollar of cash flow over the time period $n_T$ and converts this total of one dollar to the present worth at time zero. Under these conditions, $g$ equals $2/(n_T)^2$,§ and the

---

†See Chap. 9 (Depreciation) for information on the sum-of-the-years-digits method for calculating depreciation.

‡Equation (35) does not represent a true sum-of-the-years-digits factor. Normally, the constant declining rate or gradient for the sum-of-the-years-digits method of depreciation is $1/\Sigma_i^m n = 2/n_T(n_T + 1)$. For the true case of continuous cash flow declining to zero at a constant rate, $n_T$ is replaced by $n_T m$ as $m \rightarrow \infty$, and the constant gradient becomes $2/(n_T)^2$.

§By definition of terms and conditions. $\bar{R}$ is zero when $n = n_T$ and $\bar{R}$ is a when $n = 0$. Also, if a total of one dollar is the cash flow during $n_T$

\[
\int_0^{n_T} \bar{R} dn = an_T - \frac{g(n_T)^2}{2} = 10
\]

Because $\bar{R}$ is zero when $n = n_T$, $a = gn_T$. Therefore,

\[
\$1.0 = g(n_T)^2 - \frac{g(n_T)^2}{2} = \frac{g(n_T)^2}{2} \quad \text{and} \quad g = \frac{2}{(n_T)^2}
\]
As an example, if the cash flow declines at a **constant rate to zero** in 5 years and \( r \) is equivalent to 20 percent, the appropriate factor, as shown in Table 3, is

\[
\text{Factor} = \frac{2}{(0.2)(5)} \left[ 1 - \frac{1}{(0.2)(5)} \left( 1 - \frac{1}{e^{(0.2)(5)}} \right) \right]
\]

\[
= \left( 1 - \frac{1}{2.7183} \right) = 0.736
\]

(e) **Compounding factors to give future worths for cash flows which occur in an instant at a point in time before the reference point.** These factors merely show the future worth to which one dollar of principal, such as that for land purchase, will compound at continuous interest. Based on Eq. (12), the factor is

\[
\text{Factor} = 1.0e^rT = C_e
\]

For example, with \( r \) equivalent to 20 percent and a purchase made \( 1 \frac{1}{2} \) years before the reference point, the appropriate factor, as shown in Table 3, is

\[
\text{Factor} = e^{(0.2)(1.5)} = 1.350
\]

(f) **Compounding factors to give future worths for cash flows which occur uniformly before the reference point.** The basis for these factors is a uniform and continuous flow of cash amounting to a total of one dollar during the given time period of \( T \) years, such as for construction of a plant. The factor converts this one dollar to the future worth at the reference time and is based on Eq. (23).

\[
\text{Factor} = \frac{1.0}{T} \frac{e^{rT} - 1}{r} = C_f
\]

As an example, for the case of continuous compounding at \( r \) equivalent to 20 percent for a period from 3 years before the reference time, the appropriate factor, as shown in Table 3, is

\[
\text{Factor} = 3 \left( \frac{1}{2} \right) e^{(0.2)(3)} - 1 = \frac{1.8221}{(3)(0.2)} = 1.370
\]

\[\text{This can be derived by assuming an ordinary annuity with } g_nT \text{ for the first year, } g(nT - 1) \text{ for the second year, etc., to } g \text{ for year } nT. \text{ The result with discrete interest compounding is}
\]

\[
\text{Factor} = g \left[ \frac{\ln(1 + (1 + i)^{-nT})}{i^2} \right]
\]

\[\text{Replacing } i \text{ by } r/m \text{ and } nT \text{ by } mnT \text{ gives Eq. (35) for } m \to \infty.\]
TABLE 5
Discount factors \((F_a)\) with continuous interest to give present worths for cash flows which occur in an instant at a point in time after the reference point†‡

<table>
<thead>
<tr>
<th>100n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000</td>
<td>0.9991</td>
<td>0.9982</td>
<td>0.9973</td>
<td>0.9964</td>
<td>0.9955</td>
<td>0.9946</td>
<td>0.9937</td>
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</tr>
<tr>
<td>10</td>
<td>0.9048</td>
<td>0.8995</td>
<td>0.8943</td>
<td>0.8891</td>
<td>0.8839</td>
<td>0.8787</td>
<td>0.8736</td>
<td>0.8685</td>
<td>0.8634</td>
<td>0.8583</td>
</tr>
<tr>
<td>20</td>
<td>0.8187</td>
<td>0.8160</td>
<td>0.8035</td>
<td>0.7910</td>
<td>0.7786</td>
<td>0.7662</td>
<td>0.7538</td>
<td>0.7415</td>
<td>0.7292</td>
<td>0.7169</td>
</tr>
<tr>
<td>30</td>
<td>0.7408</td>
<td>0.7334</td>
<td>0.7261</td>
<td>0.7189</td>
<td>0.7118</td>
<td>0.7047</td>
<td>0.6976</td>
<td>0.6907</td>
<td>0.6839</td>
<td>0.6771</td>
</tr>
<tr>
<td>40</td>
<td>0.6730</td>
<td>0.6637</td>
<td>0.6544</td>
<td>0.6452</td>
<td>0.6360</td>
<td>0.6269</td>
<td>0.6178</td>
<td>0.6088</td>
<td>0.5999</td>
<td>0.5911</td>
</tr>
</tbody>
</table>

(Continued)
**TABLE 5**

Discount factors \((F_a)\) with continuous interest to give present worths for cash flows which occur in an instant at a point in time after the reference point†‡

(Continued)

<table>
<thead>
<tr>
<th>(100r)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.01830</td>
<td>0.01810</td>
<td>0.01790</td>
<td>0.01770</td>
<td>0.01760</td>
<td>0.01750</td>
<td>0.01740</td>
<td>0.01730</td>
<td>0.01720</td>
<td>0.01710</td>
</tr>
<tr>
<td>410</td>
<td>0.01850</td>
<td>0.01830</td>
<td>0.01810</td>
<td>0.01790</td>
<td>0.01770</td>
<td>0.01760</td>
<td>0.01750</td>
<td>0.01740</td>
<td>0.01730</td>
<td>0.01720</td>
</tr>
<tr>
<td>420</td>
<td>0.01870</td>
<td>0.01850</td>
<td>0.01830</td>
<td>0.01810</td>
<td>0.01790</td>
<td>0.01770</td>
<td>0.01760</td>
<td>0.01750</td>
<td>0.01740</td>
<td>0.01730</td>
</tr>
<tr>
<td>430</td>
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<td>0.01870</td>
<td>0.01850</td>
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<td>0.01810</td>
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<td>0.01740</td>
</tr>
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<td>440</td>
<td>0.01910</td>
<td>0.01890</td>
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<td>0.01810</td>
<td>0.01790</td>
<td>0.01770</td>
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<td>0.01750</td>
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<td>0.01930</td>
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<td>0.01870</td>
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<td>0.02200</td>
<td>0.02180</td>
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<td>0.02240</td>
<td>0.02220</td>
<td>0.02200</td>
<td>0.02180</td>
</tr>
</tbody>
</table>

† 𝑟 = nominal interest compounded continuously, percent/100; 𝑛 = number of years.
‡ See Tables 3 and 4 for information on 𝐹. 𝑎.

**TABLES FOR INTEREST AND CASH-FLOW FACTORS**

Tables of interest and cash-flow factors, such as are illustrated in Tables 1, 5, 6, 7, and 8 of this chapter, are presented in all standard interest handbooks and textbooks on the mathematics of finance as well as in appendices of most textbooks on engineering economy. Exponential functions for continuous compounding are available in the standard mathematical tables. The development of tables for any of the specialized factors is a relatively simple matter with the ready availability of digital computers, as is illustrated in Example 3 of this chapter.

The end-of-year convention is normally adopted for discrete interest factors (or for lump-sum payments) wherein the time unit of one interest period is assumed to be one year with interest compounding (or with lump-sum payments being made) at the end of each period. Thus, the effective interest rate is the form of interest most commonly understood and used by management and business executives.

In the tabulation of factors for continuous interest compounding and continuous cash flow, the nominal interest rate 𝑟 is used for calculating the
<table>
<thead>
<tr>
<th>Year</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
<th>7%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
<th>11%</th>
<th>12%</th>
<th>13%</th>
<th>14%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9756</td>
<td>0.9710</td>
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<td>0.9580</td>
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<td>0.8764</td>
<td>0.8555</td>
<td>0.8352</td>
<td>0.8156</td>
<td>0.7968</td>
<td>0.7788</td>
<td>0.7615</td>
<td>0.7450</td>
<td>0.7292</td>
<td>0.7139</td>
<td>0.7002</td>
<td>0.6872</td>
</tr>
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<td>0.8979</td>
<td>0.8764</td>
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<td>0.8156</td>
<td>0.7968</td>
<td>0.7788</td>
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<td>0.7788</td>
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<td>0.6542</td>
<td>0.6453</td>
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</table>

†Percent is nominal interest compounded continuously x 100. Year indicates one-year period in which cash flow occurs. See Tables 3 and 4 for information on $F_o$. 

**TABLE 6**

Discount factors $F_o$ with continuous interest to give present worths for cash flows which occur uniformly over one-year periods after the reference point.
### TABLE 7

Discount factors \((F_r)\) with continuous interest to give present worths for cash flows which occur uniformly over a period of \(T\) years after the reference point:

<table>
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<tr>
<th>100(T)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>0.9377</td>
<td>0.9332</td>
<td>0.9286</td>
<td>0.9241</td>
<td>0.9196</td>
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<td>0.6401</td>
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</table>

(Continued)
TABLE 7
Discount factors ($F_t$) with continuous interest to give present worths for cash flows which occur uniformly over a period of $T$ years after the reference point†‡ (Continued )

| $100rT$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|---------|---|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
| 400     | 0.24540 | 0.24490 | 0.24430 | 0.24370 | 0.24320 | 0.24260 | 0.24210 | 0.24150 | 0.24100 | 0.24040 |
| 410     | 0.23990 | 0.23930 | 0.23880 | 0.23820 | 0.23770 | 0.23720 | 0.23660 | 0.23610 | 0.23560 | 0.23500 |
| 420     | 0.23450 | 0.23400 | 0.23350 | 0.23300 | 0.23250 | 0.23200 | 0.23140 | 0.23090 | 0.23040 | 0.22990 |
| 430     | 0.22940 | 0.22890 | 0.22840 | 0.22790 | 0.22740 | 0.22690 | 0.22640 | 0.22590 | 0.22550 | 0.22500 |
| 440     | 0.22450 | 0.22400 | 0.22350 | 0.22300 | 0.22250 | 0.22200 | 0.22150 | 0.22100 | 0.22050 | 0.22000 |
| 450     | 0.21980 | 0.21930 | 0.21880 | 0.21840 | 0.21800 | 0.21750 | 0.21700 | 0.21650 | 0.21600 | 0.21550 |
| 460     | 0.21520 | 0.21480 | 0.21430 | 0.21390 | 0.21340 | 0.21300 | 0.21250 | 0.21200 | 0.21150 | 0.21100 |
| 470     | 0.21080 | 0.21040 | 0.21000 | 0.20960 | 0.20920 | 0.20870 | 0.20830 | 0.20790 | 0.20740 | 0.20700 |
| 480     | 0.20650 | 0.20620 | 0.20580 | 0.20540 | 0.20500 | 0.20460 | 0.20420 | 0.20380 | 0.20340 | 0.20300 |
| 490     | 0.20260 | 0.20220 | 0.20180 | 0.20140 | 0.20100 | 0.20060 | 0.20020 | 0.19980 | 0.19940 | 0.19900 |
| 500     | 0.19870 | 0.19490 | 0.19120 | 0.18770 | 0.18430 | 0.18100 | 0.17770 | 0.17490 | 0.17190 | 0.16900 |
| 600     | 0.16630 | 0.16360 | 0.16100 | 0.15840 | 0.15600 | 0.15360 | 0.15130 | 0.14910 | 0.14690 | 0.14480 |
| 700     | 0.14270 | 0.14070 | 0.13880 | 0.13690 | 0.13500 | 0.13330 | 0.13150 | 0.12980 | 0.12820 | 0.12650 |
| 800     | 0.12500 | 0.12340 | 0.12190 | 0.12060 | 0.11920 | 0.11790 | 0.11660 | 0.11530 | 0.11490 | 0.11360 |
| 900     | 0.11110 | 0.10990 | 0.10870 | 0.10750 | 0.10640 | 0.10530 | 0.10430 | 0.10330 | 0.10230 | 0.10130 |
| 1000    | 0.10000 | 0.09900 | 0.09800 | 0.09710 | 0.09620 | 0.09520 | 0.09430 | 0.09350 | 0.09260 | 0.09170 |
| 1100    | 0.09090 | 0.09010 | 0.08930 | 0.08850 | 0.08770 | 0.08690 | 0.08610 | 0.08530 | 0.08450 | 0.08370 |
| 1200    | 0.08330 | 0.08260 | 0.08200 | 0.08130 | 0.08060 | 0.08000 | 0.07940 | 0.07870 | 0.07810 | 0.07750 |
| 1300    | 0.07690 | 0.07630 | 0.07580 | 0.07520 | 0.07460 | 0.07400 | 0.07350 | 0.07300 | 0.07250 | 0.07200 |
| 1400    | 0.07140 | 0.07090 | 0.07040 | 0.06990 | 0.06940 | 0.06890 | 0.06850 | 0.06800 | 0.06760 | 0.06710 |
| 1500    | 0.06670 | 0.06620 | 0.06580 | 0.06540 | 0.06490 | 0.06450 | 0.06410 | 0.06370 | 0.06330 | 0.06290 |
| 1600    | 0.06250 | 0.06210 | 0.06170 | 0.06130 | 0.06100 | 0.06060 | 0.06020 | 0.05990 | 0.05950 | 0.05920 |
| 1700    | 0.05880 | 0.05850 | 0.05810 | 0.05780 | 0.05750 | 0.05710 | 0.05680 | 0.05650 | 0.05620 | 0.05590 |
| 1800    | 0.05560 | 0.05520 | 0.05490 | 0.05460 | 0.05430 | 0.05410 | 0.05380 | 0.05350 | 0.05320 | 0.05290 |
| 1900    | 0.05260 | 0.05240 | 0.05210 | 0.05180 | 0.05150 | 0.05130 | 0.05100 | 0.05080 | 0.05050 | 0.05020 |
| 2000    | 0.05000 | 0.04970 | 0.04950 | 0.04920 | 0.04900 | 0.04870 | 0.04850 | 0.04830 | 0.04810 | 0.04780 |

† $r = \text{nominal interest compounded continuously, percent}/100$; $T = n = \text{number of years in time period}$. See Tables 3 and 4 for information on $F_t$.
‡ The columns represent the unit increments from 1 to 9 or 10 to 90 for the intervals of $100rT$ shown in the left column.

Factors, but the tables are sometimes based on the effective interest rate. To avoid confusion between effective and nominal interest rates, the tables should always present a clear statement in the heading as to the type of interest basis used if there is any possibility for misunderstanding. In case such a necessary statement is not included with the continuous-interest table, the interest figures quoted are probably nominal, but it is advisable to check several of the factors by use of exponential tables to make certain that nominal, rather than effective, interest rates are quoted.
INTEREST AND INVESTMENT COSTS

TABLE 8
Discount factors (F,,) with continuous interest to give present worths

for cash

flows which decline to zero at a constant rate over a period of years ?zT
starting with the reference point?.+
0
10
20
30
40

0
1
2
3
4 '5
6
7
8
9
~--~__-___
1.00000.99670.99340.99010.98680.98350.98030.97710.97390.9707
0.96750.96430.96120.95800.95490.95180.94870.94570.94260.9396
0.93650.93350.93050.92750.92460.92160.91870.91580.91290.9100
0.90710.90420.90130.89850.89570.89290.89010.88730.88450.8818
0.87900.87630.87360.87080.86820.86550.86280.86020.85750.8549

50
60
70
80
90

0.85230.84970.84710.84450.84190.83940.83680.83430.83170.8292
0.82670.82420.82180.81930.81690.81440.81200.80960.80720.8048
0.80240.80000.79760.79530.79300.79060.78830.78600.78370.7814
0.77910.77690.77460.77240.77010.76790.76570.76350.76130.7591
0.75700.75480.75270.75050.74840.74620.74410.74200.73990.7378

100
110
120
130
140

0.73580.73370.73160.72950.72750.72550.72350.72150.71950.7175
0.71550.71350.71150.70950.70760.70570.70370.70180.69990.6980
0.69610.69420.69230.69040.68850.68670.68480.68300.68120.6794
0.67760.67580.67400.67220.67040.66860.66680.66500.66320.6615
0.65980.65800.65630.65460.65290.65120.64950.64780.64610.6444

150
160
170
180
190

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380
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0.38400.38330.38260.38190.38120.38050.37980.37910.37850.3779
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TABLE 8
Discount factors \( (F_d) \) with continuous interest to give present worths for cash flows which decline to zero at a constant rate over a period of years \( n_T \) starting with the reference point†‡ (Continued)

<table>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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† \( r \) = nominal interest compounded continuously, percent/\( 100 \); \( n_T \) = number of years in time period for cash flow to decrease to zero. See Tables 3 and 4 for information on \( F_d \).
‡ The columns represent the unit increments from 1 to 9 or 10 to 90 for the intervals of \( 100r n_T \) shown in the left column.

COSTS DUE TO INTEREST ON INVESTMENT

Money, or any other negotiable type of capital, has a time value. When a business concern invests money, it expects to receive a return during the time the money is tied up in the investment. The amount of return demanded usually is related to the degree of risk that the entire investment might be lost.

One of the duties of a design engineer is to determine the net return or profit which can be obtained by making an investment. It is necessary, therefore,
to find the total cost involved. Too often, the engineer fails to recognize the time value of money and neglects the effects of interest on cost. According to the modern definition of interest, the cost due to time value of an investment should be included as interest for that portion of the total capital investment which comes from outside sources.

**Borrowed Capital versus Owned Capital**

The question sometimes arises as to whether interest on owned capital can be charged as a true cost. The modern definition of interest permits a definite answer of “no” to this question. Court decisions and income-tax regulations verify this answer.

**Interest Effects in a Small Business**

In small business establishments, it is usually quite easy to determine the exact source of all capital. Therefore, the interest costs can be obtained with little difficulty. For example, suppose that a young chemical engineer has $20,000 and decides to set up a small plant for producing antifreeze from available raw materials. For a working-capital plus fixed-capital investment of $20,000, the chemical engineer determines that the proposed plant can provide a total yearly profit of $8000 before income taxes. Since the investment was a personal one, interest obviously could not be included as a cost. If it has been necessary to borrow the $20,000 at an annual interest rate of 10 percent, interest would have been a cost, and the total profit would have been $8000 \(- (0.10) \times (\$20,000) = \$6000\) per year.

**Interest Effects in a Large Business**

In large business establishments, new capital may come from issue of stocks and bonds, borrowing from banks or insurance companies, funds set aside for

<table>
<thead>
<tr>
<th>Source of capital</th>
<th>Approximate amount of total new capital, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>External financing (loans from banks or other concerns, issue of stocks and bonds)</td>
<td>25</td>
</tr>
<tr>
<td>Profits earned but not distributed to stockholders as dividends</td>
<td>30</td>
</tr>
<tr>
<td>Depreciation funds set aside</td>
<td>25</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>20</td>
</tr>
</tbody>
</table>
replacement of worn-out or obsolete equipment, profits received but not distributed to the stockholders, and other sources. Therefore, it is often difficult to designate the exact source of new capital, and the particular basis used for determining interest costs should be indicated when the results of a cost analysis are reported.

An approximate breakdown showing the various sources of new capital for large corporations typical of the chemical industry is presented in Table 9.

**SOURCE OF CAPITAL**

One source of new capital is outside loans. Interest on such loans is usually at a fixed rate, and the annual cost can be determined directly.

New capital may also be obtained from the issue of bonds, preferred stock, or common stock. Interest on bonds and preferred-stock dividends must be paid at fixed rates. A relatively low interest rate is paid on bonds because the bond-holder has first claim on earnings, while higher rates are paid on preferred stock because the holder has a greater chance to lose the entire investment. The holder of common stock accepts all the risks involved in owning a business. The return on common stock, therefore, is not at a fixed rate but varies depending on the success of the company which issued the stock. To compensate for this greater risk, the return on common stock may be much higher than that on bonds or preferred stock.

**Income-Tax Effects**

The effect of high income-tax rates on the cost of capital is very important. In determining income taxes, interest on loans and bonds can be considered as a cost, while the return on both preferred and common stock cannot be included as a cost. Since corporate income taxes can amount to more than half of the gross earnings, the source of new capital may have a considerable influence on the net profits.

If the annual income-tax rate for a company is 34 percent, every dollar spent for interest on loans or bonds would have a true cost after taxes of only 66 cents. Thus, after income taxes are taken into consideration, a bond issued at an annual interest rate of 6 percent would actually have an interest rate of only $6 \times \frac{66}{100} = 4.0$ percent. On the other hand, the dividends on preferred stock must be paid from net profits after taxes. If preferred stock has an annual dividend rate of 7 percent, the equivalent rate before taxes would be $7 \times \frac{100}{66} = 10.6$ percent.

Despite the fact that it may be cheaper to use borrowed capital in place of other types of capital, it is unrealistic to finance each new venture by using borrowed capital. Every corporation needs to maintain a balanced capital structure and is therefore hesitant about placing itself under a heavy burden of debt.

A comparison of interest or dividend rates for different types of externally financed capital is presented in Table 10.
TABLE 10
Typical costs for externally financed capital

Income tax rate = 34% of (total income - total pretax cost)

<table>
<thead>
<tr>
<th>Source of capital</th>
<th>Indicated interest or dividend rate, % / year</th>
<th>Actual interest or dividend rate before taxes, % / year</th>
<th>Actual interest or dividend rate after taxes, % / year</th>
</tr>
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<tr>
<td>Common stock</td>
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<td>13.6</td>
<td>9</td>
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</table>

METHODS FOR INCLUDING COST OF CAPITAL IN ECONOMIC ANALYSES

The cost of new capital obtained from bonds, loans, or preferred stock can be determined directly from the stated interest or dividend rate, adjusted for income taxes. However, the cost of new capital obtained from the issue of common stock is not so obvious, and some basis must be set for determining this cost. Probably the fairest basis is to consider the viewpoint of existing holders of common stock. If new common stock is issued, its percent return should be at least as much as that obtained from the old common stock; otherwise, the existing stockholders would receive a lower return after the issue of the new stock. Therefore, from the viewpoint of the existing stockholders, the cost of new common stock is the present rate of common-stock earnings.

A major source of new capital is from internal capital, including, primarily, undistributed profits and depreciation funds. Since this definitely is owned capital, it is not necessary to consider interest as a cost. However, some concerns prefer to assign a cost to this type of capital, particularly if comparisons of alternative investments are to be made. The reasoning here is that the owned capital could be loaned out or put into other ventures to give a definite return.

Two methods are commonly used for determining the cost of owned capital. In the first method, the capital is charged at a low interest rate on the assumption that it could be used to pay off funded debts or invest in risk-free loans. The second method requires interest to be paid on the owned capital at a rate equal to the present return on all the company’s capital.

Design-Engineering Practice for Interest and Investment Costs

Many alternative methods are used by engineers when determining interest costs in an economic analysis of a design project. In preliminary designs, one of the following two methods is usually employed:

1. No interest costs are included. This assumes that all the necessary capital comes from owned capital, and any comparisons to alternative investments must be on the same basis.
2. Interest is charged on the total capital investment at a set interest rate. Rates equivalent to those charged for bank loans or bonds are usually employed. Under these conditions, the total profit represents the increase over the return that would be obtained if the company could invest the same amount of money in an outside loan at the given interest rate.

As the design proceeds to the final stages, the actual source of new capital should be considered in detail, and more-refined methods for determining interest costs can be used.

When interest is included as a cost, there is some question as to whether the interest costs should be based on the initial investment or on the average investment over the life of the project. Although this is a debatable point, the accepted design practice is to base the interest costs on the initial investment. Because of the different methods used for treating interest as a cost, a definite statement should be made concerning the particular method employed in any given economic analysis. Interest costs become especially important when making comparisons among alternative investments. These comparisons, as well as the overall cost picture, are simplified if the role of interest in the economic analysis is clearly defined.

NOMENCLATURE FOR CHAPTER 7

- \( a \) = a constant
- \( C \) = compound interest factor
- \( C_R \) = cost for replacement or other asset, dollars
- \( C_V \) = original cost of equipment or other asset, dollars
- \( d \) = number of days in an interest period, days, or derivative
- \( e \) = base of the natural logarithm = 2.71828...
- \( F \) = discount factor
- \( g \) = constant declining rate or gradient
- \( i \) = interest rate based on the length of one interest period, percent/100
- \( i_{eff} \) = effective interest rate-exact interest rate based on an interest period of one year, percent/100
- \( Z \) = total amount of interest during \( n \) interest periods, dollars
- \( K \) = capitalized cost, dollars
- \( m \) = number of interest periods per year
- \( n \) = number of time units or interest periods
- \( n_T \) = number of time units necessary for cash flow to decrease to zero
- \( P \) = principal or present worth of capital on which interest is paid, dollars
- \( \bar{P} \) = principal or present worth considered as occurring regularly throughout the time period, dollars
- \( r \) = nominal interest rate-approximate interest rate based on an interest period of one year or continuous interest rate, percent/100
\[ R = \text{uniform periodic payments made during } n \text{ periods in an ordinary annuity, dollars/period} \]

\[ \bar{R} = \text{total of all ordinary annuity payments occurring regularly throughout the time period, dollars/period} \]

\[ S = \text{future worth-amount of principal or present worth plus interest due after } n \text{ interest periods, dollars} \]

\[ \bar{S} = \text{future worth considered as occurring continuously throughout the time period, dollars} \]

\[ T = \text{time period, years} \]

**PROBLEMS**

1. It is desired to have $9000 available 12 years from now. If $5000 is available for investment at the present time, what discrete annual rate of compound interest on the investment would be necessary to give the desired amount?

2. What will be the total amount available 10 years from now if $2000 is deposited at the present time with nominal interest at the rate of 6 percent compounded semiannually?

3. An original loan of $2000 was made at 6 percent simple interest per year for 4 years. At the end of this time, no interest had been paid and the loan was extended for 6 more years at a new, effective, compound-interest rate of 8 percent per year. What is the total amount owed at the end of the 10 years if no intermediate payments are made?

4. A concern borrows $50,000 at an annual, effective, compound-interest rate of 10 percent. The concern wishes to pay off the debt in 5 years by making equal payments at the end of each year. How much will each payment have to be?

5. The original cost for a distillation tower is $24,000 and the useful life of the tower is estimated to be 8 years. The sinking-fund method for determining the rate of depreciation is used (see Example 5), and the effective annual interest rate for the depreciation fund is 6 percent. If the scrap value of the distillation tower is $4000, determine the asset value (i.e., total book value of equipment) at the end of 5 years.

6. An annuity due is being used to accumulate money. Interest is compounded at an effective annual rate of 8 percent, and $1000 is deposited at the beginning of each year. What will the total amount of the annuity due be after 5 years?

7. By use of a digital computer, develop and present a printout of the data of effective interest versus nominal interest compounded continuously as given in Table 2.

8. By use of a digital computer, develop and present a printout of the first five lines of Table 7.

9. For total yearly payments of $5000 for 10 years, compare the compound amount accumulated at the end of 10 years if the payments are (a) end-of-year, (b) weekly, and (c) continuous. The effective (annual) interest is 20 percent and payments are uniform.

10. For the conditions of Prob. 9, determine the present worth at time zero for each of the three types of payments.

11. A heat exchanger has been designed for use in a chemical process. A standard type of heat exchanger with a negligible scrap value costs $4000 and will have a useful life
of 6 years. Another proposed heat exchanger of equivalent design capacity costs $6800 but will have a useful life of 10 years and a scrap value of $800. Assuming an effective compound interest rate of 8 percent per year, determine which heat exchanger is cheaper by comparing the capitalized costs.

12. A new storage tank can be purchased and installed for $10,000. This tank would last for 10 years. A worn-out storage tank of capacity equivalent to the new tank is available, and it has been proposed to repair the old tank instead of buying the new tank. If the tank were repaired, it would have a useful life of 3 years before the same type of repairs would be needed again. Neither tank has any scrap value. Money is worth 9 percent compounded annually. On the basis of equal capitalized costs for the two tanks, how much can be spent for repairing the existing tank?

13. Equation (28) is the expression for capitalized cost based on discrete interest compounding. For continuous interest compounding, the expression becomes

\[ K = C_v + \frac{C_R}{e^{rn} - 1} \]

Present a detailed derivation of this continuous-interest relationship going through each of the equivalent steps used in deriving Eq. (28).

14. The total investment required for a new chemical plant is estimated at $2 million. Fifty percent of the investment will be supplied from the company’s own capital. Of the remaining investment, half will come from a loan at an effective interest rate of 8 percent and the other half will come from an issue of preferred stock paying dividends at a stated effective rate of 8 percent. The income-tax rate for the company is 30 percent of pre-tax earnings. Under these conditions, how many dollars per year does the company actually lose (i.e., after taxes) by issuing preferred stock at 8 percent dividends instead of bonds at an effective interest rate of 6 percent?

15. It has been proposed that a company invest $1 million in a venture which will yield a gross income of $1 million per year. The total annual costs will be $800,000 per year including interest on the total investment at an annual rate of 8 percent. In an alternate proposal, the company can invest a total of $600,000 and receive annual net earnings (before income taxes) of $220,000 from the venture. In this case, the net earnings were determined on the basis of no interest costs. The company has $1 million of its own which it wishes to invest, and it can always obtain an effective 6 percent annual interest rate by loaning out the money. What would be the most profitable way for the company to invest its $1 million?
Expenses for taxes and insurance play an important part in determining the economic situation for any industrial process. Because Federal, state, and local taxes may amount to a major portion of a concern’s earnings, it is essential for the chemical engineer to understand the basic principles and factors underlying taxation. Insurance costs ordinarily are only a small part of the total expenditure involved in an industrial operation; however, adequate insurance coverage is necessary before any operation can be carried out on a sound economic basis.

Taxes are levied to supply funds to meet the public needs of a government, while insurance is required for protection against certain types of emergencies or catastrophic occurrences. Insurance rates and tax rates can vary considerably for business concerns as compared to the rates for individual persons. The information presented in this chapter applies generally to large business establishments.

TYPES OF TAXES

Taxes may be classified into three types: (1) property taxes, (2) excise taxes, and (3) income taxes. These taxes may be levied by the Federal government, state governments, or local governments.

Property Taxes

Local governments usually have jurisdiction over property taxes, which are commonly charged on a county basis. In addition to these, individual cities and
towns may have special property taxes for industrial concerns located within the city limits.

Property taxes vary widely from one locality to another, but the average annual amount of these charges is 1 to 4 percent of the assessed valuation. Taxes of this type are referred to as *direct* since they must be paid directly by the particular concern and cannot be passed on as such to the consumer.

**Excise Taxes**

Excise *taxes* are levied by Federal and state governments. Federal excise taxes include charges for import customs duties, transfer of stocks and bonds, and a large number of other similar items. Manufacturers’ and retailers’ excise taxes are levied by Federal and state governments on the sale of many products such as gasoline and alcoholic beverages. Taxes of this type are often referred to as *indirect* since they can be passed on to the consumer. Many business concerns must also pay excise taxes for the privilege of carrying on a business or manufacturing enterprise in their particular localities.

**Income Taxes**

In general, *income taxes* are based on gross earnings, which are defined as the difference between total income and total product cost. Revenue from income taxes is an important source of capital for both Federal and state governments. National and state laws are the basis for these levies, and the laws change from year to year. State income taxes vary from one state to another and are a function of the gross earnings for individual concerns. Depending on the particular state and the existing laws, state income taxes may range from 0 to 5 percent or more of gross earnings.

**FEDERAL INCOME TAXES**

The Federal government has set up an extremely complex system for determining income taxes for business establishments. New laws are added and old laws are changed each year, and it would be impossible to present all the rules and interpretations in a few pages. Accordingly, this section will deal only with the basic pattern of Federal income-tax regulations and give the methods generally used for determining Federal income taxes. It should be emphasized strongly that the final determination of income-tax payments should be made with the aid of legal and accounting tax experts.

†Complete details are available in “Income Tax Regulations” and periodic “Income Tax Bulletins” issued by the U.S. Treasury Department, Superintendent of Documents, Internal Revenue Service and in services published by private concerns, such as the multivolume guide entitled “Prentice-Hall Federal Taxes,” giving the latest tax laws with explanations and examples, which is published annually by Prentice-Hall Information Services, Paramus, NJ 07652.
The corporate income-tax rate in the United States has varied widely during the past 60 years. During the period from 1913 to 1935 the tax rate based on gross earnings increased from 1 to 13.75 percent. In 1938, the rate was increased to 19 percent, and, during the Second World War, it was 40 percent plus an excess-profits tax. In 1946, the standard income-tax rate for corporations was reduced to 38 percent, but the rate was increased to 42 percent in 1950 plus an excess-profits tax. During the Korean War the rate was 52 percent plus an excess-profits tax which could result in an overall tax rate of 70 percent on gross earnings. From 1954 through 1963, the corporation income-tax rate was 52 percent with reductions to 50 percent in 1964, 48 percent in 1965, and 46 percent in 1979 on gross earnings above certain base limits. Revision of the income-tax laws in 1986 resulted in a corporation income-tax rate of 34 percent on gross earnings above $75,000 beginning with taxable years starting after June 30, 1987. Table 1 summarizes the standard tax rates for corporations during the period from 1929 to 1988 and Table 2 presents a summary of Federal income taxes on corporations as applicable from 1965 to 1988 based on annual gross earnings.

The figures in Table 1 indicate the wide variations in income-tax rates caused by national emergencies, the prevailing economic situation, and the desires of lawmakers in office at any particular time. Figure 8-1 presents a

<table>
<thead>
<tr>
<th>Year</th>
<th>Regular tax rate, %</th>
<th>Effective limit with wartime excess-profits tax, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1929</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>1930-1931</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>1932-1935</td>
<td>13.75</td>
<td></td>
</tr>
<tr>
<td>1936-1937</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>1938-1939</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>1940</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>1941</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>1942-1943</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>1944-1945</td>
<td>40</td>
<td>72</td>
</tr>
<tr>
<td>1946-1949</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>42</td>
<td>52</td>
</tr>
<tr>
<td>1951</td>
<td>50.75</td>
<td>68</td>
</tr>
<tr>
<td>1952-1953</td>
<td>52</td>
<td>70</td>
</tr>
<tr>
<td>1954-1963</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>1964</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>1965-1978</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>1979-1987</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>1988-</td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 2
Federal income taxes on corporations (1965 to 1990)

<table>
<thead>
<tr>
<th>Year</th>
<th>Taxes</th>
<th>Limitations</th>
<th>Percent of gross earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-1974</td>
<td>Normal tax</td>
<td>On gross earnings</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td><strong>Surtax</strong></td>
<td>On gross earnings above $25,000</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Combined rate</td>
<td>On gross earnings above $25,000</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Capital-gains tax</td>
<td>Varies depending on accounting methods</td>
<td>25-30</td>
</tr>
<tr>
<td>1975-1978</td>
<td>Normal tax</td>
<td>On first $25,000 of gross earnings</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Surtax</td>
<td>On gross earnings over $25,000</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Combined rate</td>
<td>On gross earnings over $50,000</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Capital-gains tax</td>
<td>Varies depending on accounting methods</td>
<td>48</td>
</tr>
<tr>
<td>1979-1981</td>
<td>Normal tax</td>
<td>On first $25,000 of gross earnings</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Surtax</td>
<td>On second $25,000 of gross earnings</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Combined rate</td>
<td>On third $25,000 of gross earnings</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Capital-gains tax</td>
<td>Varies depending on accounting methods</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Surtax</td>
<td>On fourth $25,000 of gross earnings</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Combined rate</td>
<td>Graduated as shown to reach a combined rate of 46 percent on gross earnings above $100,000</td>
<td>46</td>
</tr>
<tr>
<td>1982-1987</td>
<td>Same as for 1979 to 1981 except the graduated tax-rate percentages were 16%, 19%, 30%, 40% and 46% for 1982 and 15%, 18%, 30%, 40%, and 46% for years from 1983 to 1987. Starting in 1983, the graduated tax rate was phased out for gross earnings above $1,000,000 by increasing taxes for those cases by the lesser of 5% of gross earnings in excess of $1,000,000 or $20,250.</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>1988-</td>
<td>Normal tax</td>
<td>On first $50,000 of gross earnings</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Surtax</td>
<td>On gross earnings of $50,000 to $75,000</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Combined rate</td>
<td>Graduated as shown to reach a combined rate of 34 percent on gross earnings above $75,000</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>Capital-gains tax</td>
<td>Varies depending on accounting methods</td>
<td>34</td>
</tr>
</tbody>
</table>

The graduated tax rate was phased out for gross earnings above $100,000 by increasing taxes for those cases by the lesser of 5% of gross earnings or $11,750. But it is generally the same as the tax rate on all gross earnings for companies.

Graphical representation showing the changes in income-tax rates for a typical chemical company for 1943 to 1988.

Many industries have special tax exemptions because of the type of product, market, or service involved in their business, or because the government wishes to offer particular support and inducement to concerns producing essential materials. Even for large concerns with high profits, the overall tax rate can vary widely from year to year depending on the size of available tax
FIGURE 8-1
Example of variation in income-tax rate with time for a chemical company. (Based on annual reports by Pfizer Inc.)
deductions or because of carry-back or carry-forward provisions. These possible variations in income-tax effects can have an important influence on the optimum timing for expenditures or other financial transactions.

**Normal Tax**

A so-called normal tax has been levied by the Federal government on the earnings of corporations. This tax was at a rate set by the national lawmakers. For taxable years of 1965 to 1974, the normal tax in the United States was 22 percent of gross earnings. For the years 1975 through 1978, the normal tax was 20 percent of the first $25,000 of gross earnings and 22 percent of gross earnings above $25,000. For 1979, the normal tax was 17 percent of the first $25,000 of gross earnings, 20 percent of the second $25,000, 30 percent of the third $25,000, and 40 percent of the fourth $25,000. These percentages for 1979 were changed to 16, 19, 30, and 40 for 1982 and to 15, 18, 30, and 40 for 1982 to 1987. In 1988, the normal tax was changed to 15 percent of the first $50,000 of gross earnings and 25 percent of gross earnings of $50,000 to $75,000.

**Surtax**

In addition to the normal tax, corporations have had to pay a second Federal income tax on gross earnings above a certain base limit. This additional tax is known as a surtax. The base limit was $25,000 per year for taxable years of 1965 to 1974 and $50,000 per year for taxable years of 1975 through 1978. In this period, the surtax on gross earnings above the limit was at a rate of 26 percent, resulting in a tax rate of 48 percent for gross earnings above $25,000 in 1965 to 1974 and $50,000 in 1975 to 1978. For 1979 to 1987, a graduated tax rate was in effect for the normal tax with the base limit being $100,000 per year and a tax rate of 46 percent applying to gross earnings above this figure. As of 1988, the base limit was changed to $75,000 with a tax rate of 34 percent applying to annual gross earnings above this figure.

**Example 1 Determination of annual income** taxes. With the Federal income-tax regulations in effect in 1988, the graduated tax rate was phased out for gross earnings above $100,000 by increasing taxes for gross earnings above $100,000 by the lesser of 5 percent of the gross earnings in excess of $100,000 per year or $11,750 (see Table 2). Show that this results in a flat, overall tax rate of 34 percent for all cases of annual gross earnings exceeding $335,000.

**Solution.** From Table 1, annual tax on gross earnings of $335,000 is

\[

tax = 0.15 \times 50,000 + 0.25 \times 25,000 + 0.34 \times 260,000 + 0.05 \times 235,000 = 11,750
\]

or $11,750, whichever is smaller.
Thus, total tax is $7500 + $6250 + $88,400 + $11,750 = \$113,900 \text{ per year, Flat, overall tax rate is } \frac{\$113,900}{\$335,000} \times 100 = 34 \text{ percent on gross earnings of } \$335,000 \text{ or on any gross earnings greater than this because the limit of } \$11,750 \text{ for use in adjusting the tax will always be less than 5 percent of gross earnings greater than } \$335,000 \approx \$100,000.

Capital-Gains Tax

A capital-gains tax is levied on profits made from the sale of capital assets, such as land, buildings, or equipment. The profit is known as long-term capital gain if the asset was held for more than one year if it was acquired after December 31, 1987 or between December 31, 1977 and June 23, 1984 (six months for property acquired between June 23, 1984 and December 31, 1987, nine months in 1977, and six months before 1977). The profit is known as short-term capital gain if the time held is less than those indicated for long-term capital gain.

The net capital gain is the total of short-term and long-term capital gains, and this total is generally taxed for corporations at the same rate as ordinary income in the year the gain occurred. With this simplified definition of taxes on capital gains, the tax rate for corporations capital gains would be 34 percent starting in 1988. However, there are many special tax regulations for capital gains for corporations relative to items such as carry-back and carry-forward of net capital losses, alternative tax calculation methods, and minimum tax requirements. The tax regulations change regularly and are relatively complicated for corporation capital gains.

Tax Exemption for Dividends Received

Corporations are given a partial tax exemption for dividends received. In general, only 15 percent of such dividends are considered as taxable income for corporations, with the remaining 85 percent being tax exempt.

Contributions

Corporate contributions to appropriate organizations, as defined by the income-tax laws, can be deducted as an expense up to 10 percent of the taxable income figured without regard to the contribution deduction and other special deductions. Thus, for a corporation which is paying income tax at a 34 percent rate, a contribution of $10,000 would represent an actual cost to the corporation after taxes of only $6600.

\[\text{For a complete and up-to-date description, see the most recent annual issue of “Prentice-Hall Federal Tax Handbook,” Prentice-Hall Information Services, Paramus, NJ 07652.}\]
Carry-back and Carry-forward of Losses

The preceding analyses of taxes have been based on the assumption that the corporations involved were operating at a profit. In case the situation was one in which a loss resulted, some method of tax accounting needs to be available for this case of negative taxable income. To handle this possible situation, tax regulations permit the corporation to use the loss to offset profits in other years by *carry-back* or *carry-forward* of losses. Tax laws permit a corporation to carry its losses back as charges against profits for as many as three years before the loss or, if necessary, to carry the losses forward as charges against profits for as many as five years after the loss.

Investment Credit

The 1971 Revenue Act of the United States provided for a special first-year tax deduction on new investments for machinery, equipment, and certain other assets used in production processes, in the form of a 7 percent “investment credit” for the first year of the life for assets with over 7 years of estimated service life. The investment credit rate was increased to 10 percent for the years 1975 and later, with the possibility of a higher rate. The investment credit amount was limited to the first $25,000 of the corporation’s tax liability for the year plus 50 to 90 percent (depending on the year) of the corporation’s tax liability above $25,000. This investment credit was repealed in 1986 for properties placed in service after December 31, 1985 with special transition rules applying to carry-forwards, carry-backs, and certain types of property.

Tax revisions, such as those referred to in the preceding sections, are often made for the primary purpose of stimulating or controlling investments and the national economy. Accordingly, one can expect regular changes in the tax regulations, and the assistance of responsible tax experts who keep up with the latest developments is recommended for final evaluation of economic effects.

Taxes and Depreciation

Because Federal income taxes are based on gross earnings, which means that all costs have been deducted, the U.S. Treasury Department has devoted considerable effort to controlling one of the major costs in industrial operations, i.e., the cost for depreciation. The subject of depreciation costs is considered in Chap. 9, where some of the tax regulations by the U.S. Treasury Department are discussed.

In determining the influence of depreciation costs on income taxes, it should be clear that depreciation costs represent a deduction from taxable gross earnings. Thus, if $d$ is the depreciation cost for the year and $\phi$ is the fractional tax rate,

\[
\text{Tax “credit” for depreciation} = \phi d
\] (1)
Funds set aside for depreciation, although they represent a cost, normally go directly into the corporation treasury. Therefore, if $S$ represents the total annual income or revenue and $C$ represents the total annual costs with the exceptions of depreciation and taxes,

\[
\text{Net annual cash flow to company after taxes} = (S - C - d)(1 - \phi) + d = (S - C)(1 - \phi) + \phi d \quad (2)
\]

The preceding equation is applied to various situations of cash flow in Table 3 of Chap. 10.

Excess-Profits Tax

During times of national emergency, certain types of business concerns can realize extremely high income and profit. This is true in particular for concerns producing military necessities during wartime. An *excess-profits tax* may be levied, therefore, to supply the national government with part of these profits.

The system for determining excess-profits taxes is extremely complex. In general, the amount of the tax is based on the normal past earnings of a concern or on the total capital investment. Special provisions are made for new corporations or for concerns which do not have a normal past history to use as a basis. The excess-profits taxes are very unpopular with businessmen, and there is always considerable opposition to the levying of these taxes.

Tax Returns

Income-tax returns may be reported on a cash basis or on an accrual basis. When the cash basis is used, only money actually received or paid out during the year is reported. With the accrual method, income and expenses are included as of the time they were incurred, even though final payment has not yet been made.

Returns may be based on a standard calendar year or on a fiscal year. Any date may be chosen as the end of the fiscal year, and it is usually advisable to choose a time when the work of assembly and determination of the tax will be the most convenient. The tax payment itself is usually made in installments.

OTHER TAXES

The Federal Insurance Contribution Act levies a social security tax on most employers and also requires a certain percentage of employees’ wages to be withheld. Special local assessments for tax purposes are often encountered, and concerns doing business in foreign countries must pay taxes based on the laws of the foreign countries involved.

The question sometimes arises in cost accounting whether certain service charges and license fees can be considered as taxes. If the charge can be regarded as part of a public duty to support government, it is legally correct to
designate the charge as a tax. When the exaction is for a service and the amount charged is a reasonable fee for the service actually received, the cost cannot be considered as a tax. Fees for building permits, government inspections, formation of corporations, bridge and road tolls, and certain types of licenses cannot be charged as taxes because the primary purpose of these fees is to serve for control and regulation rather than to support government.

If a corporation is organized by an individual for the purpose of avoiding high personal-income taxes, the organization is classed as a personal-holding company, and special tax rates apply. The amount of income tax which must be paid by a private business exceeds that required of an equivalent corporation at surprisingly small gross earnings. Consequently, a private business should make a periodic analysis of the advantages and disadvantages of becoming incorporated.

INSURANCE

The annual insurance cost for ordinary industrial concerns is approximately 1 percent of the capital investment. Despite the fact that insurance costs may represent only a small fraction of total costs, it is necessary to consider insurance requirements carefully to make certain the economic operation of a plant is protected against emergencies or unforeseen developments.

The design engineer can aid in reducing insurance requirements if he or she understands the factors which must be considered in obtaining adequate insurance. In particular, the engineer should be aware of the different types of insurance available and the legal responsibilities of a concern with regard to accidents or other unpredictable emergencies.

LEGAL RESPONSIBILITY

A concern can obtain insurance to protect itself against loss of property owing to any of a number of different causes. In case a property loss occurs and the loss is covered by insurance, payment will be made for the damage even though the loss was caused by the owner’s negligence.

Protection against unforeseen emergencies, other than direct property loss, can also be obtained through insurance. For example, injuries to employees or persons near the danger area may occur due to a fire or explosion, and the concern involved should have insurance adequate to handle claims made in these cases. It is, of course, impossible to insure against every possible emergency, but it is necessary to consider the results of a potential occurrence, and the legal responsibility for various types of events should be understood. The payments required for settling a case in which legal responsibility has been proved may be much greater than any costs due to direct property damage.

An assumed liability is one which the concern accepts in the form of a written contract or statement, while a legal liability is always in effect whether or not it is stated in writing. Legal liabilities include civil responsibility for events
occurring because of damage or injuries due to negligence. A stronger type of legal liability is known as criminal liability. This is involved in cases where gross negligence or reckless disregard for the life and property of others is claimed.

The design engineer should be familiar with the legal aspects of any laws or regulations governing the type of plant or process involved in a design. In case of an accident, failure to comply with the definite laws involved is a major factor in fixing legal responsibility. Compliance with all existing laws, however, is not a sufficient basis for disallowance of legal liability. Every known safety feature should be included and extraordinary care in the complete operation must be proved before a good case can be presented for disallowing legal liability.

Many contracts include hold-harmless agreements wherein the legal responsibility for an accident or other type of event is indicated as part of a written agreement. Any new lease or contract should be examined by an expert to make certain all hold-harmless agreements are clearly stated and understood by both parties.

Any concern producing a product which may be dangerous to life or property has a legal responsibility to indicate the potential hazard by use of warning labels or other protective methods. The manufacturer must supply safe shipping containers and make certain that any hazards involved in their handling or use are clearly indicated. Legal liability also holds for defective or misrepresented products.

A manufacturing establishment may have on its property some object which would be highly attractive as a place for children to play. Two examples would be a quarry pit and a sand pile. An object of this type is known as an attractive nuisance, and the concern may be liable for injuries to children if the injuries are a result of their playing around or in the object. The liability would apply even though the children were obviously trespassing. High fences or some other effective safety measure should be used to keep children from gaining admittance to an attractive nuisance.

An industrial concern has a legal responsibility for property belonging to others as long as the property is on the concern’s premises. This responsibility is known as a bailee’s liability. The property may be stored equipment or materials, finished products, or products in process. If the property is damaged or destroyed, the bailee’s liability is roughly a function of the degree of care used in safeguarding the property. In case the damaged or destroyed property is insured by the owner, the insurance company will pay the claim; however, the insurance company can then exercise its subrogation rights and attempt to force the bailee to pay the full amount received by the owner.

TYPES OF INSURANCE

Many different types of insurance are available for protection against property loss or charges based on legal liability. Despite every precaution, there is always the possibility of an unforeseen event causing a sudden drain on a company’s
finances, and an efficient management protects itself against such potential emergencies by taking out insurance. In order to make an intelligent analysis of insurance requirements for any kind of operation, it is necessary to understand the physical factors involved in carrying out the process and to be aware of the types of insurance available.

The major insurance requirements for manufacturing concerns can be classified as follows:

1. Fire insurance and similar emergency coverage on buildings, equipment, and all other owned, used, or stored property. Included in this category would be losses caused by lightning, wind- or hailstorms, floods, automobile accidents, explosions, earthquakes, and similar occurrences.
2. Public-liability insurance, including bodily injury and property loss or damage, on all operations such as those involving automobiles, elevators, attractive nuisances, bailee’s charges, aviation products, or any company function carried on at a location away from the plant premises.
3. Business-interruption insurance. The loss of income due to a business interruption caused by a fire or other emergency may far exceed any loss in property. Consequently, insurance against a business interruption of this type should be given careful consideration.
4. Power-plant, machinery, and special-operations hazards.
5. Workmen’s-compensation insurance.
6. Marine and transportation insurance on all property in transit.
7. Comprehensive crime coverage.
8. Employee-benefit insurance, including life, hospitalization, accident, health, personal property, and pension plans.

Self-Insurance

On an average basis, insurance companies pay out loss claims amounting to 55 to 60 cents for each dollar received. The balance is used for income taxes, salaries, commissions, administrative costs, inspection costs, and various overhead costs. Theoretically, a saving of 40 to 45 cents per dollar paid for insurance could be achieved by self-insurance. If insurance requirements are great, this saving could amount to a very large sum, and it would be worthwhile to consider the possibilities of self-insurance.

A careful analysis of all risks involved is necessary when considering self-insurance on possible losses or emergencies. The final decision should not be based on whether or not the insurable event will occur, because this is impossible to predict. Instead, the decision should be based on the total loss involved if the event or a series of such events were to occur. If an industrial concern has a number of widespread interests and sufficient funds available to handle simultaneous major losses in several of these interests, it might be
reasonable to consider self-insurance on some of the potential hazards. On the other hand, if a single potential event could ruin the economic standing of the company, it would be very inadvisable to assume the risk involved in self-insurance.

There are several different ways of applying self-insurance. One method involves depositing money equivalent to an insurance premium into a special company fund. This fund can then be used to handle any losses or emergencies which may occur. At the outset, this fund would be small and would be inadequate to handle any major losses. Consequently, if this method is used, it may be necessary to supply an original base fund or else assume a disproportionate amount of risk until the fund has built up to a practical value. Under ordinary conditions, the premiums paid into a self-insurance reserve are not tax-deductible.

A second method may be used in which the company assumes all the risk and no payments are made into a reserve fund. This method is designated as "self-assumption of risk." Partial self-insurance may be obtained through the purchase of deductible insurance from regular agencies. The purchaser assumes the risk up to a certain amount and the insurance company agrees to pay for any additional losses.

The effects of income taxes should be considered when making a final decision regarding insurance. Because the premiums for standard insurance are tax-deductible, the actual cost after taxes for the protection may be much less than the direct premium charge. Another advantage of standard insurance is the inspection services supplied by the insurance companies. These companies require periodic inspections by specialists to make certain that the insurance rates are adequate, and the reports of these inspectors often indicate new ideas or methods for increasing the safety of the operation.

The overall policies of the particular manufacturing concern dictate the type and amount of insurance which will be held. It should be realized, however, that a well-designed insurance plan requires a great deal of skilled and informed investigation by persons who understand all the aspects of insurance as well as the problems involved in the manufacturing operation.

PROBLEMS

1. The fixed-capital investment for an existing chemical plant is $20 million. Annual property taxes amount to 1 percent of the fixed-capital investment, and state income taxes are 5 percent of the gross earnings. The net income per year after all taxes is $2 million, and the Federal income taxes amount to 34 percent of gross earnings. If the same plant had been constructed at a location where property taxes were 4 percent of the fixed-capital investment and state income taxes were 2 percent of the gross earnings, what would be the net income per year after taxes, assuming all other cost factors were unchanged?

2. The gross earnings for a small corporation were $54,000 in 1970. What would have been the percent reduction in Federal income taxes paid by the company if the tax
rates in effect in 1988 had been in effect in 1970? (See Table 2 for Federal income-tax rules in effect.)

3. During the period of one taxable year at a manufacturing plant, the total income on a cash basis was $21 million. Five million dollars of immediate debts due the company was unpaid at the end of the year. The company paid out $15 million on a cash basis during the year, and all of this amount was tax-deductible as a product cost. The company still owed $3 million of tax-deductible bills at the end of the year. If the total Federal income tax for the company amounts to 34 percent of the gross earnings, determine the amount of Federal income tax due for the year on a cash basis and also on an accrual basis.

4. Complete fire and allied-coverage insurance for one unit of a plant requires an annual payment of $700 based on an investment value of $100,000. If income taxes over a 10-year period average 30 percent of gross earnings, by how much is the net income, after taxes, reduced during this lo-year period owing to the cost of the insurance?

5. Self-insurance is being considered for one portion of a chemical company. The fixed-capital investment involved is $50,000, and insurance costs for complete protection would amount to $400 per year. If self-insurance is used, a reserve fund will be set up under the company’s jurisdiction, and annual insurance premiums of $300 will be deposited in this fund under an ordinary annuity plan. All money in the fund can be assumed to earn interest at a compound annual rate of 5 percent. Neglecting any charges connected with administration of the fund, how much money should be deposited in the fund at the beginning of the program in order to have enough money accumulated to replace a complete $50,000 loss after 10 years?

6. A corporation shows a gross earnings or net profit before Federal income taxes of $200,000 in the taxable years of 1978, 1981, and 1988. The taxable income for the corporation in all 3 years is the $200,000 given, and there are no special tax exemptions for the corporation so that the situation described in this chapter under the headings of normal tax and surtax applies (see Table 2). Determine the Federal income tax paid by the corporation in each of the 3 years and find the amount of tax saved in 1988 because of the tax regulations in effect that year as compared to 1981 and 1978. Repeat for the case of a gross earnings of $50,000 instead of $200,000.

7. Locate the most recent issues of the Annual Report for Pfizer Inc. in your local business library past 1988 and extend the data for gross earnings and income tax as percent of gross earnings presented in Fig. 8-1 of this chapter.
An analysis of costs and profits for any business operation requires recognition of the fact that physical assets decrease in value with age. This decrease in value may be due to physical deterioration, technological advances, economic changes, or other factors which ultimately will cause retirement of the property. The reduction in value due to any of these causes is a measure of the depreciation.

The economic function of depreciation, therefore, can be employed as a means of distributing the original expense for a physical asset over the period during which the asset is in use.

Because the engineer thinks of depreciation as a measure of the decrease in value of property with time, depreciation can immediately be considered from a cost viewpoint. For example, suppose a piece of equipment had been put into use 10 years ago at a total cost of $31,000. The equipment is now worn out and is worth only $1000 as scrap material. The decrease in value during the 10-year period is $30,000; however, the engineer recognizes that this $30,000 is in reality a cost incurred for the use of the equipment. This depreciation cost was spread over a period of 10 years, and sound economic procedure would require part of this cost to be charged during each of the years. The application

†According to the Internal Revenue Service, depreciation is defined as “A reasonable allowance for the exhaustion, wear, and tear of property used in the trade or business including a reasonable allowance for obsolescence.” The terms amortization and depreciation are often used interchangeably. Amortization is usually associated with a definite period of cost distribution, while depreciation usually deals with an unknown or estimated period over which the asset costs are distributed. Depreciation and amortization are of particular significance as an accounting concept which serves to reduce taxes.
of depreciation in engineering design, accounting, and tax studies is almost always based on costs prorated throughout the life of the property.

Meaning of Value

From the viewpoint of the design engineer, the total cost due to depreciation is the original or new value of a property minus the value of the same property at the end of the depreciation period. The original value is usually taken as the total cost of the property at the time it is ready for initial use. In engineering design practice, the total depreciation period is ordinarily assumed to be the length of the property's useful life, and the value at the end of the useful life is assumed to be the probable scrap or salvage value of the components making up the particular property.

It should be noted here that the engineer cannot wait until the end of the depreciation period to determine the depreciation costs. These costs must be prorated throughout the entire life of the property, and they must be included as an operating charge incurred during each year. The property value at the end of the depreciation period and the total length of the depreciation period cannot be known with certainty when the initial yearly costs are determined. Consequently, it is necessary to estimate the final value of the property as well as its useful life. In estimating property life, the various factors which may affect the useful-life period, such as wear and tear, economic changes, or possible technological advances, should be taken into consideration.

When depreciation is not used in a prorated-cost sense, various meanings can be attached to the word value. One of these meanings involves appraisal of both initial and final values on the basis of conditions at a certain time. The difference between the estimated cost of new equivalent property and the appraised value of the present asset is known as the appraised depreciation. This concept involves determination of the values of two assets at one date as compared with the engineering-cost concept, which requires determination of the value of one asset at two different times.

Purpose of Depreciation as a Cost

Consideration of depreciation as a cost permits realistic evaluation of profits earned by a company and, therefore, provides a basis for determination of Federal income taxes. Simultaneously, the consideration of depreciation as a cost provides a means whereby funds are set aside regularly to provide recovery of the invested capital. When accountants deal with depreciation, they must follow certain rules which are established by the U.S. Bureau of Internal Revenue for determination of income taxes. These rules deal with allowable life for the depreciable equipment and acceptable mathematical procedures for allocating the depreciation cost over the life of the asset.

Although any procedure for depreciation accounting can be adopted for internal company evaluations, it is highly desirable to keep away from the
necessity of maintaining two sets of accounting books. Therefore, the engineer should be familiar with Federal regulations relative to depreciation and should follow these regulations as closely as possible in evaluating depreciation as a cost.

**TYPES OF DEPRECIATION**

The causes of depreciation may be physical or functional. **Physical depreciation** is the term given to the measure of the decrease in value due to changes in the physical aspects of the property. Wear and tear, corrosion, accidents, and deterioration due to age or the elements are all causes of physical depreciation. With this type of depreciation, the serviceability of the property is reduced because of physical changes. Depreciation due to all other causes is known as **functional depreciation**.

One common type of functional depreciation is **obsolescence**. This is caused by technological advances or developments which make an existing property obsolete. Even though the property has suffered no physical change, its economic serviceability is reduced because it is inferior to improved types of similar assets that have been made available through advancements in technology.

Other causes of functional depreciation could be (1) change in demand for the service rendered by the property, such as a decrease in the demand for the product involved because of saturation of the market, (2) shift of population center, (3) changes in requirements of public authority, (4) inadequacy or insufficient capacity for the service required, (5) termination of the need for the type of service rendered, and (6) abandonment of the enterprise. Although some of these situations may be completely unrelated to the property itself, it is convenient to group them all under the heading of functional depreciation.

Because depreciation is measured by decrease in value, it is necessary to consider all possible causes when determining depreciation. Physical losses are easier to evaluate than functional losses, but both of these must be taken into account in order to make fair allowances for depreciation.

**Depletion**

Capacity loss due to materials actually consumed is measured as **depletion**. Depletion cost equals the initial cost times the ratio of amount of material used to original amount of material purchased. This type of depreciation is particularly applicable to natural resources, such as stands of timber or mineral and oil deposits.

**Costs for Maintenance and Repairs**

The term **maintenance** conveys the idea of constantly keeping a property in good condition; **repairs** connotes the replacing or mending of broken or worn
parts of a property. The costs for maintenance and repairs are direct operating expenses which must be paid from income, and these costs should not be confused with depreciation costs.

The extent of maintenance and repairs may have an effect on depreciation cost, because the useful life of any property ought to be increased if it is kept in good condition. However, a definite distinction should always be made between costs for depreciation and costs for maintenance and repairs.

SERVICE LIFE

The period during which the use of a property is economically feasible is known as the service life of the property. Both physical and functional depreciation are taken into consideration in determining service life, and, as used in this book, the term is synonymous with economic or useful life. In estimating the probable service life, it is assumed that a reasonable amount of maintenance and repairs will be carried out at the expense of the property owner.

Many data are available concerning the probable life of various types of property. Manufacturing concerns, engineers, and the U.S. Internal Revenue Service (IRS) have compiled much information of this sort. All of these data are based on past records, and there is no certainty that future conditions will be unchanged. Nevertheless, by statistical analysis of the various data, it is possible to make fairly reliable estimates of service lives.

The U.S. Internal Revenue Service recognizes the importance of depreciation as a legitimate expense, and the IRS has issued formal statements which list recommended service lives for many types of properties. Prior to July 12, 1962, Federal regulations for service lives and depreciation rates were based on the so-called Bulletin “F,” “Income Tax Depreciation and Obsolescence-Estimated Useful Lives and Depreciation Rates” as originally published by the U.S. Internal Revenue Service in 1942. In July, 1962, the Bulletin “F” regulations were replaced by a set of new guidelines based on four groups of depreciable assets. The 1971 Revenue Act of the United States provided more flexibility in choosing depreciation life by allowing a choice of depreciation life of 20 percent longer or shorter than the guideline lives called for by earlier tax laws for machinery, equipment, or other assets put in service after December 31, 1970. This is known as the Class Life Asset Depreciation Range System (ADR).

Table 1 presents estimated service lives for equipment based on the four group guidelines as recommended by the Internal Revenue Service in the 1962 Federal regulations. These values, along with similar values as presented in Bulletin “F,” can serve as an indication of acceptable and useful lives to those not using other procedures.

†For an up-to-date presentation of Federal income-tax regulations as related to depreciation, including estimation of service lives, see the latest annual issue of “Prentice-Hall Federal Taxes,” Prentice-Hall Information Services, Paramus, NJ 07652.
TABLE 1
Estimated life of equipment

The following tabulation for estimating the life of equipment in years is an abridgement of information from “Depreciation-Guidelines and Rules” (Rev. Proc. 62-21) issued by the Internal Revenue Service of the U.S. Treasury Department as Publication No. 456 (7-62) in July, 1962. See Table 2 for an extended and more flexible interpretation including repair allowance as approved by the Federal regulations in 1971.

<table>
<thead>
<tr>
<th>Group</th>
<th>Description</th>
<th>Life, years</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>General business assets</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Office furniture, fixtures, machines, equipment</td>
<td>10</td>
</tr>
<tr>
<td>2.</td>
<td>Transportation</td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>Aircraft</td>
<td>6</td>
</tr>
<tr>
<td>b.</td>
<td>Automobile</td>
<td>3</td>
</tr>
<tr>
<td>c.</td>
<td>Buses</td>
<td>9</td>
</tr>
<tr>
<td>d.</td>
<td>General-purpose trucks</td>
<td>4-6</td>
</tr>
<tr>
<td>e.</td>
<td>Railroad cars (except for railroad companies)</td>
<td>15</td>
</tr>
<tr>
<td>f.</td>
<td>Tractor units</td>
<td>4</td>
</tr>
<tr>
<td>g.</td>
<td>Trailers</td>
<td>6</td>
</tr>
<tr>
<td>h.</td>
<td>Water transportation equipment</td>
<td>18</td>
</tr>
<tr>
<td>3.</td>
<td>Land and site improvements (not otherwise covered)</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>Buildings (apartments, banks, factories, hotels, stores, warehouses)</td>
<td>40-60</td>
</tr>
<tr>
<td>II</td>
<td>Nonmanufacturing activities (excluding transportation, communications, and public utilities)</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Agriculture</td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>Machinery and equipment</td>
<td>10</td>
</tr>
<tr>
<td>b.</td>
<td>Animals</td>
<td>3-10</td>
</tr>
<tr>
<td>c.</td>
<td>Trees and vines</td>
<td>variable</td>
</tr>
<tr>
<td>d.</td>
<td>Farm buildings</td>
<td>25</td>
</tr>
<tr>
<td>2.</td>
<td>Contract construction</td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>General</td>
<td>5</td>
</tr>
<tr>
<td>b.</td>
<td>Marine</td>
<td>12</td>
</tr>
<tr>
<td>3.</td>
<td>Fishing</td>
<td>variable</td>
</tr>
<tr>
<td>4.</td>
<td>Logging and sawmilling</td>
<td>6-10</td>
</tr>
<tr>
<td>5.</td>
<td>Mining (excluding petroleum refining and smelting and refining of minerals)</td>
<td>10</td>
</tr>
<tr>
<td>6.</td>
<td>Recreation and amusement</td>
<td>10</td>
</tr>
<tr>
<td>7.</td>
<td>Services to general public</td>
<td>10</td>
</tr>
<tr>
<td>8.</td>
<td>Wholesale and retail trade</td>
<td>10</td>
</tr>
<tr>
<td>III</td>
<td>Manufacturing</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Aerospace industry</td>
<td>8</td>
</tr>
<tr>
<td>2.</td>
<td>Apparel and textile products</td>
<td>9</td>
</tr>
<tr>
<td>3.</td>
<td>Cement (excluding concrete products)</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>Chemicals and allied products</td>
<td>11</td>
</tr>
<tr>
<td>5.</td>
<td>Electrical equipment</td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>Electrical equipment in general</td>
<td>12</td>
</tr>
<tr>
<td>b.</td>
<td>Electronic equipment</td>
<td>8</td>
</tr>
<tr>
<td>6.</td>
<td>Fabricated metal products</td>
<td>12</td>
</tr>
<tr>
<td>7.</td>
<td>Food products, except grains, sugar and vegetable oil products</td>
<td>12</td>
</tr>
<tr>
<td>8.</td>
<td>Glass products</td>
<td>14</td>
</tr>
<tr>
<td>9.</td>
<td>Grain and grain-mill products</td>
<td>17</td>
</tr>
</tbody>
</table>

(Continued)
<table>
<thead>
<tr>
<th>Group III: Manufacturing (continued)</th>
<th>Life, years</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. Knitwear and knit products</td>
<td>9</td>
</tr>
<tr>
<td>11. Leather products</td>
<td>11</td>
</tr>
<tr>
<td>12. Lumber, wood products, and furniture</td>
<td>10</td>
</tr>
<tr>
<td>13. Machinery unless otherwise listed</td>
<td>12</td>
</tr>
<tr>
<td>14. Metalworking machinery</td>
<td>12</td>
</tr>
<tr>
<td>15. Motor vehicles and parts</td>
<td>12</td>
</tr>
<tr>
<td>16. Paper and allied products</td>
<td></td>
</tr>
<tr>
<td>a. Pulp and paper</td>
<td></td>
</tr>
<tr>
<td>b. Paper conversion</td>
<td></td>
</tr>
<tr>
<td>17. Petroleum and natural gas</td>
<td></td>
</tr>
<tr>
<td>a. Contract drilling and field service</td>
<td>6</td>
</tr>
<tr>
<td>b. Company exploration, drilling, and production</td>
<td>14</td>
</tr>
<tr>
<td>c. Petroleum refining</td>
<td></td>
</tr>
<tr>
<td>d. Marketing</td>
<td></td>
</tr>
<tr>
<td>18. Plastic products</td>
<td></td>
</tr>
<tr>
<td>19. Primary metals</td>
<td></td>
</tr>
<tr>
<td>a. Ferrous metals</td>
<td></td>
</tr>
<tr>
<td>b. Nonferrous metals</td>
<td></td>
</tr>
<tr>
<td>20. Printing and publishing</td>
<td></td>
</tr>
<tr>
<td>21. Scientific instruments, optical and clock manufacturing</td>
<td>12</td>
</tr>
<tr>
<td>22. Railroad transportation equipment</td>
<td>12</td>
</tr>
<tr>
<td>23. Rubber products</td>
<td></td>
</tr>
<tr>
<td>24. Ship and boat building</td>
<td></td>
</tr>
<tr>
<td>25. Stone and clay products</td>
<td></td>
</tr>
<tr>
<td>26. Sugar products</td>
<td></td>
</tr>
<tr>
<td>27. Textile mill products</td>
<td>12-14</td>
</tr>
<tr>
<td>28. Tobacco products</td>
<td></td>
</tr>
<tr>
<td>29. Vegetable oil products</td>
<td></td>
</tr>
<tr>
<td>30. Other manufacturing in general</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group IV: Transportation, communications, and public utilities</th>
<th>Life, years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Air transport</td>
<td>6</td>
</tr>
<tr>
<td>2. Central steam production and distribution</td>
<td>28</td>
</tr>
<tr>
<td>3. Electric utilities</td>
<td></td>
</tr>
<tr>
<td>a. Hydraulic</td>
<td>50</td>
</tr>
<tr>
<td>b. Nuclear</td>
<td>20</td>
</tr>
<tr>
<td>c. Steam</td>
<td>28</td>
</tr>
<tr>
<td>d. Transmission and distribution</td>
<td>30</td>
</tr>
<tr>
<td>4. Gas utilities</td>
<td></td>
</tr>
<tr>
<td>a. Distribution</td>
<td>35</td>
</tr>
<tr>
<td>b. Manufacture</td>
<td>30</td>
</tr>
<tr>
<td>c. Natural-gas production</td>
<td>14</td>
</tr>
<tr>
<td>d. Trunk pipelines and storage</td>
<td>22</td>
</tr>
<tr>
<td>5. Motor transport (freight)</td>
<td>8</td>
</tr>
<tr>
<td>6. Motor transport (passengers)</td>
<td>8</td>
</tr>
<tr>
<td>7. Pipeline transportation</td>
<td>22</td>
</tr>
<tr>
<td>8. Radio and television broadcasting</td>
<td>6</td>
</tr>
</tbody>
</table>

(Continued)
Table 1 gives a partial listing of the **Class Life Asset Depreciation Range System (CLADR)** as recommended for use by Federal regulations in 1971. The table shows the basic guideline life period as recommended in the earlier regulations along with the 20 percent variation allowed plus recommended guidelines for repair and maintenance allowance. Although these values are recommended by the Internal Revenue Service, the IRS does not require taxpayers to use the indicated lives. However, if other life periods are used, the taxpayer must be prepared to support the claim.

Tax-law changes put into effect with the 1981 Economic Recovery Act and modified in 1986 have instituted a new system of depreciation known as the **Accelerated Cost Recovery System (ACRS)**. The latter has replaced the former ADR system for most tangible depreciable property used in a trade or business placed in service on or after January 1, 1981. In the ACRS [or **Modified Accelerated Cost Recovery System (MACRS)** which went into effect for property put into service on or after January 1, 1987], the recovery of capital costs as depreciation was determined over statutory periods of time using statutory percentages depending on the **class life** of the property and the number of years since the property was placed in service. The statutory periods of time were generally shorter than the useful life of the asset or the period for which it was used to produce income.

The statutory class lives for the Modified Accelerated Cost Recovery System are as follows where the key factor is the ADR (Asset Depreciation Range) midpoint designation, which corresponds in general to the asset guideline period shown in Table 2:

Three-year class-ADR midpoint of 4 years and less. This includes items such as machinery and equipment used in research, some automobiles, and certain types of trailers.

Five-year class-ADR midpoint of 4 to 10 years. This includes most production machinery, heavy trucks, and some automobiles and light trucks.
<table>
<thead>
<tr>
<th>Description of class life asset</th>
<th>Lower period (Midpoint)</th>
<th>Upper limit</th>
<th>Annual asset guideline repair allowance, percentage of cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets used in business activities:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Office furniture, <strong>fixtures</strong>, and equipment</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Information systems, computers, peripheral equipment</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Data handling equipment, except computers</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Airplanes, except commercial</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Automobiles, taxis</td>
<td>2.5</td>
<td>3</td>
<td>3.5</td>
</tr>
<tr>
<td>Buses</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>Light general-purpose trucks</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Heavy general-purpose trucks</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Railroad cars and locomotives, except owned by railroad transportation companies</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Tractor units for use over-the-road</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Trailers and trailer-mounted containers</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Vessels, barges, tugs and similar <strong>water-transportation equipment</strong></td>
<td>14.5</td>
<td>18</td>
<td>21.5</td>
</tr>
<tr>
<td>Land improvements</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial steam and electricity generation and/or distribution systems</td>
<td>22.5</td>
<td>33.5</td>
<td>33.5</td>
</tr>
<tr>
<td><strong>Assets used in agriculture</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets used in agriculture</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td><strong>Assets used in mining</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets used in mining</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td><strong>Assets used in drilling of oil and gas wells</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets used in exploration for and production of petroleum and natural gas deposits</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td><strong>Assets used in petroleum refining</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets used in petroleum refining</td>
<td>13</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td><strong>Assets used in marketing of petroleum and petroleum products</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets used in contract construction other than marine</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td><strong>Assets used in marine contract construction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets used in marine contract construction</td>
<td>9.5</td>
<td>12</td>
<td>14.5</td>
</tr>
</tbody>
</table>

*Values were excerpted from the listing given with full description of each category in the 1988 “Prentice Hall Federal Taxes” guide as updated from the original Federal regulation to the March 21, 1977 Revenue Procedure in the Internal Revenue Bulletin. The official documents originally setting up the ADR system were U.S. Treasury Decision 7128 in 1971 and Revenue Procedure 71-25 in 1971.*
### Table 2
Class life asset depreciation range (Continued)

<table>
<thead>
<tr>
<th>Description of class life asset</th>
<th>Asset depreciation range (ADR) (in years)</th>
<th>Annual asset guideline repair allowance, percentage of cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower limit</td>
<td>Upper limit</td>
</tr>
<tr>
<td><strong>Assets used in the manufacture of:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grain and grain-mill products</td>
<td>9.5</td>
<td>12</td>
</tr>
<tr>
<td>Sugar and sugar products</td>
<td>13.5</td>
<td>17</td>
</tr>
<tr>
<td>Vegetable oils and vegetable-oil products</td>
<td>14.5</td>
<td>18</td>
</tr>
<tr>
<td>Other food and kindred products</td>
<td>14.5</td>
<td>18</td>
</tr>
<tr>
<td>Tobacco and tobacco products</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Knitted goods</td>
<td>6</td>
<td>7.5</td>
</tr>
<tr>
<td><strong>Nonwoven fabrics</strong></td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Wood products and furniture</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Pulp and paper</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Chemicals and allied products</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>Rubber products</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Finished plastic products</td>
<td>9</td>
<td>11</td>
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<tr>
<td>Glass products</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Cement</td>
<td>16</td>
<td>20</td>
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<tr>
<td>Machinery</td>
<td>8</td>
<td>10</td>
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<tr>
<td>Electrical equipment</td>
<td>9.5</td>
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<tr>
<td>Motor vehicles</td>
<td>9.5</td>
<td>12</td>
</tr>
<tr>
<td><strong>Assets used in electric, gas, water, and steam utility services:</strong></td>
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<tr>
<td>Electric utility nuclear production plant</td>
<td>16</td>
<td>20</td>
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<tr>
<td>Electric utility steam production plant</td>
<td>22.5</td>
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<tr>
<td>Electric utility transmission and distribution plant</td>
<td>24</td>
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<tr>
<td>Gas utility distribution facilities</td>
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<td>35</td>
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<tr>
<td>Gas utility manufactured gas production plant</td>
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<tr>
<td><strong>Substitute natural gas-coal gasification (Lurgi process with advanced methanation)</strong></td>
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<td>Natural gas production plant</td>
<td>14.5</td>
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<td>Liquefied natural gas plant</td>
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<td>14</td>
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<tr>
<td>Water utilities</td>
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<td>22</td>
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<tr>
<td>Central steam utility production and distribution</td>
<td>40</td>
<td>50</td>
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<tr>
<td></td>
<td>22.5</td>
<td>28</td>
</tr>
</tbody>
</table>

†Values were excerpted from the listing given with full description of each category in the 1988 “Prentice Hall Federal Taxes” guide as updated from the original Federal regulation to the March 21, 1977 Revenue Procedure in the Internal Revenue Bulletin. The official documents originally setting up the ADR system were U.S. Treasury Decision 7128 in 1971 and Revenue Procedure 71-25 in 1971.
Seven-year class-ADR midpoint of 10 to 16 years. Included here are items such as office furniture and equipment.

Ten-year class-ADR midpoint of 16 to 20 years. This includes properties such as tank cars and assets used in petroleum refining and food manufacturing.

Fifteen-year class-ADR midpoint of 20 to 25 years. Included here are items related to certain chemical production processes and some utilities.

Twenty-year class-ADR midpoint of 25 years or more. This includes many utilities and electrical distribution systems.

For the Accelerated Cost Recovery System in effect after 1980 and before 1987, the statutory classes were 3 year, 5 year, 10 year, and 15 year. The two classes of 7 year and 20 year were added in the Modified Accelerated Cost Recovery System for properties put into service on January 1, 1987 or later.

There has been considerable demand for a wider choice of service lives for properties, and the widespread revision and reinterpretation of the national income-tax laws in 1954, 1962, 1971, 1981, and 1986 met part of this demand. During times of national emergencies, the United States Congress may approve rapid-amortization policies to make it more attractive for concerns to invest in additional plants and equipment needed for the national welfare. Certificates of necessity can be obtained for certain types of industries, and these certificates permit writing off various percentages of the value of new equipment over selected periods of time.

**SALVAGE VALUE**

_Salvage_ value is the net amount of money obtainable from the sale of used property over and above any charges involved in removal and sale. If a property is capable of further service, its salvage value may be high. This is not necessarily true, however, because other factors, such as location of the property, existing price levels, market supply and demand, and difficulty of dismantling, may have an effect. The term salvage _value_ implies that the asset can give some type of further service and is worth more than merely its scrap or junk value.

If the property cannot be disposed of as a useful unit, it can often be dismantled and sold as junk to be used again as a manufacturing raw material. The profit obtainable from this type of disposal is known as the _scrap_, or _junk_, _value_.

Salvage value, scrap value, and service life are usually estimated on the basis of conditions at the time the property is put in use. These factors cannot be predicted with absolute accuracy, but improved estimates can be made as the property increases in age. It is advisable, therefore, to make new estimates from time to time during the service life and make any necessary adjustments in the
depreciation costs. Because of the difficulties involved in making reliable estimates of salvage and scrap values, engineers often neglect the small error involved and designate these values as zero. Federal tax regulations generally limit salvage or scrap values to 10 percent or less of the initial value of the property.

**PRESENT VALUE**

The *present value* of an asset may be defined as the value of the asset in its condition at the time of valuation. There are several different types of present values, and the standard meanings of the various types should be distinguished.

**Book Value, or Unamortized Cost**

The difference between the original cost of a property, and all the depreciation charges made to date is defined as the *book value* (sometimes called *unamortized cost*). It represents the worth of the property as shown on the owner’s accounting records.

**Market Value**

The price which could be obtained for an asset if it were placed on sale in the open market is designated as the *market value*. The use of this term conveys the idea that the asset is in good condition and that a buyer is readily available.

**Replacement Value**

The cost necessary to replace an existing property at any given time with one at least equally capable of rendering the same service is known as the *replacement value*.

It is difficult to predict future market values or replacement values with a high degree of accuracy because of fluctuations in market demand and price conditions. On the other hand, a future book value can be predicted with absolute accuracy as long as a constant method for determining depreciation costs is used. It is quite possible for the market value, replacement value, and book value of a property to be widely different from one another because of unrealistic depreciation allowances or changes in economic and technological factors.

†See Chap. 11 (Optimum Design and Design Strategy) for a discussion on inflation and the strategy for considering it.
METHODS FOR DETERMINING DEPRECIATION

Depreciation costs can be determined by a number of different methods, and the design engineer should understand the bases for the various methods. The Federal government has definite rules and regulations concerning the manner in which depreciation costs may be determined. These regulations must be followed for income-tax purposes as well as to obtain most types of governmental support. Since the methods approved by the government are based on sound economic procedures, most industrial concerns use one of the government-sanctioned methods for determining depreciation costs, both for income-tax calculations and for reporting the concern’s costs and profits. It is necessary, therefore, that the design engineer keep abreast of current changes in governmental regulations regarding depreciation allowances.

In general, depreciation accounting methods may be divided into two classes: (1) arbitrary methods giving no consideration to interest costs, and (2) methods taking into account interest on the investment. Straight-line, declining-balance, and sum-of-the-years-digits methods are included in the first class, while the second class includes the sinking-fund and the present-worth methods.

Straight-Line Method

In the straight-line method for determining depreciation, it is assumed that the value of the property decreases linearly with time. Equal amounts are charged for depreciation each year throughout the entire service life of the property. The annual depreciation cost may be expressed in equation form as follows:

\[
d = \frac{V - V_s}{n}
\]

where
\[d\] = annual depreciation, $/year
\[V\] = original value of the property at start of the service-life period, completely installed and ready for use, dollars
\[V_s\] = salvage value of property at end of service life, dollars
\[n\] = service life, years

The asset value (or book value) of the equipment at any time during the service life may be determined from the following equation:

\[
V_a = V - a d
\]

†An alternate procedure often used by industrial concerns is to use straight-line depreciation for reporting profits and one of the accelerated-depreciation methods as approved by Federal regulations for income-tax calculations.
where \( V_a \) = asset or book value, dollars, and \( a \) = the number of years in actual use.

Because of its simplicity, the straight-line method is widely used for determining depreciation costs. In general, design engineers report economic evaluations on the basis of straight-line depreciation unless there is some specific reason for using one of the other methods.

Because it is impossible to estimate exact service lives and salvage values when a property is first put into use, it is sometimes desirable to reestimate these factors from time to time during the life period of the property. If this is done, straight-line depreciation can be assumed during each of the periods, and the overall method is known as multiple straight-line depreciation. Figure 9-1 shows how the asset value of a property varies with time using the straight-line and the multiple straight-line methods for determining depreciation.

The straight-line method may be applied on the basis of units of production or predicted amount of service output, instead of life years. The depreciation may be based on miles, gallons, tons, number of unit pieces produced, or other measures of service output. This so-called unit-of-production or service-output method is particularly applicable when depletion occurs, as in the exploitation of natural resources. It should also be considered for properties having useful lives that are more dependent on the number of operations performed than on calendar time.
Declining-Balance (or Fixed Percentage) Method

When the declining-balance method is used, the annual depreciation cost is a fixed percentage of the property value at the beginning of the particular year. The fixed-percentage (or declining-balance) factor remains constant throughout the entire service life of the property, while the annual cost for depreciation is different each year. Under these conditions, the depreciation cost for the first year of the property’s life is $V f$, where $f$ represents the fixed-percentage factor.

At the end of the first year

$$ Asset\ value = V_a = V(1 - f) \quad (3) $$

At the end of the second year

$$ V_a = V(1 - f)^2 \quad (4) $$

At the end of $a$ years

$$ V_a = V(1 - f)^a \quad (5) $$

At the end of $n$ years (i.e., at the end of service life)

$$ V_a = V(1 - f)^n = V_s \quad (6) $$

Therefore,

$$ f = 1 - \left( \frac{V_s}{V} \right)^{1/n} \quad (7) $$

Equation (7) represents the textbook method for determining the fixed-percentage factor, and the equation is sometimes designated as the Matheson formula. A plot showing the change of asset value with time using this declining-balance depreciation method is presented in Fig. 9-1. Comparison with the straight-line method shows that declining-balance depreciation permits the investment to be paid off more rapidly during the early years of life. The increased depreciation costs in the early years are very attractive to concerns just starting in business, because the income-tax load is reduced at the time when it is most necessary to keep all pay-out costs at a minimum.

The textbook relationship presented in Eq. (7) is seldom used in actual practice, because it places too much emphasis on the salvage value of the property and is certainly not applicable if the salvage value is zero. To overcome this disadvantage, the value of the fixed-percentage factor is often chosen arbitrarily using a sound economic basis.

Prior to 1954, the United States government would not accept any depreciation method which permitted depreciation rates more than 50 percent greater than those involved in the straight-line method. In 1954, the laws were changed to allow rates up to twice (200 percent) those for the straight-line method. Under these conditions, one arbitrary method for choosing the value of
The fixed-percentage factor is based on the straight-line rate of depreciation during the first year.

Based on the 1954 tax revision for depreciation accounting, any method could be used if the depreciation for the first two-thirds of the useful life of the property did not exceed the total of such allowances if they had been computed by the double declining-balance method.
property than in the later life. They are particularly applicable for units in which the greater proportion of the production occurs in the early part of the useful life or when operating costs increase markedly with age.

Example 1 Determination of depreciation by straight-line and declining-balance methods. The original value of a piece of equipment is $22,000, completely installed and ready for use. Its salvage value is estimated to be $2000 at the end of a service life estimated to be 10 years. Determine the asset (or book) value of the equipment at the end of 5 years using:
(a) Straight-line method.
(b) Textbook declining-balance method.
(c) Double declining-balance (200 percent) method (i.e., the declining-balance method using a fixed-percentage factor giving a depreciation rate equivalent to twice the minimum rate with the straight-line method).

Solution
(a) Straight-line method:

\[ V = \$22,000 \]
\[ V_s = \$2000 \]
\[ n = 10 \text{ years} \]
\[ d = \frac{V - V_s}{n} = \frac{20,000}{10} = \$2000/\text{year} \]

Asset value after 5 years = \( V_a \), where \( a = 5 \), or
\[ V_a = V - ad = 22,000 - (5)(2000) = \$12,000 \]

(b) Textbook declining-balance method:
\[ f = 1 - \left( \frac{V_s}{V} \right)^{1/n} = 1 - \left( \frac{2000}{22,000} \right)^{1/10} = 0.2131 \]

Asset value after 5 years is
\[ V_a = V (1 - f)^a = (22,000)(1 - 0.2131)^5 = \$6650 \]

(c) Double declining-balance (200 percent) method:

Using the straight-line method, the minimum depreciation rate occurs in the first year when \( V = \$22,000 \) and the depreciation = \$2000. This depreciation rate is \( \frac{2000}{22,000} \), and the double declining-balance (or double fixed-percentage) factor is \( (2)(\frac{2000}{22,000}) = 0.1818 = f \). (It should be noted that the double declining-balance method is often applied to cases where the salvage value is considered to be zero. Under this condition, the double fixed-percentage factor for this example would be 0.2000.)

Asset value after 5 years is
\[ V_a = V (1 - f)^a = (22,000)(1 - 0.1818)^5 = \$8060 \]
Sum-of-the-Years-Digits Method

The **sum-of-the-years-digits method** is an arbitrary process for determining depreciation which gives results similar to those obtained by the declining-balance method. Larger costs for depreciation are allotted during the early-life years than during the later years. This method has the advantage of permitting the asset value to decrease to zero or a given salvage value at the end of the service life.

In the application of the sum-of-the-years-digits method, the annual depreciation is based on the number of service-life years remaining and the sum of the arithmetic series of numbers from 1 to \( n \), where \( n \) represents the total service life. The yearly depreciation factor is the number of useful service-life years remaining divided by the sum of the arithmetic series. This factor times the total depreciable value at the start of the service life gives the annual depreciation cost.

As an example, consider the case of a piece of equipment costing $20,000 when new. The service life is estimated to be 5 years and the scrap value $2000. The sum of the arithmetic series of numbers from 1 to \( n \) is \( 1 + 2 + 3 + 4 + 5 = 15 \). The total depreciable value at the start of the service life is $20,000 - $2000 = $18,000. Therefore, the depreciation cost for the first year is \((18,000)(\frac{5}{15}) = 6000\), and the asset value at the end of the first year is $14,000. The depreciation cost for the second year is \((18,000)(\frac{4}{15}) = 4800\). Similarly, the depreciation costs for the third, fourth, and fifth years, respectively, would be $3600, $2400, and $1200. Figure 9-1 presents a curve showing the change with time in asset value when the sum-of-the-years-digits method is used for determining depreciation.

### Sinking-Fund Method

The use of compound interest is involved in the **sinking-fund method**. It is assumed that the basic purpose of depreciation allowances is to accumulate a sufficient fund to provide for the recovery of the original capital invested in the property. An ordinary annuity plan is set up wherein a constant amount of money should theoretically be set aside each year. At the end of the service life,

\[ d_a = \text{depreciation for year} \quad a = \frac{(n - a + 1)}{n} (V - V_s) \]

\[ = \frac{2(n - a + 1)}{n(n + 1)} (V - V_s) \]

†Equations which apply for determining annual depreciation by the sum-of-the-years-digits method are
the sum of all the deposits plus accrued interest must equal the total amount of
depreciation.

Derivation of the formulas for the sinking-fund method can be accom-
plished by use of the following notations in addition to those already given:

\[ i = \text{annual interest rate expressed as a fraction} \]
\[ R = \text{uniform annual payments made at end of each year (this is the annual}
\text{depreciation cost), dollars} \]
\[ V - V_s = \text{total amount of the annuity accumulated in an estimated service life of}
\text{n years (original value of property minus salvage value at end of}
\text{service life), dollars} \]

According to the equations developed for an ordinary annuity in Chap. 7
(Interest and Investment Costs),

\[ R = \frac{(V - V_s)i}{(1 + i)^n - 1} \quad (8) \]

The amount accumulated in the fund after \( a \) years of useful life must be
equal to the total amount of depreciation up to that time. This is the same as
the difference between the original value of the property \( V \) at the start of the
service life and the asset value \( V_a \) at the end of \( a \) years. Therefore,

Total amount of depreciation after \( a \) years = \( V - V_a \) \hspace{1cm} (9)

\[ V - V_a = R \frac{(1 + i)^a - 1}{i} \quad (10) \]

Combining Eqs. (8) and (10),

\[ V - V_a = (V - V_s) \frac{(1 + i)^a - 1}{(1 + i)^n - 1} \quad (11) \]

Asset (or book) value after \( a \) years = \( V_a \)

\[ V_a = V - (V - V_s) \frac{(1 + i)^a - 1}{(1 + i)^n - 1} \quad (12) \]

Since the value of \( R \) represents the annual depreciation cost, the yearly
cost for depreciation is constant when the sinking-fund method is used. As
shown in Fig. 9-3, this method results in book values which are always greater

\[ \text{Exactly the same result for asset value after a years is obtained if an annuity due (i.e., equal}
\text{periodic payments at beginning of each year) is used in place of an ordinary annuity. The periodic}
\text{payment with an annuity due would be } \frac{R}{(1 + i)}. \text{ In accepted engineering practice, the sinking-fund}
\text{method is based on an ordinary annuity plan.} \]
than those obtained with the straight-line method. Because of the effects of interest in the sinking-fund method, the annual decrease in asset value of the property is less in the early-life years than in the later years.

Although the sinking-fund viewpoint assumes the existence of a fund into which regular deposits are made, an actual fund is seldom maintained. Instead, the money accumulated from the depreciation charges is put to work in other interests, and the existence of the hypothetical fund merely serves as a basis for this method of depreciation accounting.

The sinking-fund theory of cost accounting is now used by few concerns, although it has seen considerable service in the public-utilities field. Theoretically, the method would be applicable for depreciating any property that did not undergo heavy service demands during its early life and stood little chance of becoming obsolete or losing service value due to other functional causes.

The same approach used in the sinking-fund method may be applied by analyzing depreciation on the basis of reduction with time of future profits obtainable with a property. When this is done, it is necessary to use an interest rate equivalent to the annual rate of return expected from the use of the property. This method is known as the present-worth method and gives results similar to those obtained with the conventional sinking-fund approach. The sinking-fund and the present-worth methods are seldom used for depreciation cost accounting but are occasionally applied for purposes of comparing alternative investments.
Accelerated Cost Recovery System

The Accelerated Cost Recovery System (ACRS) is a system for determining depreciation allowances based on statutory annual percentages and class life periods established for the United States by Federal income-tax regulations. The basis for the statutory percentage factors is the declining-balance method of depreciation combined with the straight-line method. The original ACRS was in effect by Federal tax laws from 1981 through 1986 with a Modified Accelerated Cost Recovery System (MACRS) going into effect in 1987.

In general, ACRS allowed one-half of a full year’s deduction for property in the year it was placed in service and no deduction in the year when the property was anticipated to be disposed of, although special month-by-month rules could be applied for some cases. Similarly, this so-called “half-year convention” applied for MACRS as one-half of a full year’s deduction for property in the year it was placed in service, but it also allowed one-half of a full year’s deduction during the year of disposal. Thus, the years of depreciable values equaled the class years for ACRS and equaled one more than the class years for MACRS.

The bases of calculation for the statutory percentage factors which were applied to the values of the original property to determine the yearly deductions are as follows:

For ACRS, the statutory percentages were based on a HO-percent declining balance with a switch to straight-line depreciation at the time appropriate to

<table>
<thead>
<tr>
<th>Applicable recovery year</th>
<th>Applicable recovery percentage to give annual depreciation for class life of 3 years</th>
<th>5 years</th>
<th>10 years</th>
<th>15 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
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<tr>
<td>15</td>
<td></td>
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</table>
maximize the deduction. The half-year convention applied for the first year when property was placed in service. Salvage value was taken as zero.

For MACRS, the statutory percentages were based on a 200-percent declining balance for class lives of 3, 5, 7, and 10 years and a 150-percent declining balance for class lives of 15 and 20 years with a switch to straight-line depreciation at the time appropriate to maximize the deduction. The half-year convention applied for the first year when property was placed in service and also for the year of disposal. Salvage value was taken as zero.

For both ACRS and MACRS, the statutory percentages have been calculated for each of a group of class years, and these, in turn, have been related to values of the Class Life Accelerated Depreciation Range (CLADR) as noted earlier. Results are given in Table 3 for ACRS and Table 4 for MACRS.

Details, such as shown in Tables 3 and 4, are presented annually as part of the United States Federal Income Tax Regulations. Tables are also given with conventions other than the half-year conventions for MACRS, such as mid-

### TABLE 4

<table>
<thead>
<tr>
<th>Applicable recovery year</th>
<th>Applicable recovery percentage to give annual depreciation for class life of</th>
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</thead>
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<td>1</td>
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<td>20</td>
<td>4.461</td>
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<td>21</td>
<td>2.231</td>
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quarter with property placed in service in the first, second, third, or fourth quarter.?

During the period from 1981 through 1986, instead of using the applicable ACRS percentages to determine annual depreciation deductions, corporations were allowed to use straight-line depreciation over the recovery period under the following conditions:

<table>
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<th>For</th>
<th>Use recovery period of</th>
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<tbody>
<tr>
<td>3-year class-life property</td>
<td>3, 5, or 12 years</td>
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<tr>
<td>5-year class-life property</td>
<td>5, 12, or 25 years</td>
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<tr>
<td>10-year class-life property</td>
<td>10, 25, or 35 years</td>
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<tr>
<td>15-year class-life property</td>
<td>15, 35, or 45 years</td>
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</table>

Businesses were expected, in general, to conform with the Modified Accelerated Cost Recovery System to determine depreciation deductions for real and tangible property put into service after 1986 when such deductions were involved in income-tax determinations. Figure 9-4 gives a comparison of depreciation results using methods of ACRS, MACRS, and double declining balance (200 percent) with no salvage value combined with straight line.

Example 2 Determination of percentage factors as given for Modified Accelerated Cost Recovery System. Calculate the percentage factors for a class life of 10 years as presented in Table 4 of this chapter for the Modified Accelerated Cost Recovery System (MACRS). Note that MACRS is based on a 200 percent declining balance for this class life with a switch to straight-line depreciation at the time appropriate to maximize the deduction. It is also based on salvage value being zero. The half-year convention in the first and last years applies. Use an initial property value of $22,000 to permit comparison of results to Fig. 9-4.

Solution. The declining-balance equation to use is (value of property at start of year)(1 - f) = value of property at end of year with f being the declining-balance factor. The 200 percent declining-balance factor is based on two times the minimum depreciation rate which occurs in the first year when \( V = \frac{22,000}{10} = 2200 \). Thus, the 200-percent declining-balance factor is \( \frac{2(2200)}{22,000} = 0.20 = f \) which applies to each full year being considered.

For the first year, with the half-year convention, the investment is considered as being made at the midpoint of the year. Thus, the f which applies for the first year is \( \frac{2(1100)}{22,000} = 0.10 \).

Value at start of 1st year or at midpoint of 1st year = $22,000.
Value at end of 1st year = $22,000(1 - 0.10) = $19,800.
Percentage factor for 1st year = \( \frac{(22,000 - 19,800)}{22,000}(100) = 10.00\% \).

\(^\text{†See} \) Prentice-Hall Federal Taxes for the current year, Prentice-Hall Information Services, Paramus, NJ 07652.
Value at end of 2nd year = $19,800(1 - 0.20) = $15,840.
Percentage factor for 2nd year = $[(19,800 - 15,840) / 22,000]$ (100) = 18.00%.
Value at end of 3rd year = $15,840(1 - 0.20) = $12,672.
Percentage factor for 3rd year = $[(15,840 - 12,672) / 22,000]$ (100) = 14.40%.
Value at end of 4th year = $12,672(1 - 0.20) = $10,138.
Percentage factor for 4th year = $[(12,672 - 10,138) / 22,000]$ (100) = 11.52%.
Value at end of 5th year = $10,138(1 - 0.20) = $8110.
Percentage factor for 5th year = $[(10,138 - 8110) / 22,000]$ (100) = 9.22%.
Value at end of 6th year = $8110(1 - 0.20) = $6488.
Percentage factor for 6th year = $[(8110 - 6488) / 22,000]$ (100) = 7.37%.
At this point, any further use of the declining-balance factor for the remaining 4.5 years of property life results in a deduction less than that obtained with the straight-line depreciation method. If one stays with the **200-percent** reducing-balance for the 7th year, the amount of depreciation in the 7th year would be $6488 - 6488(1 - 0.20) = $1298 compared to $6488/4.5 = $1442 if the switch is made to straight-line depreciation. Therefore, switch to straight-line method for the remaining 4.5 years of life with the annual depreciation to reduce the property value to zero after 4.5 years being $1442/year.

Value at end of 7th year $= 6488 - 1442 = $5046.$
Percentage factor for 7th year $= (1442/22,000)(100) = 6.55\%$ or 6.56\%.
Value at end of 8th year $= 5046 - 1442 = $3604.$
Percentage factor for 8th, 9th, and 10th years $= (1442/22,000)(100) = 6.55\%.$
Value at end of 10th year $= 3604 - (2)(1442) = $720.$
Percentage factor for 11th year up to half year $= (720/22,000)(100) = 6.55/2 = 3.28\%.$

These percentages agree with those presented in Table 4 of this Chapter, and the year-end values agree with those shown in Fig. 9-4.

**SINGLE-UNIT AND GROUP DEPRECIATION**

In depreciation accounting procedures, assets may be depreciated on the basis of individual units or on the basis of various types of property groups or classifications. The **single-unit method** requires keeping records on each individual asset. Although the application of this method is simple, the large number of detailed records required makes the accounting expenses very high.

To simplify the accounting procedures, many concerns combine their various assets into groups for depreciation purposes. There are several types of group accounts employed, and the most common among these are composite accounts, classified accounts, and vintage-group accounts.

A **composite** account includes all depreciable assets in one single group, and an overall depreciation rate is applied to the entire account. With this method, the composite depreciation rate must be redetermined when important changes occur in the relative distribution of the service lives of the individual assets.

Instead of including all assets in a single depreciation account, it is possible to classify properties into general types, such as machinery and equipment, office furniture and fixtures, buildings, and transportation equipment. The records for these groups are known as **classified accounts**. A classified account is similar to a composite account because many items are included in the same group, regardless of life characteristics.
Another approach to group depreciation is to include in each account all similar assets having approximately the same service lives. These accounts are known as *vintage-group accounts*. A separate record is kept for each group and the same depreciation rate is applied to all the items included in each account. With this method, the advantages of single-unit depreciation are obtained since life characteristics serve as the basis. If a large number of items are contained in a vintage-group account, the overall depreciation results can be quite accurate because the law of averages will apply to the true service lives as compared with the estimated service lives.

**ADJUSTMENT OF DEPRECIATION ACCOUNTS**

The estimated service life and salvage value of a property are seldom exactly equal to the actual service life and salvage value. It is, therefore, advisable to adjust depreciation accounts by making periodic reestimations of the important variables. When a property is retired under conditions which do not permit exact agreement between estimated and actual values, the difference between the book depreciation and the actual depreciation may be handled in one of the following ways: (1) The gain or loss may be credited or charged on the financial record for the current period; (2) the difference may be credited or charged to a special depreciation reserve; or (3) the difference may be carried on the books for amortization during a reasonable future period.

According to the Federal income-tax laws, any gain on the retirement of a property is taxed as a capital gain. However, losses cannot be subtracted from the taxable income unless the maximum expected life was used. Because of the losses involved when a property must be retired before the end of its estimated service life, some concerns prefer to use a combination of methods 2 and 3 indicated in the preceding paragraph. A special depreciation reserve is built up by continuing the book depreciation of properties whose actual service lives exceed the estimated service lives. This fund is then used to handle losses due to early retirement of assets. The final choice of method for adjusting depreciation accounts depends on the accounting policies of the individual concern and on income-tax regulations.

**EVALUATION OF DEPRECIATION METHODS**

Comparison of the various depreciation methods shows that the declining-balance and the sum-of-the-years-digits methods give similar results. In both cases, the depreciation costs are greater in the early-life years of the property than in the later years. Annual depreciation costs are constant when the straight-line, sinking-fund, or present-worth method is used. Because interest effects are included in the sinking-fund and present-worth methods, the annual decrease in asset value with these two methods is lower in the early-life years than in the
later years. The straight-line method is widely used for depreciation cost accounting because it is very simple to apply, both to groups and single units, and it is acceptable for cost-accounting purposes and for some income-tax determinations.

From the viewpoint of financial protection, it is desirable to make a greater charge for property depreciation during early life than during later life. This can be accomplished by use of the declining-balance or sum-of-the-years-digits method. The difficulties of accurate application to group accounts and income-tax restrictions have served to suppress the widespread usage of these methods. However, in recent years, a large number of industrial concerns have started using declining-balance and sum-of-the-years-digits depreciation, with many companies finding it desirable to use the combination method approved by Federal income-tax regulations of declining balance plus straight-line with statutory percentages for each year based on property life class.

The liberalized tax laws passed in 1954 first permitted use of double declining-balance depreciation as well as sum-of-the-years-digits depreciation for income-tax calculations. In general, these laws gave approval to any depreciation method which did not give faster write-offs during the first two-thirds of an asset’s useful life than the double declining-balance method. These regulations were not applicable to assets with service lives of less than 3 years.

The final choice of the best depreciation method depends on a number of different factors. The type and function of the property involved is, of course, one important factor. Also, it is desirable to use a simple formula giving results as accurate as the estimated values involved. The advisability of keeping two separate sets of books, one for income-tax purposes and one for company purposes, should be considered. The final decision involves application of good judgment and an analysis of the existing circumstances.

**NOMENCLATURE FOR CHAPTER 9**

\[ a = \text{length of time in actual use, years} \]
\[ d = \text{annual depreciation, $/year} \]
\[ f = \text{fixed-percentage or declining-balance factor} \]
\[ i = \text{annual interest rate expressed as a fraction, percent/100} \]
\[ n = \text{service life, years} \]
\[ R = \text{uniform annual payments made in an ordinary annuity, dollars} \]
\[ V = \text{original value of a property at start of service-life period, completely installed and ready for use, dollars} \]
\[ V_a = \text{asset or book value, dollars} \]
\[ V_s = \text{salvage value of property at end of service life, dollars} \]

**PROBLEMS**

1. A reactor of special design is the major item of equipment in a small chemical plant. The initial cost of the completely installed reactor is $60,000, and the salvage value at the end of the useful life is estimated to be $10,000. Excluding depreciation costs
for the reactor, the total annual expenses for the plant are $100,000. How many years of useful life should be estimated for the reactor if 12 percent of the total annual expenses for the plant are due to the cost for reactor depreciation? The straight-line method for determining depreciation should be used.

2. The initial installed cost for a new piece of equipment is $10,000, and its scrap value at the end of its useful life is estimated to be $2000. The useful life is estimated to be 10 years. After the equipment has been in use for 4 years, it is sold for $7000. The company which originally owned the equipment employs the straight-line method for determining depreciation costs. If the company had used an alternative method for determining depreciation costs, the asset (or book) value for the piece of equipment at the end of 4 years would have been $5240. The total income-tax rate for the company is 34 percent of all gross earnings. Capital-gains taxes amount to 34 percent of the gain. How much net saving after taxes would the company have achieved by using the alternative (in this case, reducing-balance) depreciation method instead of the straight-line depreciation method?

3. A piece of equipment originally costing $40,000 was put into use 12 years ago. At the time the equipment was put into use, the service life was estimated to be 20 years and the salvage and scrap value at the end of the service life were assumed to be zero. On this basis, a straight-line depreciation fund was set up. The equipment can now be sold for $10,000, and a more advanced model can be installed for $55,000. Assuming the depreciation fund is available for use, how much new capital must be supplied to make the purchase?

4. The original investment for an asset was $10,000, and the asset was assumed to have a service life of 12 years with $2000 salvage value at the end of the service life. After the asset has been in use for 5 years, the remaining service life and final salvage value are reestimated at 10 years and $1000, respectively. Under these conditions, what is the depreciation cost during the sixth year of the total life if straight-line depreciation is used?

5. A property has an initial value of $50,000, service life of 20 years, and final salvage value of $4000. It has been proposed to depreciate the property by the text-book declining-balance method. Would this method be acceptable for income-tax purposes if the income-tax laws do not permit annual depreciation rates greater than twice the minimum annual rate with the straight-line method?

6. A piece of equipment having a negligible salvage and scrap value is estimated to have a service life of 10 years. The original cost of the equipment was $40,000. Determine the following:
   (a) The depreciation charge for the fifth year if double declining-balance depreciation is used.
   (b) The depreciation charge for the fifth year if sum-of-the-years-digits depreciation is used.
   (c) The percent of the original investment paid off in the first half of the service life using the double declining-balance method.
   (d) The percent of the original investment paid off in the first half of the service life using the sum-of-the-years-digits method.

7. The original cost of a property is $30,000, and it is depreciated by a 6 percent sinking-fund method. What is the annual depreciation charge if the book value of the property after 10 years is the same as if it had been depreciated at $2500/year by the straight-line method?
8. A concern has a total income of $1 million/year, and all expenses except depreciation amount to $600,000/year. At the start of the first year of the concern’s operation, a composite account of all depreciable items shows a value of $850,000, and the overall service life is estimated to be 20 years. The total salvage value at the end of the service life is estimated to be $50,000. Thirty percent of all profits before taxes must be paid out as income taxes. What would be the reduction in income-tax charges for the first year of operation if the sum-of-the-years-digits method were used for depreciation accounting instead of the straight-line method?

9. The total value of a new plant is $2 million. A certificate of necessity has been obtained permitting a write-off of 60 percent of the initial value in 5 years. The balance of the plant requires a write-off period of 15 years. Using the straight-line method and assuming negligible salvage and scrap value, determine the total depreciation cost during the first year.

10. A profit-producing property has an initial value of $50,000, a service life of 10 years, and zero salvage and scrap value. By how much would annual profits before taxes be increased if a 5 percent sinking-fund method were used to determine depreciation costs instead of the straight-line method?

11. In order to make it worthwhile to purchase a new piece of equipment, the annual depreciation costs for the equipment cannot exceed $3000 at any time. The original cost of the equipment is $30,000, and it has zero salvage and scrap value. Determine the length of service life necessary if the equipment is depreciated (a) by the sum-of-the-years-digits method, and (b) by the straight-line method.

12. The owner of a property is using the unit-of-production method for determining depreciation costs. The original value of the property is $55,000. It is estimated that this property can produce 5500 units before its value is reduced to zero: i.e., the depreciation cost per unit produced is $10. The property produces 100 units during the first year, and the production rate is doubled each year for the first 4 years. The production rate obtained in the fourth year is then held constant until the value of the property is paid off. What would have been the annual depreciation cost if the straight-line method based on time had been used?

13. Calculate the percentage factors for a class life of 10 years as presented in Table 3 of this chapter for the Accelerated Cost Recovery System (ACRS). Note that ACRS is based on a HO-percent declining balance with switch to straight-line depreciation at the time appropriate to maximize the deduction. It is also based on salvage value being zero. The half-year convention in the first year applies, but the last half-year deduction cannot be claimed as such. Use an initial property value of $22,000 to permit comparison to Fig. 9-4 and Example 2.

14. A materials-testing machine was purchased for $20,000 and was to be used for 5 years with an expected residual salvage value of $5000. Graph the annual depreciation charges and year-end book values obtained by using:
   (a) Straight-line depreciation.
   (b) Sum-of-digits depreciation.
   (c) Double-declining-balance depreciation.
   (d) ACRS with 5-year property recovery.

15. An asset with an original cost of $10,000 and no salvage value has a depreciation charge of $2381 during its second year of service when depreciated by the sum-of-digits method. What is its expected useful life?
The word **profitability** is used as the general term for the measure of the amount of profit that can be obtained from a given situation. Profitability, therefore, is the common denominator for all business activities.

Before capital is invested in a project or enterprise, it is necessary to know how much profit can be obtained and whether or not it might be more advantageous to invest the capital in another form of enterprise. Thus, the determination and analysis of profits obtainable from the investment of capital and the choice of the best investment among various alternatives are major goals of an economic analysis.

There are many reasons why capital investments are made. Sometimes, the purpose is merely to supply a service which cannot possibly yield a monetary profit, such as the provision of recreation facilities for free use of employees. The profitability for this type of venture cannot be expressed on a positive numerical basis. The design engineer, however, usually deals with investments which are expected to yield a tangible profit.

Because profits and costs are considered which will occur in the future, the possibilities of inflation or deflation affecting future profits and costs must be recognized. The strategy for handling effects of inflation or deflation is discussed in Chap. 11.

Investments may be made for replacing or improving an existing property, for developing a completely new enterprise, or for other purposes wherein a
profit is expected from the outlay of capital. For cases of this sort, it is extremely important to make a careful analysis of the capital utilization.

**PROFITABILITY STANDARDS**

In the process of making an investment decision, the profits anticipated from the investment of funds should be considered in terms of a minimum profitability standard. This profitability standard, which can normally be expressed on a direct numerical basis, must be weighed against the overall judgment evaluation for the project in making the final decision as to whether or not the project should be undertaken.

The judgment evaluation must be based on the recognition that a quantified profitability standard can serve only as a guide. Thus, it must be recognized that the profit evaluation is based on a prediction of future results so that assumptions are necessarily included. Many intangible factors, such as future changes in demand or prices, possibility of operational failure, or premature obsolescence, cannot be quantitized. It is in areas of this type that judgment becomes critical in making a final investment decision.

A primary factor in the judgment decision is the consideration of possible alternatives. For example, the alternatives to continuing the operation of an existing plant may be to replace it with a more efficient plant, to discontinue the operation entirely, or to make modifications in the existing plant. In reaching the final decision, the alternatives should be considered two at a time on a mutually exclusive basis.

An obvious set of alternatives involves either making the capital investment in a project or investing the capital in a safe venture for which there is essentially no risk and a guaranteed return. In other words, the second alternative involves the company’s decision as to the cost of capital.

**Cost of Capital**

Methods for including the cost of capital in economic analyses have been discussed in Chap. 7. Although the management and stockholders of each company must establish the company’s characteristic cost of capital, the simplest approach is to assume that investment of capital is made at a hypothetical cost or rate of return equivalent to the total profit or rate of return over the full expected life of the particular project. This method has the advantage of putting the profitability analysis of all alternative investments on an equal basis, thereby permitting a clear comparison of risk factors. This method is particularly useful for preliminary estimates, but it may need to be refined further to take care of income-tax effects for final evaluation.

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†One often-used basis for the minimum profitability standard is the value of money to the company, expressed as a rate, based on earnings after taxes.
BASES FOR EVALUATING PROJECT PROFITABILITY

Total profit alone cannot be used as the deciding profitability factor in determining if an investment should be made. The profit goal of a company is to maximize income above the cost of the capital which must be invested to generate the income. If the goal were merely to maximize profits, any investment would be accepted which would give a profit, no matter how low the return or how great the cost. For example, suppose that two equally sound investments can be made. One of these requires $100,000 of capital and will yield a profit of $10,000/year, and the second requires $1 million of capital and will yield $25,000/year. The second investment gives a greater yearly profit than the first, but the annual rate of return on the second investment is only

\[
\left( \frac{25,000}{1,000,000} \right) \times 100 = 2.5 \text{ percent}
\]

while the annual rate of return on the $100,000 investment is 10 percent. Because reliable bonds and other conservative investments will yield annual rates of return in the range of 6 to 9 percent, the $1 million investment in this example would not be very attractive; however, the 10 percent return on the $100,000 capital would make this investment worthy of careful consideration. Thus, for this example, the rate of return, rather than the total amount of profit, is the important profitability factor in determining if the investment should be made.

The basic aim of a profitability analysis is to give a measure of the attractiveness of the project for comparison to other possible courses of action. It is, therefore, very important to consider the exact purpose of a profitability analysis before a standard reference or base case is chosen. If the purpose is merely to present the total profitability of a given project, a simple statement of total profit per year or annual rate of return may be satisfactory. On the other hand, if the purpose is to permit comparison of several different projects in which capital might be invested, the method of analysis should be such that all cases are on the same basis so that direct comparison can be made among the appropriate alternatives.

Mathematical Methods for Profitability Evaluation

The most commonly used methods for profitability evaluation, as illustrated in Fig. 10-1, can be categorized under the following headings:

1. Rate of return on investment
2. Discounted cash flow based on full-life performance
3. Net present worth
4. Capitalized costs
5. Payout period
Each of these methods has its advantages and disadvantages, and much has been written on the virtues of the various methods. Because no single method is best for all situations, the engineer should understand the basic ideas involved in each method and be able to choose the one best suited to the needs of the particular situation.

RATE OF RETURN ON INVESTMENT. In engineering economic studies, rate of return on investment is ordinarily expressed on an annual percentage basis. The yearly profit divided by the total initial investment necessary represents the fractional return, and this fraction times 100 is the standard percent return on investment.†

Profit is defined as the difference between income and expense. Therefore, profit is a function of the quantity of goods or services produced and the selling price. The amount of profit is also affected by the economic efficiency of the operation, and increased profits can be obtained by use of effective methods which reduce operating expenses.

To obtain reliable estimates of investment returns, it is necessary to make accurate predictions of profits and the required investment. To determine the profit, estimates must be made of direct production costs, fixed charges including depreciation, plant overhead costs, and general expenses. Profits may be expressed on a before-tax or after-tax basis, but the conditions should be indicated. Both working capital and fixed capital should be considered in determining the total investment.§

Returns Incorporating Minimum Profits as an Expense

The standard method for reporting rate of return on investment has been outlined in the preceding paragraphs. Another method which is sometimes used for reporting rate of return is based on the assumption that it must be possible to obtain a certain minimum profit or return from an investment before the necessary capital outlay will be desirable. This minimum profit is included as a

†The normal procedure is to base the percent return on investment on the total initial investment. However, because equipment depreciates during its useful life, it is sometimes convenient to base the rate of return on the average estimated investment during the life of the project. With this method, the rate of return is determined by dividing the average annual profit or saving by one-half the initial fixed-capital investment (or initial fixed-capital investment minus the estimated salvage value at the end of the useful life) plus the working-capital investment.

‡An article by J. Linsley, Return on Investment: Discounted and Undiscounted, Chem. Eng., 86(11):201 (May 21, 1979), suggests that “return on investment” can be defined as net, after-tax profit plus depreciation divided by capital investment. This definition of return on investment where depreciation cash flow is included as part of the return is not used in this book. Instead, this method of handling cash flow is included in the profitability methods reported for discounted-cash-flow Profitability Index and Net Present Worth.

§Under some conditions, such as a profitability analysis based on a small component of an overall operation, the return on investment can be based on the fixed-capital investment instead of the total investment.
fictitious expense along with the other standard expenses. When return on investment is determined in this manner, the result shows the risk earning rate, and represents the return over and above that necessary to make the capital expenditure advisable. If the return is zero or larger, the investment will be attractive. This method is sometimes designated as return based on capital recovery with minimum profit.

The inclusion of minimum profit as an expense is rather unrealistic, especially when it is impossible to designate the exact return which would make a given investment worthwhile. One difficulty with this method is the tendency to use a minimum rate of return equal to that obtained from present investments. This, of course, gives no consideration to the element of risk involved in a new venture. Despite these objections, the use of returns incorporating
minimum profits as an expense is acceptable providing the base, or minimum return and the general method employed are clearly indicated.

Example 1 Determination of rate of return on investment—consideration of income-tax effects. A proposed manufacturing plant requires an initial fixed-capital investment of $900,000 and $100,000 of working capital. It is estimated that the annual income will be $800,000 and the annual expenses including depreciation will be $520,000 before income taxes. A minimum annual return of 15 percent before income taxes is required before the investment will be worthwhile. Income taxes amount to 34 percent of all pre-tax profits.

Determine the following:

(a) The annual percent return on the total initial investment before income taxes.
(b) The annual percent return on the total initial investment after income taxes.
(c) The annual percent return on the total initial investment before income taxes based on capital recovery with minimum profit.
(d) The annual percent return on the average investment before income taxes assuming straight-line depreciation and zero salvage value.

Solution

(a) Annual profit before income taxes = $800,000 − $520,000 = $280,000.
Annual percent return on the total initial investment before income taxes = \( \frac{280,000}{(900,000 + 100,000)} \times 100 = 28\% \).
(b) Annual profit after income taxes = ($280,000)(0.66) = $184,800.
Annual percent return on the total initial investment after income taxes = \( \frac{184,800}{(900,000 + 100,000)} \times 100 = 18.5\% \).
(c) Minimum profit required per year before income taxes = ($900,000 + $100,000)(0.15) = $150,000.
Fictitious expenses based on capital recovery with minimum profit = $520,000 + $150,000 = $670,000/year. Annual percent return on the total investment based on capital recovery with minimum annual rate of return of 15 percent before income taxes = \( \frac{(800,000 - 670,000)}{(900,000 + 100,000)} \times 100 = 13\% \).
(d) Average investment assuming straight-line depreciation and zero salvage value = $900,000/2 + $100,000 = $550,000.
Annual percent return on average investment before income taxes = \( \frac{280,000}{550,000} \times 100 = 51\% \).

The methods for determining rate of return, as presented in the preceding sections, give “point values” which are either applicable for one particular year or for some sort of “average” year. They do not consider the time value of money, and they do not account for the fact that profits and costs may vary significantly over the life of the project.

One example of a cost that can vary during the life of a project is depreciation cost. If straight-line depreciation is used, this cost will remain constant; however, it may be advantageous to employ a declining-balance or sum-of-the-years-digits method to determine depreciation costs, which will immediately result in variations in costs and profits from one year to another. Other predictable factors, such as increasing maintenance costs or changing sales volume, may also make it necessary to estimate year-by-year profits with
variation during the life of the project. For these situations, analyses of project profitability cannot be made on the basis of one point on a flat time-versus-earning curve, and profitability analyses based on discounted cash flow may be appropriate. Similarly, time-value-of-money considerations may make the discounted-cash-flow approach desirable when annual profits are constant.

DISCOUNTED CASH FLOW

Rate of Return Based on Discounted Cash Flow†

The method of approach for a profitability evaluation by discounted cash flow takes into account the time value of money and is based on the amount of the investment that is unreturned at the end of each year during the estimated life of the project. A trial-and-error procedure is used to establish a rate of return which can be applied to yearly cash flow so that the original investment is reduced to zero (or to salvage and land value plus working-capital investment) during the project life. Thus, the rate of return by this method is equivalent to the maximum interest rate (normally, after taxes) at which money could be borrowed to finance the project under conditions where the net cash flow to the project over its life would be just sufficient to pay all principal and interest accumulated on the outstanding principal.

To illustrate the basic principles involved in discounted-cash-flow calculations and the meaning of rate of return based on discounted cash flow, consider the case of a proposed project for which the following data apply:

- Initial fixed-capital investment = $100,000
- Working-capital investment = $10,000
- Service life = 5 years
- Salvage value at end of service life = $10,000

<table>
<thead>
<tr>
<th>Year</th>
<th>Predicted after-tax cash flow to project based on total income minus all costs except depreciation, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(110,000)</td>
</tr>
<tr>
<td>1</td>
<td>30,000</td>
</tr>
<tr>
<td>2</td>
<td>31,000</td>
</tr>
<tr>
<td>3</td>
<td>36,000</td>
</tr>
<tr>
<td>4</td>
<td>40,000</td>
</tr>
<tr>
<td>5</td>
<td>43,000</td>
</tr>
</tbody>
</table>

†Common names of methods of return calculations related to the discounted-cash-flow approach are profitability index, interest rate of return, true rate of return, and investor’s rate of return.
### TABLE 1

**Computation of discounted-cash-flow rate of return**

<table>
<thead>
<tr>
<th>Year (n')</th>
<th>Estimated cash flow to project, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^n} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-1}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-2}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-3}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-4}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-5}} )</th>
<th>Present value, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>110,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30,000</td>
<td>0.8696</td>
<td>26,100</td>
<td>0.8333</td>
<td>25,000</td>
<td>0.8000</td>
<td>24,000</td>
<td>0.7692</td>
<td>23,000</td>
<td>0.7389</td>
<td>22,000</td>
<td>0.7100</td>
<td>21,000</td>
</tr>
<tr>
<td>2</td>
<td>31,000</td>
<td>0.7561</td>
<td>23,400</td>
<td>0.6944</td>
<td>21,500</td>
<td>0.6400</td>
<td>19,800</td>
<td>0.5904</td>
<td>18,200</td>
<td>0.5429</td>
<td>16,500</td>
<td>0.4971</td>
<td>15,000</td>
</tr>
<tr>
<td>3</td>
<td>36,000</td>
<td>0.6575</td>
<td>23,300</td>
<td>0.5787</td>
<td>20,700</td>
<td>0.5120</td>
<td>18,400</td>
<td>0.4571</td>
<td>16,200</td>
<td>0.4096</td>
<td>14,100</td>
<td>0.3647</td>
<td>12,000</td>
</tr>
<tr>
<td>4</td>
<td>40,000</td>
<td>0.5718</td>
<td>22,900</td>
<td>0.4623</td>
<td>19,300</td>
<td>0.4096</td>
<td>16,400</td>
<td>0.3612</td>
<td>13,800</td>
<td>0.3125</td>
<td>11,300</td>
<td>0.2665</td>
<td>9,700</td>
</tr>
<tr>
<td>5</td>
<td>43,000</td>
<td>0.4971</td>
<td>31,300</td>
<td>0.4019</td>
<td>25,300</td>
<td>0.3277</td>
<td>20,600</td>
<td>0.2683</td>
<td>15,900</td>
<td>0.2207</td>
<td>11,400</td>
<td>0.1762</td>
<td>7,900</td>
</tr>
<tr>
<td>Total</td>
<td>127,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio = total present value / initial investment</td>
<td>1.155</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Trial for i = 0.15**

**Trial for i = 0.20**

**Trial for i = 0.25**

**Trial for i = 0.207**

<table>
<thead>
<tr>
<th>Year (n')</th>
<th>Estimated cash flow to project, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^n} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-1}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-2}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-3}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-4}} )</th>
<th>Present value, $</th>
<th>Discount factor, ( \frac{1}{(1+i)^{n-5}} )</th>
<th>Present value, $</th>
</tr>
</thead>
</table>

+ As illustrated in Fig. 10-2, interpolation to determine the correct rate of return can be accomplished by plotting the ratio (total present value/initial investment) versus the trial interest rate for three bracketing values and reading the correct rate from the curve where the ratio = 1.0.

**Note:** In this example, interest was compounded annually on an end-of-year basis and continuous interest compounding was ignored. Also, construction period and land value were not considered. The preceding effects could have been included in the analysis for a more sophisticated treatment using the methods presented in Examples 2 and 3 of this chapter.

Designate the discounted-cash-flow rate of return as \( i \). This rate of return represents the after-tax interest rate at which the investment is repaid by proceeds from the project. It is also the maximum after-tax interest rate at which funds could be borrowed for the investment and just break even at the end of the service life.

At the end of five years, the cash flow to the project, compounded on the basis of end-of-year income, will be

\[
(30,000)(1 + i)^4 + (31,000)(1 + i)^3 + (36,000)(1 + i)^2 + (40,000)(1 + i) + 43,000 = S
\]  

The symbol \( S \) represents the future worth of the proceeds to the project and must just equal the future worth of the initial investment compounded at an interest rate \( i \) corrected for salvage value and working capital. Thus,

\[
S = (110,000)(1 + i)^5 - 10,000 - 10,000
\]
Setting Eq. (1) equal to Eq. (2) and solving by trial and error for \( i \) gives \( i = 0.207 \), or the discounted-cash-flow rate of return is 20.7 percent.

Some of the tedious and time-consuming calculations can be eliminated by applying a discount factor to the annual cash flows and summing to get a present value equal to the required investment. The discount factor for end-of-year payments and annual compounding is

\[
d_{n'} = \frac{1}{(1 + i)^{n'}} = \text{discount factor}
\]

where \( i \) = rate of return
\( n' \) = year of project life to which cash flow applies

This discount factor, \( d_{n'} \), is the amount that would yield one dollar after \( n' \) years if invested at an interest rate of \( i \). The discounted-cash-flow rate of return can be determined by the trial-and-error method illustrated in Table 1, where the annual cash flows are discounted by the appropriate discount factor to a total present value equal to the necessary initial investment.

Example 2 Discounted-cash-flow calculations based on continuous interest compounding and continuous cash flow. Using the discount factors for continuous interest and continuous cash flow presented in Tables 5 to 8 of Chapter 7, determine the continuous discounted-cash-flow rate of return \( r \) for the example presented in the preceding section where yearly cash flow is continuous. The data follow.

Initial tied-capital investment = $100,000
Working-capital investment = $10,000
Service life = 5 years
Salvage value at end of service life = $10,000

†The significance of the use of discount factors, as illustrated in Table 1 and Example 2, can be seen by dividing both sides of Eq. (1) and Eq. (2) by \((1 + i)^n\), or by \((1 + i)^n\) for the general case where \( n \) is the estimated service life in years.
Predicted after-tax cash flow to project based on total income minus all costs except depreciation with cash flow occurring continuously, $ (total of continuous cash flow for year indicated)

<table>
<thead>
<tr>
<th>Year</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30,000</td>
</tr>
<tr>
<td>2</td>
<td>31,000</td>
</tr>
<tr>
<td>3</td>
<td>36,000</td>
</tr>
<tr>
<td>4</td>
<td>40,000</td>
</tr>
<tr>
<td>5</td>
<td>43,000</td>
</tr>
</tbody>
</table>

**Solution.** The following tabulation shows the final result of the trial-and-error solution using the factors $F_a$ and $F_b$ from Tables 5 and 6 in Chap. 7:

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimated continuous cash flow to project, $</th>
<th>Trial for $r = 0.225$</th>
<th>Present value, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>O-l</td>
<td>(110,000) In an instant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>30,000</td>
<td>0.8954</td>
<td>26,850</td>
</tr>
<tr>
<td>2-3</td>
<td>31,000</td>
<td>0.7151</td>
<td>22,200</td>
</tr>
<tr>
<td>3-4</td>
<td>36,000</td>
<td>0.5710</td>
<td>20,550</td>
</tr>
<tr>
<td>4-5</td>
<td>40,000</td>
<td>0.4560</td>
<td>18,250</td>
</tr>
<tr>
<td>5</td>
<td>43,000 +20,000 In an instant</td>
<td>0.3648</td>
<td>15,650</td>
</tr>
</tbody>
</table>

Total 110,000

Because the assumed trial value of $r = 0.225$ discounted all the cash flows to the present worth of $110,000, the continuous interest rate of 22.5 percent represents the discounted-cash-flow rate of return for this example which can be compared to the value of 20.7 percent shown in Table 1 for the case of discrete interest compounding and instantaneous cash flow.

**NET PRESENT WORTH**

In the preceding treatment of discounted cash flow, the procedure has involved the determination of an index or interest rate which discounts the annual cash flows to a zero present value when properly compared to the initial investment. This index gives the rate of return which includes the profit on the project, payoff of the investment, and normal interest on the investment. A related approach, known as the method of net present worth (or net present value or venture worth), substitutes the cost of capital at an interest rate $i$ for the
discounted-cash-flow rate of return. The cost of capital can be taken as the average rate of return the company earns on its capital, or it can be designated as the minimum acceptable return for the project. The net present worth of the project is then the difference between the present value of the annual cash flows and the initial required investment.

To illustrate the method for determining net present worth, consider the example presented in Table 1 for the case where the value of capital to the company is at an interest rate of 15 percent. Under these conditions, the present value of the cash flows is $127,000 and the initial investment is $110,000. Thus, the net present worth of the project is

\[ \frac{127,000}{110,000} = 17,000 \]

Work Sheet for Calculating Present Value and Net Present Worth

An example of a work sheet that can be used for handling discounted-cash-flow presentations to determine present value and net present worth is given in Table 2. The definitions as given in lines 16 and 17 of this table clearly show the preferred distinction between the terms net present worth and present value as used in this text. The table is particularly useful because it makes certain the user handles depreciation cash flow correctly by subtracting depreciation costs to determine tax costs (see lines 10 and 11) and including depreciation cash flow to determine the annual cash income (see lines 9 and 12). Line 14 shows four values of discount factors for 15 percent interest based on (a) continuous uniform cash flow and continuous interest compounding, (b) continuous uniform cash flow and finite (year-end) interest compounding, (c) finite (year-end) cash flow and continuous interest compounding, and (d) finite (year-end) cash flow and finite (year-end) interest compounding.

Lines 1, 2, and 3 (investments) in Table 2 would normally only be filled in for the first column (discount factor of 1.000) which is designated as the zero year for the operation, with the unit actually going into operation at the start of the so-called first year. It is assumed that working capital and salvage value will be recovered in a lump sum at the end of the estimated service life, so these values are listed on lines 1, 2, and 13 as positive (incoming funds) numbers in the end-of-life column. Since these are lump-sum instantaneous values, the discount factor to apply to them is the finite (year-end) cash flow factor as shown in line 14 in the end-of-life column.

Line 13 gives the annual cash flows for each of the operating years with the zero-year column giving only the total capital investment. In line 16, the present value of the annual cash flows to the project is obtained by summing the individual present values for each year of operation including the present value of the working-capital and salvage-value recovery at the end of the service life. Line 17 merely applies the definition of net present worth as used in this text as the difference between the total present value of the annual cash flows to the project and the initial required investment.
TABLE 2
Work sheet for presenting discounted-cash flow, present-value, and net-present-worth determinations

Project Title:

Notes: 1. Dollar values can be in thousands of dollars and rounded to the nearest $1,000.
2. For lines 11 and 14, company policies will dictate which tax rate, interest, and discount factors to use.
3. The estimated service life for this example is taken as 5 years.
4. For lines 5, 6, and 7, see Table 27 of Chapter 6 for estimating information and basis.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fixed-capital investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Working capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Total capital investment (1 + 2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Annual income (sales)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Annual manufacturing cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) Raw materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Labor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Utilities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) Maintenance and repairs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(e) Operating supplies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(f) Laboratory charges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(g) Patents and royalties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(h) Local taxes and insurance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(i) Plant overhead</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(j) Other (explain in Notes)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-T.</td>
<td>Total of line 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. Annual general expenses
   (a) Administrative
   (b) Distribution and selling
   (c) Research and development
   (d) Interest
   (e) Other (explain in Notes)

6-T. Total of line 6

7. Total product cost (5-T + 6-T)

8. Annual operating income (4 - 7)

9. Annual depreciation

10. Income before tax (8 - 9)

11. Income after 34% tax (0.66 x 10)

12. Annual cash income (9 + 11)

13. Annual cash flow (3 + 12) (see heading above)

14. Discount factors for 15% interest
   (a) See footnote †
   (b) See footnote ¶
   (c) See footnote §§
   (d) See footnote 11

<table>
<thead>
<tr>
<th></th>
<th>1.000</th>
<th>0.929</th>
<th>0.799</th>
<th>0.688</th>
<th>0.592</th>
<th>0.510</th>
<th>0.472</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>1.000</td>
<td>0.933</td>
<td>0.812</td>
<td>0.706</td>
<td>0.614</td>
<td>0.534</td>
<td>0.472</td>
</tr>
<tr>
<td>(c)</td>
<td>1.000</td>
<td>0.861</td>
<td>0.741</td>
<td>0.638</td>
<td>0.549</td>
<td>0.472</td>
<td>0.472</td>
</tr>
<tr>
<td>(d)</td>
<td>1.000</td>
<td>0.870</td>
<td>0.756</td>
<td>0.658</td>
<td>0.572</td>
<td>0.497</td>
<td>0.497</td>
</tr>
</tbody>
</table>

15. Annual present value (13 x 14)

16. TOTAL present value of annual cash flows (sum of line 15 not including 0 year)
   = in dollars or thousands of dollars

17. Net present worth = total present value of annual cash flows - total capital investment = line 16 - line 3 = in dollars or thousands of dollars

† Continuous uniform cash flow and continuous nominal interest (r) of 15%.
¶ Continuous uniform cash flow and finite effective interest (i) of 15%.
§§ Finite (year-end) cash flow and continuous nominal interest (r) of 15%.
¶¶ Finite (year-end) cash flow and finite effective interest (i) of 15%.
Notes should be included with the table to explain the basis for special factors used, such as escalation factors, startup costs, and depreciation method. The notes can also be used to explain the methods used for estimating the various items as, for example, note 4 in Table 2 showing the methods used for estimating lines 5, 6, and 7.

The format shown in Table 2 is intended as an example, and a real case would undoubtedly include more columns to represent a life of more than five years. Similarly, capital is normally spent during the period of one or two years before operations begin and sales are made. Thus, the factors in the zero-year column could be changed to values other than 1.000 using methods presented in Chap. 7 (Interest and Investment Costs) as illustrated in Example 3 of this chapter.

**CAPITALIZED COSTS**

The *capitalized-cost* profitability concept is useful for comparing alternatives which exist as possible investment choices within a single overall project. For example, if a decision based on profitability analysis were to be made as to whether stainless steel or mild steel should be used in a chemical reactor as one part of a chemical plant, capitalized-cost comparison would be a useful and appropriate approach. This particular case is illustrated in Example 9 of Chap. 7.

Capitalized cost related to investment represents the amount of money that must be available initially to purchase the equipment and simultaneously provide sufficient funds for interest accumulation to permit perpetual replacement of the equipment. If only one portion of an overall process to accomplish a set objective is involved and operating costs do not vary, then the alternative giving the least capitalized cost would be the desirable economic choice.

The basic equation for capitalized cost for equipment was developed in Chap. 7 as Eq. (28), which can be written as follows:

\[
K = C_V + \frac{C_R}{(1 + i)^n - 1} = \frac{C_R(1 + i)^n}{(1 + i)^n - 1} + V_s
\]

where

- \( K \) = capitalized cost
- \( C_V \) = original cost of equipment
- \( C_R \) = replacement cost
- \( V_s \) = salvage value at end of estimated useful life
- \( n \) = estimated useful life of equipment
- \( i \) = interest rate

\[
\frac{(1 + i)^n}{(1 + i)^n - 1} = \text{capitalized-cost factor}
\]

†For an analysis of the meaning of capitalized costs, development of related equations, and references, see Chap. 7 (Interest and Investment Costs).
Inclusion of Operating Costs in Capitalized-Costs Profitability Evaluation

The capitalized-costs concept can be extended to include operating costs by adding an additional capitalized cost to cover operating costs during the life of the project. Each annual operating cost is considered as equivalent to a necessary piece of equipment that will last one year.

The procedure is to determine the present or discounted value of each year’s cost by the method illustrated in Table 1. The sum of these present values is then capitalized by multiplying by the capitalized-cost factor given with Eq. (4). The total capitalized cost is the sum of the capitalized cost for the initial investment and that for the operating costs plus the working capital. This procedure is illustrated as part of Example 5 in this chapter.

PAYOUT PERIOD

Payout period, or payout time,† is defined as the minimum length of time theoretically necessary to recover the original capital investment in the form of cash flow to the project based on total income minus all costs except depreciation. Generally, for this method, original capital investment means only the original, depreciable, fixed-capital investment, and interest effects are neglected. Thus,

\[
\text{Payout period in years} = \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr + avg depreciation/yr}}
\]  

(5)

Another approach to payout period takes the time value of money into consideration and is designated as payout period including interest. With this method, an appropriate interest rate is chosen representing the minimum acceptable rate of return. The annual cash flows to the project during the estimated life are discounted at the designated interest rate to permit computation of an average annual figure for profit plus depreciation which reflects the time value of money. The time to recover the tied-capital investment plus compounded interest on the total capital investment during the estimated life by

†If annual operating cost is constant and the cost is considered as an end-of-year cost, the capitalized cost of operation is equal to the annual operating cost divided by \( i \). Continuous interest compounding can be used to resolve the problem of whether an operating cost is an end-of-year or start-of-year cost. The effects of depreciation methods and taxes may be very important when capitalized costs are used to compare design alternatives involving operating costs.

‡Other equivalent names are payback period, payback time, payoff period, payoff time, and cash-recovery period.

§This discounting procedure is similar to that illustrated in the footnote to part (a) of Example 5 in this chapter. Continuous interest tables, such as Tables 5 to 8 in Chap. 7, can also be used.
means of the average annual cash flow is the payout period including interest, or

Payout period including interest

\[
\frac{\text{depreciable fixed-capital investment}}{\text{interest on total capital investment during estimated service life}} + \frac{\text{avg profit/yr} + \text{avg depreciation/yr}}{(\text{avg profit/yr} + \text{avg depreciation/yr})_{\text{as constant annuity}}} = \text{payout period including interest}
\]

(6)

This method tends to increase the payout period above that found with no interest charge and reflects advantages for projects that earn most of their profits during the early years of the service life.

**USE OF CONTINUOUS INTEREST COMPOUNDING**

In the preceding presentation of methods for profitability evaluation, where interest was considered, it was generally treated as finite-period interest compounded annually. By use of the relationships developed in Chap. 7 (Interest and Investment Costs), it is a simple matter to convert to the case of continuous interest compounding in place of finite interest compounding.

For example, the discount factor \( d_{n'} = 1/(1 + i)^n \), given as Eq. (3), becomes

\[
d_{n'} = \frac{1}{e^{rn'}}
\]

(7)

for the case of continuous interest compounding with \( r \) representing the nominal continuous interest. The preceding equation follows directly from Eq. (18) of Chap. 7.

The application of continuous interest compounding, along with a method of profitability evaluation which includes construction costs and other prestartup costs, is illustrated in the following example.

**Example 3** Determination of profitability index with continuous interest compounding and prestartup costs. Determine the discounted-cash-flow rate of return (i.e., the profitability index) for the overall plant project described in the following, and present a plot of cash position versus time to illustrate the solution.

One year prior to startup of the plant, the necessary land is purchased at a cost of $200,000.

During the year prior to the startup, the plant is under construction with money for the construction and related activities flowing out uniformly during the entire year starting at zero dollars and totaling $600,000 for the year.

A working-capital investment of $200,000 is needed at the time the plant starts operation and must be retained indefinitely.

Salvage value for the plant at the end of the estimated useful life is $100,000.

The estimated useful life is 10 years.

Estimations of operating costs, income, and taxes indicate that the annual cash flow to the project (i.e., net profit plus depreciation per year) will be $310,000 flowing uniformly throughout the estimated life. This is an after-tax figure.
The concept of continuous interest compounding and continuous cash flow will be used. Neglect any effects due to inflation or deflation.

Solution. The procedure for this problem is similar to that illustrated in Table 1 in that a trial-and-error method is used with various interest rates until a rate is found which decreases the net cash position to zero at the end of the useful life. Let \( r \) represent the profitability index or discounted-cash-flow rate of return with continuous cash flow and continuous interest compounding.

1. Determination of cash position at zero time (i.e., at time of plant startup) in terms of unknown profitability index \( r \).

**Land value.** The in-an-instant value of the land is $200,000 one year before the zero reference point of plant startup time. The land value at zero time, therefore, is the future worth of this $200,000 after one year with continuous interest compounding. Thus, by Eq. (36) of Chap. 7 or part \((e)\) of Table 3 in Chap. 7.

\[
\text{Compounded land value at zero time} = 200,000(e^r)
\]

**Construction cost.** The total construction cost of the plant during the one year prior to startup is $600,000 occurring uniformly during the year. The compounded construction cost at zero time, therefore, is the future worth of this $600,000 after one year flowing uniformly throughout the year with continuous compounding. Thus, by Eq. (37) of Chap. 7 or part \((f)\) of Table 3 in Chap. 7.

\[
\text{Compounded construction cost at zero time} = 600,000 \frac{e^r - 1}{r}
\]

**Working-capital investment.** The working-capital investment of $200,000 must be supplied at the time of plant startup or at the reference point of zero time.

**Summary of cash position at zero time.**

Total cash position at zero time

\[
CP_{\text{zero time}} = 200,000(e^r) + 600,000 \frac{e^r - 1}{r} + 200,000
\]

2. Determination of cash position at end of estimated useful life (i.e., ten years from zero time) in terms of profitability index \( r \). At the end of the useful life with the correct value of \( r \), the total cash position, taking into account the working-capital investment, the salvage value, and the land value, must be zero.

After plant startup, the annual cash flow to the project (i.e., net profit plus depreciation) is $310,000 flowing continuously and uniformly, and this annual figure is constant throughout the estimated useful life.

The following procedure for evaluating the total cash position at the end of the estimated useful life is analogous to the procedure used in establishing Eqs. (1), (2), (3), and (7) of this chapter.

At the end of each year, the compounded cash flow to the project, with continuous uniform flow and continuous compounding, gives, by Eq. (37) of
Chap. 7 or part (f) of Table 3 in Chap. 7, a future worth \( S_{\text{each year}} \) oft

\[
S_{\text{each year}} = \$310,000 \frac{e^r - 1}{r} \tag{A}
\]

At the end of 10 years, the total future worth \( S \) of the cash flow to the project, by Eq. (36) of Chap. 7, or part (e) of Table 3 in Chap. 7 becomes

\[
S = (\$310,000) \frac{e^r - 1}{r} (e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1) \tag{B}
\]

The future worth of the total flow to the project after 10 years must be equal to the future worth of the total cash position at zero time \( CP_{\text{zero time}} \) compounded continuously for 10 years minus salvage value, land value, and working-capital investment. Therefore, by Eq. (36) of Chap. 7 or part (e) of Table 3 in Chap. 7,

\[
S = (CP_{\text{zero time}})(e^{10r}) = \$100,000 - \$200,000 - \$200,000 \tag{C}
\]

3. Determination of profitability index \( r \). Equating Eq. (B) to Eq. (C) gives the following result with \( r \) as the only unknown, and a trial-and-error solution will give the profitability index \( r \).

\[
(\$310,000) \frac{e^r - 1}{r} (e^{9r} + e^{8r} + e^{7r} + \cdots + e^r + 1) - (CP_{\text{zero time}})(e^{10r}) + \$100,000 + \$200,000 + \$200,000 = 0 \tag{D}
\]

The trial-and-error approach can be simplified by dividing Eq. (D) by \( e^{10r} \) and substituting the expression for \( CP_{\text{zero time}} \) to give the present-value or discounted-cash-flow equation as follows:

\[
(\$310,000) \frac{e^r - 1}{r} \sum_{n'=1}^{10} \frac{1}{e^{nr}} - \$200,000(e^r) - \$600,000 \frac{e^r - 1}{r} - \$200,000 + (\$100,000 + \$200,000 + \$200,000) \frac{1}{e^{10r}} = 0 \tag{E}
\]

where \( 1/e^{nr} \) represents the discount factor for continuous cash flow and continuous compounding as given in Eq. (7) of this chapter.

Because the compounded annual flow to the project is constant for each year at \( (\$310,000)(e^r - 1)/r \), the year-by-year use of the discount factor, as illustrated in Table 1 of this chapter, can be replaced by a one-step process wherein the equivalent present value is determined from Eq. (25) of Chap. 7, \( P = \frac{R(e^{rn} - 1)/re^{rn}}{e^{rn}} \), with \( R = \$310,000 \), so that

\[
310,000 \sum_{n'=1}^{10} \frac{1}{e^{nr}} = 310,000 \sum_{n'=1}^{10} \frac{1}{e^{rn'}} = 310,000 \sum_{n'=1}^{10} \frac{1}{e^{rn'}}
\]

†The concept of continuous and uniform cash flow with continuous interest compounding is obviously an assumption which is reasonable for some cash flow, such as costs for raw material and labor. However, it is clear that some major portions of the cash flow may not approximate continuous flow. For this reason, the annual cash flow is often estimated as an end-of-year-figure, and the interest factor in Eq. (A) is eliminated.
Similarly, in Eq. (D), the future-worth expression for the cash flow, 
$310,000[(e^r - 1)/r](e^9r + e^8r + e^7r + \cdots + e^r + 1)$, can be replaced by 
$310,000[(e^{rn} - 1)/r]$, as shown by Eq. (23) of Chap. 7.

With these simplifications, either Eq. (D) or (E) can be used for the trial-and-error solution for \( r \). Table 3 shows the method of solution using the present-value Eq. (E) to give the correct value of \( r = 0.26 \).

Thus, the profitability index or discounted-cash-flow rate of return for this example is 26%.

**TABLE 3**
Computation of profitability index for Example 3†

![](https://example.com/table3.png)

n = 10 years
Basis: Eq. (E) and zero (present-value) time at plant startup

<table>
<thead>
<tr>
<th>Trial for</th>
<th>( r = 0.20 )</th>
<th>( r = 0.25 )</th>
<th>( r = 0.40 )</th>
<th>( r = 0.26 )‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Present value of cash flow to project</td>
<td>( e^r - 1 ) ( r ) = 1.107</td>
<td>1.136</td>
<td>1.230</td>
<td>1.142</td>
</tr>
<tr>
<td>( ($310,000) \sum_{n=1}^{10} 1 ) ( e^r ) ( r ) ( e^{rn} - 1 ) ( \text{rem} )</td>
<td>3.68</td>
<td>2.45</td>
<td>3.565</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td>( ($310,000) \sum_{n=1}^{10} 1 ) ( e^r ) ( r ) ( e^{rn} - 1 ) ( \text{rem} )</td>
<td>$1,335,000</td>
<td>$1,140,000</td>
<td>$760,000</td>
</tr>
<tr>
<td>b. Present value of land ( ($200,000) ) ( e^r )</td>
<td>$244,000</td>
<td>$257,000</td>
<td>$298,000</td>
<td>$259,000</td>
</tr>
<tr>
<td>c. Present value of construction cost ( ($600,000) ) ( e^r - 1 ) ( r )</td>
<td>$664,000</td>
<td>$682,000</td>
<td>$738,000</td>
<td>$685,000</td>
</tr>
<tr>
<td>d. Present value of working-capital investment</td>
<td>$200,000</td>
<td>$200,000</td>
<td>$200,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>e. Present value of terminal land, working capital, and salvage value based on interest compounded continuously for ( n ) years ( ($500,000) ) ( 1 ) ( e^{rn} )</td>
<td>$68,000</td>
<td>$41,000</td>
<td>$9,000</td>
<td>$37,000</td>
</tr>
<tr>
<td>f. Total of all present values with interest of ( r ). Should be zero at correct value of ( r ).</td>
<td>$295,000</td>
<td>$42,000</td>
<td>$467,000</td>
<td>$0</td>
</tr>
</tbody>
</table>

† Graphical methods, special tables for particular cases, MAP1 worksheets and terminology, computer solutions, and rules of thumb are available to simplify the type of calculations illustrated in this table. For example, see G. A. Taylor, "Managerial and Engineering Economy: Economic Decision-Making, 2d ed., D. Van Nostrand Company, Inc., Princeton, New Jersey, 1975.

‡ See Fig. 10-2 for example of graphical interpolation procedure.
**PLANT DESIGN AND ECONOMICS FOR CHEMICAL ENGINEERS**

Cumulative cash position =

\[
\text{net profit + depreciation) = total investment}
\]

Land value + salvage value + W.C.I.

No Interest

Based on Eq. (F) \( r = 0.26 \)

Figure 10-3

Illustrative plot showing cash position versus time to explain graphically the solution to Example 3. Dashed line is with interest or profitability index of 26 percent. Solid line is with no interest charge. (Note that method for calculating depreciation is not important except for income taxes.)

4. Graphical representation of problem solution. Equation (D) can be generalized, with the simplifications indicated in the preceding section of this problem, to give Table 3 (note that direct land value, salvage value, and working-capital investment are now included in the cash composition).

Cash position at time \( n \)

\[
= \frac{e^{rn} - 1}{r}
\]

\[
= \text{(land value)}e^{r(n + y)} = \text{(construction cost)}\frac{e^{yr} = 1}{yr}e^{rn'}
\]

\[-\text{(working-capital investment)}(e^{rn'}) (F)
\]

where \( Z \) is the time period in years the land is owned before startup and \( Y \) is the time period in years required for construction, In this example, \( Z \) and \( Y \) are both 1.0.

Figure 10-3 is the requested plot of cash position versus time for the case of \( r = 0.26 \) based on Eq. (F) and illustrates the concepts involved in the solution of this problem showing the cases with interest (or profitability index) and without interest.

**DETERMINING ACCEPTABLE RETURNS**

It is often possible to make a profit by the investment of capital, but it is not always easy to determine if a given return is sufficient to justify an investment. Many factors must be considered when deciding if a return is acceptable, and it is not possible to give one figure which will apply for all cases.

When dealing with ordinary industrial operations, profits cannot be predicted with absolute accuracy. Risk factors, therefore, must be given careful
consideration, and the degree of uncertainty involved in predicted returns on investments plays an important role in determining what returns are acceptable.

A certain amount of risk is involved in any type of investment, but the degree of risk varies widely for different types of enterprises. For example, there is very little uncertainty in predicting returns on capital invested in government bonds, and the chances of losing the original capital are very small. However, money invested in a wildcat mining enterprise would stand a good chance of being lost completely with no return whatsoever.

If capital is available for investment in a proposed enterprise, it would also be available for use in other ventures. Therefore, a good basis for determining an acceptable return is to compare the predicted return and the risks involved with returns and risks in other types of investments.

Very conservative investments, such as government bonds, pay low returns in the range of 5 to 7 percent, but the risk involved is practically negligible. Preferred stocks yield returns of about 7 to 9 percent. There is some risk involved in preferred-stock investments since a business depression or catastrophe could cause reduction in returns or even a loss of the major portion of the capital investment. Common stocks may yield very high returns; however, the returns fluctuate considerably with varying economic conditions, and there is always the possibility of losing much or all of the original investment.

It can be stated that moderate risks are involved in common-stock investments. Certainly, at least moderate risks are involved in most industrial projects. In general, a 20 percent return before income taxes would be the minimum acceptable return for any type of business proposition, even if the economics appeared to be completely sound and reliable. Many industrial concerns demand a predicted pretax return of at least 30 percent based on reliable economic estimates before they will consider investing capital in projects that are known to be well engineered and well designed.

The final decision as to an acceptable return depends on the probable accuracy of the predicted return and on the amount of risk the investor wishes to take. Availability of capital, personal opinions, and intangible factors, such as the response of the public to changes or appearances, may also have an important effect on the final decision.

ALTERNATIVE INVESTMENTS

In industrial operations, it is often possible to produce equivalent products in different ways. Although the physical results may be approximately the same, the capital required and the expenses involved can vary considerably depending on the particular method chosen. Similarly, alternative methods involving varying capital and expenses can often be used to carry out other types of business ventures. It may be necessary, therefore, not only to decide if a given business venture would be profitable, but also to decide which of several possible methods would be the most desirable.
The final decision as to the best among alternative investments is simplified if it is recognized that each dollar of additional investment should yield an adequate rate of return. In practical situations, there are usually a limited number of choices, and the alternatives must be compared on the basis of incremental increases in the necessary capital investment.

The following simple example illustrates the principle of investment comparison. A chemical company is considering adding a new production unit which will require a total investment of $1,200,000 and will yield an annual profit of $240,000. An alternative addition has been proposed requiring an investment of $2 million and yielding an annual profit of $300,000. Although both of these proposals are based on reliable estimates, the company executives feel that other equally sound investments can be made with at least a 14 percent annual rate of return. Therefore, the minimum rate of return required for the new investment is 14 percent.

The rate of return on the $1,200,000 unit is 20 percent, and that for the alternative addition is 15 percent. Both of these returns exceed the minimum required value, and it might appear that the $2 million investment should be recommended because it yields the greater amount of profit per year. However, a comparison of the incremental investment between the two proposals shows that the extra investment of $800,000 gives a profit of only $60,000, or an incremental return of 7.5 percent. Therefore, if the company has $2 million to invest, it would be more profitable to accept the $1,200,000 proposal and put the other $800,000 in another investment at the indicated 14 percent return.

A general rule for making comparisons of alternative investments can be stated as follows: The minimum investment which will give the necessary functional results and the required rate of return should always be accepted unless there is a specific reason for accepting an alternative investment requiring more initial capital. When alternatives are available, therefore, the base plan would be that requiring the minimum acceptable investment. The alternatives should be compared with the base plan, and additional capital would not be invested unless an acceptable incremental return or some other distinct advantage could be shown.

**Alternatives When an Investment Must Be Made**

The design engineer often encounters situations where it is absolutely necessary to make an investment and the only choice available is among various alternatives. An example of this might be found in the design of a plant requiring an evaporation operation. The evaporator units must have a given capacity based on the plant requirements, but there are several alternative methods for carrying out the operation. A single-effect evaporator would be satisfactory. However, the operating expenses would be lower if a multiple-effect evaporator were used, because of the reduction in steam consumption. Under these conditions, the best number of effects could be determined by comparing the increased savings with the investment required for each additional effect. A
igraphical representation showing this kind of investment comparison is presented in Fig. 10-4.

The base plan for an alternative comparison of the type discussed in the preceding paragraph would be the minimum investment which gives the necessary functional results. The alternatives should then be compared with the base plan, and an additional investment would be recommended only if it would give a definite advantage.

When investment comparisons are made, alternatives requiring more initial capital are compared only with lower investments which have been found to be acceptable. Consider an example in which an investment of $50,000 will give a desired result, while alternative investments of $60,000 and $70,000 will give the same result with less annual expense. Suppose that comparison between the $60,000 and the $50,000 cases shows that $60,000 investment to be unacceptable. Certainly, there would be no reason to give further consideration to the $60,000 investment, and the next comparison should be between the $70,000 and the $50,000 cases. This type of reasoning, in which alternatives are compared in pairs on a mutually exclusive basis, is illustrated in the following simplified example.

AN EXAMPLE TO ILLUSTRATE PRINCIPLES OF ALTERNATIVE INVESTMENT ANALYSIS. In making a choice among various alternative investments, it is necessary to recognize the need to compare one investment to
another on a mutually exclusive basis in such a manner that the *return on the incremental investment* is satisfactory. The following example illustrates this principle.

An existing plant has been operating in such a way that a large amount of heat is being lost in the waste gases. It has been proposed to save money by recovering the heat that is now being lost. Four different heat exchangers have been designed to recover the heat, and all prices, costs, and savings have been calculated for each of the designs. The results of these calculations are presented in the following:

<table>
<thead>
<tr>
<th>Design</th>
<th>No. 1</th>
<th>No. 2</th>
<th>No. 3</th>
<th>No. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total initial installed cost, $</td>
<td>10,000</td>
<td>16,000</td>
<td>20,000</td>
<td>26,000</td>
</tr>
<tr>
<td>Operating costs, $/yr</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Fixed charges, % of initial cost/yr</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Value of heat saved, $/yr</td>
<td>4,100</td>
<td>6,000</td>
<td>6,900</td>
<td>8,850</td>
</tr>
</tbody>
</table>

The company in charge of the plant demands at least a 10 percent annual return based on the initial investment for any unnecessary investment. Only one of the four designs can be accepted. Neglecting effects due to income taxes and the time value of money, which (if any) of the four designs should be recommended?

The first step in the solution of this example problem is to determine the amount of money saved per year for each design, from which the annual percent return on the initial investment can be determined. The net annual savings equals the value of heat saved minus the sum of the operating costs and fixed charges; thus,

For design No. 1,

\[
\text{Annual savings} = 4100 - (0.2)(10,000) - 100 = 2000
\]

\[
\text{Annual percent return} = \frac{2000}{10,000} (100) = 20\%
\]

For design No. 2,

\[
\text{Annual savings} = 6000 - (0.2)(16,000) - 100 = 2700
\]

\[
\text{Annual percent return} = \frac{2700}{16,000} (100) = 16.9\%
\]
For design No. 3,

Annual savings = \( 6900 - (0.2)(20,000) - 100 = 2800 \)

Annual percent return = \( \frac{2800}{20,000}(100) = 14\% \)

For design No. 4,

Annual savings = \( 8850 - (0.2)(26,000) - 100 = 3550 \)

Annual percent return = \( \frac{3550}{26,000}(100) = 13.6\% \)

Because the indicated percent return for each of the four designs is above the minimum of 10 percent required by the company, any one of the four designs would be acceptable, and it now becomes necessary to choose one of the four alternatives.

ALTERNATIVE ANALYSIS BY METHOD OF RETURN ON INCREMENTAL INVESTMENT. Analysis by means of return on incremental investment is accomplished by a logical step-by-step comparison of an acceptable investment to another which might be better. If design No. 1 is taken as the starting basis, comparison of design No. 2 to design No. 1 shows that the annual saving of $2700 - $2000 = $700 results by making an additional investment of $16,000 - $10,000 = $6,000. Thus, the percent return on the incremental investment is \( \frac{700}{6000}(100) = 11.7\% \), and design No. 2 is acceptable by company policy in preference to design No. 1. This logical procedure results in the following tabulation and the choice of design No. 2 as the final recommendation:

Design No. 1 is acceptable.
Comparing design No. 1 to design No. 2, annual percent return = \( \frac{700}{6000}(100) = 11.7\% \). Thus, design No. 2 is acceptable and is preferred over design No. 1.
Comparing design No. 2 to design No. 3, annual percent return = \( \frac{100}{4000}(100) = 2.5\% \). Thus, design No. 3 compared to design No. 2 shows that the return is unacceptable and design No. 2 is preferred.
Comparing design No. 2 to design No. 4, annual percent return = \( \frac{850}{10000}(100) = 8.5\% \). Thus, design No. 4 is not acceptable when compared to design No. 2, and design No. 2 is the alternative that should be recommended.

ALTERNATIVE ANALYSIS INCORPORATING MINIMUM RETURN AS A COST. Identical results to the preceding are obtained by choosing the alternative giving the greatest annual profit or saving if the required return is included as a cost for each case. For the heat-exchanger example, the annual cost for the required
return would be 10 percent of the total initial investment; thus,

For design No. 1, annual savings above required return = 2000 - (0.1) \times 10,000 = $1000.
For design No. 2, annual savings above required return = 2700 - (0.1) \times 16,000 = $1100.
For design No. 3, annual savings above required return = 2800 - (0.1) \times 20,000 = $800.
For design No. 4, annual savings above required return = 3550 - (0.1) \times 26,000 = $950.

Because annual saving is greatest for design No. 2, this would be the recommended alternative which is the same result as was obtained by the direct analysis based on return on incremental investment.

This simplified example has been used to illustrate the basic concepts involved in making comparisons of alternative investments. The approach was based on using the simple return on initial investment in which time value of money is neglected. Although this method may be satisfactory for preliminary and rough estimations, for final evaluations a more sophisticated approach is needed in which the time value of money is considered along with other practical factors to assure the best possible chance for future success. Typical more advanced approaches of this type are presented in the following sections.

**ANALYSIS WITH SMALL INVESTMENT INCREMENTS**

The design engineer often encounters the situation in which the addition of small investment increments is possible. For example, in the design of a heat exchanger for recovering waste heat, each square foot of additional heat-transfer area can cause a reduction in the amount of heat lost, but the amount of heat recovered per square foot of heat-transfer area decreases as the area is increased. Since the investment for the heat exchanger is a function of the heat-transfer area, a plot of net savings (or net profit due to the heat exchanger) versus total investment can be made. A smooth curve of the type shown in Fig. 10-5 results.

The point of maximum net savings, as indicated by 0 in Fig. 10-5, represents a classical optimum condition. However, the last incremental investment before this maximum point is attained is at essentially a zero percent return. On the basis of alternative investment comparisons, therefore, some investment less than that for maximum net savings should be recommended.

The exact investment where the incremental return is a given value occurs at the point where the slope of the curve of net savings versus investment equals the required return. Thus, a straight line with a slope equal to the necessary return is tangent to the net-savings-versus-investment curve at the point representing the recommended investment. Such a line for an annual return on incremental investment of 20 percent is shown in Fig. 10-5, and the recommended investment for this case is indicated by $RI$. If an analytical expression
relating net savings and investment is available, it is obvious that the recommended investment can be determined directly by merely setting the derivative of the net savings with respect to the investment equal to the required incremental return.

The method described in the preceding paragraph can also be used for continuous curves of the type represented by the dashed curve in Fig. 10-4. Thus, the line \( ab \) in Fig. 10-4 is tangent to the dashed curve at the point representing the recommended investment for the case of a 15 percent incremental return.

**Example 4  Investment comparison for required operation with limited number of choices.** A plant is being designed in which 450,000 lb per 24-h day of a water-caustic soda liquor containing 5 percent by weight caustic soda must be concentrated to 40 percent by weight. A single-effect or multiple-effect evaporator will be used, and a single-effect evaporator of the required capacity requires an initial investment of $18,000. This same investment is required for each additional effect. The service life is estimated to be 10 years, and the salvage value of each effect at the end of the service life is estimated to be $6000. Fixed charges minus depreciation amount to 20 percent yearly, based on the initial investment. Steam costs $0.60 per 1000 lb, and administration, labor, and miscellaneous costs are $40 per day, no matter how many evaporator effects are used.

Where \( X \) is the number of evaporator effects, \( 0.9X \) equals the number of pounds of water evaporated per pound of steam. There are 300 operating days per year. If the minimum acceptable return on any investment is 15 percent, how many effects should be used?

**Solution.** Basis: 1 operating day

\[ X = \text{total number of evaporator effects} \]
Depreciation per operating day (straight-line method) = \( \frac{X(18,000 - 6000)}{(10)(300)} \)  
= $4.00X/day

Fixed charges - depreciation = \( \frac{X(18,000)(0.20)}{300} \)  
= $12.00X/day

Pounds of water evaporated per day = \( (450,000)(0.05)(\frac{24}{25}) - (450,000)(0.05)(\frac{60}{60}) \)  
= 393,800 lb/day

\[
\text{Steam costs} = \frac{(393,800)(0.60)}{X(0.9)(1000)} = \frac{$262.5}{X} \text{ per day}
\]

<table>
<thead>
<tr>
<th>X = no. of effects</th>
<th>Steam costs per day</th>
<th>Fixed charges minus depreciation per day</th>
<th>Depreciation per day</th>
<th>Labor, etc., per day</th>
<th>Total cost per day</th>
</tr>
</thead>
</table>
| 1
| $262.5             | $12                 | $4                                      | $40                  | $318.5              |
| 2
| 131.3              | 24                  | 8                                       | 40                   | 203.3               |
| 3
| 87.5               | 36                  | 12                                      | 40                   | 175.5               |
| 4
| 65.6               | 48                  | 16                                      | 40                   | 169.6               |
| 5
| 52.5               | 60                  | 20                                      | 40                   | 172.5               |
| 6
| 43.8               | 72                  | 24                                      | 40                   | 179.8               |

Comparing two effects with one effect,

\[
\text{Percent return} = \frac{(318.5 - 203.3)(300)(100)}{36,000 - 18,000} = 192\%
\]

Therefore, two effects are better than one effect.

Comparing three effects with two effects,

\[
\text{Percent return} = \frac{(203.3 - 175.5)(300)(100)}{54,000 - 36,000} = 46.3\%
\]

Therefore, three effects are better than two effects.

Comparing four effects with three effects,

\[
\text{Percent return} = \frac{(175.5 - 169.6)(300)(100)}{72,000 - 54,000} = 9.8\%
\]

Since a return of at least 15 percent is required on any investment, three effects are better than four effects, and the four-effect evaporator should receive no further consideration.
Comparing five effects with three effects,

\[
\text{Percent return} = \frac{(175.5 - 172.5)(300)(100)}{90,000 - 54,000} = 2.5\%
\]

Therefore, three effects are better than five effects.

Since the total daily costs continue to increase as the number of effects is increased above five, no further comparisons need to be made.

A three-effect evaporator should be used.

ANALYSIS OF ADVANTAGES AND DISADVANTAGES OF VARIOUS PROFITABILITY MEASURES FOR COMPARING ALTERNATIVES

Of the methods presented for profitability evaluation and the economic comparison of alternatives, net present worth and discounted cash flow are the most generally acceptable, and these methods are recommended. Capitalized costs have limited utility but can serve to give useful and correct results when applied to appropriate situations. Payout period does not adequately consider the later years of the project life, does not consider working capital, and is generally useful only for rough and preliminary analyses. Rates of return on original investment and average investment do not include the time value of money, require approximations for estimating average income, and can give distorted results of methods used for depreciation allowances.

It is quite possible to compare a series of alternative investments by each of the profitability measures outlined in the early part of this chapter and find that different alternatives would be recommended depending on the evaluation technique used. If there is any question as to which method should be used for a final determination, net present worth should be chosen, as this will be the most likely to maximize the future worth of the company.

Investment costs due to land can be accounted for in all the methods except payout period. Costs incurred during the construction period prior to startup can be considered correctly in both the net-present-worth and the discounted-cash-flow methods, while they are ignored in the return-on-investment methods and are seldom taken into account in determining payout period.

None of the methods gives a direct indication of the magnitude of the project, although net present worth does give a general idea of the magnitude if interpreted correctly. In certain cases, such as for alternatives of different economic lives, the discounted-cash-flow rate-of-return method is very difficult to use for comparing investments correctly. The discounted-cash-flow rate-of-return method may give multiple or impossible answers for unusual cash-flow situations, such as a case where no cash investment is needed at the start and for certain replacement situations.

†This situation is illustrated in Example 5 of this chapter.
TABLE 4
Definitions to clarify income-tax situation for profitability evaluation

<table>
<thead>
<tr>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue = total income (or total savings)</td>
</tr>
<tr>
<td>Net profits = revenue - all expenses - income tax</td>
</tr>
<tr>
<td>All expenses = cash expenses + depreciation</td>
</tr>
<tr>
<td>Income tax = (revenue - all expenses) * tax rate</td>
</tr>
<tr>
<td>Cash flow = net profits + depreciation</td>
</tr>
<tr>
<td>Cash flow = (revenue) * (1 - tax rate) - (cash expenses) * (1 - tax rate) + (depreciation * tax rate)</td>
</tr>
</tbody>
</table>

For the case of a 34% tax rate

$1.00 of revenue (either as sales income or savings) yields a cash flow of $0.66.
$1.00 of cash expenses (as raw materials, labor, etc.) yields a cash outflow of $0.66.
$1.00 of depreciation yields a cash inflow of $0.34.

Consideration of Income Taxes

Income-tax effects can be included properly in all the profitability methods discussed in this chapter by using appropriate definitions of terms, such as those presented in Table 4. The methods of discounted-cash-flow rate of return and present worth are limited to consideration of cash income and cash outgo over the life of the project. Thus, depreciation, as a cost, does not enter directly into the calculations except as it may affect income taxes.

Net cash flow represents the difference between all cash revenues and all cash expenses with taxes included as a cash expense. Thus, discounted-cash-flow rate of return and present worth should be calculated on an after-tax basis, unless there is some particular reason for a pretax basis, such as comparison to a special alternate which is presented on a pre-tax basis.

Example 5 Comparison of alternative investments by different profitability methods. A company has three alternative investments which are being considered. Because all three investments are for the same type of unit and yield the same service, only one of the investments can be accepted. The risk factors are the same for all three cases. Company policies, based on the current economic situation, dictate that a minimum annual return on the original investment of 15 percent after taxes must be predicted for any unnecessary investment with interest on investment not included as a cost. (This may be assumed to mean that other equally sound investments yielding a 15 percent return after taxes are available.) Company policies also dictate that, where applicable, straight-line depreciation is used and, for time-value of money interpretations, end-of-year cost and profit analysis is used. Land value and prestartup costs can be ignored.

Given the following data, determine which investment, if any, should be made by alternative-analysis profitability-evaluation methods of

(a) Rate of return on initial investment
(b) Minimum payout period with no interest charge
(c) Discounted cash flow
(d) Net present worth
(e) Capitalized costs
### PROFITABILITY. ALTERNATIVE INVESTMENTS, AND REPLACEMENTS

<table>
<thead>
<tr>
<th>Investment number</th>
<th>Total initial fixed-capital investment, $</th>
<th>Working-capital investment, $</th>
<th>Salvage value at end of service life, $</th>
<th>Service life, years</th>
<th>Annual cash flow to project after taxes, $</th>
<th>Annual cash expenses (constant for each year), $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100,000</td>
<td>10,000</td>
<td>10,000</td>
<td>5</td>
<td>See yearly tabulation $</td>
<td>44,000</td>
</tr>
<tr>
<td>2</td>
<td>170,000</td>
<td>10,000</td>
<td>15,000</td>
<td>7</td>
<td>52,000 (constant)</td>
<td>28,000</td>
</tr>
<tr>
<td>3</td>
<td>210,000</td>
<td>15,000</td>
<td>20,000</td>
<td>8</td>
<td>59,000 (constant)</td>
<td>21,000</td>
</tr>
</tbody>
</table>

† This is total annual income or revenue minus all costs except depreciation and interest cost for investment.

‡ This is annual cost for operation, maintenance, taxes, and insurance. Equals total annual income minus annual cash flow.

§ For investment number 1, variable annual cash flow to project is: year 1 = $30,000, year 2 = $31,000, year 3 = $36,000, year 4 = $40,000, year 5 = $43,000.

### Solution

(a) Method of rate of return on initial investment.

Average annual profit = annual cash flow − annual depreciation cost

The average annual profits for investment No. 1, using straight-line depreciation are as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Average annual profit, dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30,000 − (100,000 − 10,000) / 5 = 30,000 − 18,000 = 12,000</td>
</tr>
<tr>
<td>2</td>
<td>31,000 − 18,000</td>
</tr>
<tr>
<td>3</td>
<td>36,000 − 18,000</td>
</tr>
<tr>
<td>4</td>
<td>40,000 − 18,000</td>
</tr>
<tr>
<td>5</td>
<td>43,000 − 18,000</td>
</tr>
<tr>
<td>Total</td>
<td>90,000</td>
</tr>
</tbody>
</table>

For investment No. 1, the arithmetic average of the annual profits is $90,000/5 = $18,000.

The annual average rate of return on the first investment is

\[
\frac{18,000}{100,000 + 10,000} (100) = 16.4\% \text{ after taxes}
\]

An alternate method to obtain the average of the annual profits would be to determine the amount of the annuity R based on the end-of-year payments that would compound to the same future worth as the individual profits using an interest rate i of 0.15. With this approach, the average of the annual profits for investment No. 1 would be $17,100.

The method for determining this $17,100 is to apply the series compound-amount factor \([(1 + i)^n - 1]/i\) [see Eq. (21) in Chap. 71 to the annuity to give the future worth S of the annual incomes. The expression is \((12,000)(1 + i)^4 + (13,000)(1 + i)^3 + (18,000)(1 + i)^2 + (22,000)(1 + i) + 25,000 = R(1 + i)^5 - 1]/i\). Solving for the case of i = 0.15 gives \(R = 17,100\).
Because this return is greater than 15 percent, one of the three investments will be recommended, and it is only necessary to compare the three investments.

<table>
<thead>
<tr>
<th>For investment number</th>
<th>Total initial investment</th>
<th>Average annual profit, dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$110,000</td>
<td>$18,000</td>
</tr>
<tr>
<td>2</td>
<td>$180,000</td>
<td>$29,900</td>
</tr>
<tr>
<td>3</td>
<td>$225,000</td>
<td>$135,200</td>
</tr>
</tbody>
</table>

Comparing investment No. 2 with investment No. 1,

Percent return $= \frac{29,900 - 18,000}{180,000 - 110,000} \times 100 = 17.0\%$

Therefore, investment No. 2 is preferred over investment No. 1. Comparing investment No. 3 with investment No. 2,

Percent return $= \frac{35,200 - 29,900}{225,000 - 180,000} \times 100 = 11.8\%$

This return is not acceptable, and investment No. 2 should be recommended.

The same result would have been obtained if a minimum return of 15 percent had been incorporated as an expense.

(b) Method of minimum payout period with no interest charge.

Payout period (with no interest charge) $= \frac{\text{depreciable fixed-capital investment}}{\text{avg profit/yr} + \text{avg depreciation/yr}}$

For investment number

<table>
<thead>
<tr>
<th>For investment number</th>
<th>Payout period, years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{90,000}{18,000 + 18,000} = 2.50$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{155,000}{29,900 + 22,100} = 2.98$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{190,000}{35,200 + 23,800} = 3.22$</td>
</tr>
</tbody>
</table>

The payout period for investment No. 1 is least; therefore, by this method, investment No. 1 should be recommended.

(c) Method of discounted cash flow. For investment No. 1, as illustrated in Table 1, the rate of return based on discounted cash flow is 20.7 percent.

For investment No. 2, the discounted-cash-flow equation is

\[
(52,000) \left[ \frac{1}{1 + i} + \frac{1}{(1 + i)^2} + \cdots + \frac{1}{(1 + i)^7} \right] + (10,000 + 15,000) \frac{1}{(1 + i)^8} = $180,000
\]

By trial-and-error solution, the discounted-cash-flow rate of return is 22.9 percent
Similarly, for investment No. 3,

\[
(59,000) \left[ \frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^8} \right] + (15,000 + 20,000) \frac{1}{(1+i)^8}
\]

\[
= 225,000
\]

By trial-and-error solution, the discounted-cash-flow rate of return is 21.5 percent.

To make a choice among the three alternatives, it is necessary to make a comparison among the three possible choices. This comparison can be made in a relatively straightforward manner using discounted-cash-flow rates of return by comparing pairs of investments on a mutually exclusive basis if the various alternatives have the same economic service lives. When different lengths of service life are involved, as in this problem, the best approach is to avoid the calculated rates of return and make the investment comparison by the net present-worth method as shown in part (d) of this problem. It would be possible to use discounted-cash-flow rates of return for comparison between investments with different service lives by assuming that each investment could be repeated at the end of its service life until a common end point was obtained for the investments being compared; however, this method becomes very involved mathematically and is not realistic.

If the service lives of the investments being compared are not widely different, the following approximate method using discounted-cash-flow rate of return can be employed for the comparison.†

In comparing a pair of alternatives, the base time is chosen as the longer of the two service lives. For the case of the investment with the shorter life, it is assumed that the accumulated amount at the end of its life can be invested at the minimum acceptable rate for the remaining time to equalize the two lives. The rate of return on the incremental investment can then be determined.

**Comparison of investment No. 2 to investment No. 1.** At the end of its 7-year service life, the net value of investment No. 2 is

\[
(180,000)(1 + 0.229)^7 + 10,000 + 15,000 = 785,000
\]

With investment No. 1, the net value after 7 years is the amount accumulated in 5 years times the factor to let this accumulated amount be invested at 15 percent for 2 more years, or

\[
[(110,000)(1 + 0.207)^5 + 10,000 + 10,000](1 + 0.15)^2 = 398,000
\]

Therefore, a gain of $785,000 - $398,000 = $387,000 is made in 7 years by an added investment of $70,000 if investment No. 2 is made instead of investment No. 1. The discounted-cash-flow rate of return for this incremental investment is found by

\[
(70,000)(1 + i)^7 = 387,000
\]

\[
i = 0.277 \text{ or } 27.7\%
\]

This return is greater than 15 percent; so investment No. 2 is preferred over investment No. 1.

†The method is shown to illustrate the use of discounted-cash-flow rates of return for investment comparisons. It is correct only for comparisons involving equal service lives. If service lives are different, this method tends to favor the investment with the longest service life.
Comparison of investment No. 3 to investment No. 2. At the end of its 8-year service life, the net value of investment No. 3 is

$$(225,000)(1 + 0.215)^8 + 15,000 + 20,000 = \$1,105,000$$

For comparison, $180,000 invested in investment No. 2 would, with the last year at a 15 percent return, accumulate in 8 years to

$$\left[(180,000)(1 + 0.229)^8 + 10,000 + 15,000\right](1 + 0.15) = \$903,060$$

Therefore, a gain of $1,105,000 - $903,000 = $202,000 is made in 8 years by an added investment of $45,000 by making investment No. 3 instead of investment No. 2. The discounted-cash-flow rate of return for this incremental investment is found by

$$(45,000)(1 + i)^8 = 202,000$$

$$i = 0.208 \text{ or } 20.8\%$$

This return is greater than 15 percent; so investment No. 3 is preferred over investment No. 2. Therefore, investment No. 3 should be recommended.

(d) Method of net present worth. For investment No. 1, as illustrated in Table 1, the present value of the cash flow to the project, discounted at an interest rate of 15 percent, is $127,000. Therefore, the net present worth of investment No. 1 is $127,000 - $110,000 = $17,000.

For investments 2 and 3, the present values of the cash flows to the projects are determined from the first two equations under part (c) of this problem, with \(i = 0.15\). The resulting net present worth are:

For investment No. 2, net present worth = $226,000 - $180,000 = $46,000

For investment No. 3, net present worth = $278,000 - $225,000 = $53,000

The greatest net present worth is found for investment No. 3; therefore, investment No. 3 should be recommended.

(e) Method of capitalized costs. Capitalized costs for each investment situation must include the capitalized cost for the original investment to permit an indefinite number of replacements plus the capitalized present value of the cash expenses plus working capital.

Capitalized present value of cash expenses is determined as follows:

Let \(C_{n'}^{\prime}\) be the annual cash expense in year \(n'\) of the project life. The present value of the annual cash expenses is then

$$\sum_{d=1}^{n'-n} \frac{1}{(1 + i)^d} C_{n'}^{\prime}$$

and the capitalized present value is

$$\frac{(1 + i)^n \sum_{n'=1}^{n'-n} C_{n'}^{\prime} (1 + i)^{n'-n}}{(1 + i)^n - 1}$$
If \( C_r \) is constant, as is the case for this example, the capitalized present value becomes (annual cash expenses)/\( i \). Therefore,

\[
\text{Capitalized cost} = \frac{C_r (1 + i)^n}{(1 + i)^n - 1} + \frac{V_s}{i} + \text{annual cash expenses} + \text{working capital}
\]

where \( n \) = service life
\( i \) = annual rate of return
\( C_r \) = replacement cost
\( V_s \) = salvage value

For investment No. 1,

\[
\text{Capitalized cost} = \frac{(90,000)(1 + 0.15)^5}{(1 + 0.15)^5 - 1} + 10,000 + \frac{44,000}{0.15} + 10,000 = $492,000
\]

For investment No. 2,

\[
\text{Capitalized cost} = \frac{(155,000)(1 + 0.15)^7}{(1 + 0.15)^7 - 1} + 15,000 + \frac{28,000}{0.15} + 10,000 = $460,000
\]

For investment No. 3,

\[
\text{Capitalized cost} = \frac{(190,000)(1 + 0.15)^8}{(1 + 0.15)^8 - 1} + 20,000 + \frac{21,000}{0.15} + 15,000 = $457,000
\]

The capitalized cost based on a minimum rate of return of 15 percent is least for investment No. 3; therefore, **investment No. 3 should be recommended**.

**Note:** Methods (a) and (b) in this problem give incorrect results because the time value of money has not been included. Although investment No. 3 is recommended by methods (c), (d), and (e), it is a relatively narrow choice over investment No. 2. Consequently, for a more accurate evaluation, it would appear that the company management should be informed that certain of their policies relative to profitability evaluation are somewhat old fashioned and do not permit the presentation of a totally realistic situation. For example, the straight-line depreciation method may not be the best choice, and a more realistic depreciation method may be appropriate. The policy of basing time-value-of-money interpretations on end-of-year costs and profits is a simplification, and it may be better to permit the use of continuous interest compounding and continuous cash flow where appropriate. For a final detailed analysis involving a complete plant, variations in prestartup costs among alternatives may be important, and this factor should not be ignored.

**REPLACEMENTS**

The term “replacement,” as used in this chapter, refers to a special type of alternative in which facilities are currently in existence and it may be desirable to replace these facilities with different ones. Although intangible factors may
have a strong influence on decisions relative to replacements, the design engineer must understand the tangible economic implications when a recommendation is made as to whether or not existing equipment or facilities should be replaced.

The reasons for making replacements can be divided into two general classes, as follows:

1. An existing property must be replaced or changed in order to continue operation and meet the required demands for service or production. Some examples of reasons for this type of necessary replacement are:
   a. The property is worn out and can give no further useful service.
   b. The property does not have sufficient capacity to meet the demand placed upon it.
   c. Operation of the property is no longer economically feasible because changes in design or product requirements have caused the property to become obsolete.

2. An existing property is capable of yielding the necessary product or service, but more efficient equipment or property is available which can operate with lower expenses.

When the reason for a replacement falls in the first general type, the only alternatives are to make the necessary changes or else go out of business. Under these conditions, the final economic analysis is usually reduced to a comparison of alternative investments.

The correct decision as the the desirability of replacing an existing property which is capable of yielding the necessary product or service depends on a clear understanding of theoretical replacement policies plus a careful consideration of many practical factors. In order to determine whether or not a change is advisable, the operating expenses with the present equipment must be compared with those that would exist if the change were made. Practical considerations, such as amount of capital available or benefits to be gained by meeting a competitor’s standards, may also have an important effect on the final decision.

Methods of Profitability Evaluation for Replacements

The same methods that were explained and applied earlier in this chapter are applicable for replacement analyses. Net-present-worth and discounted-cash-flow methods give the soundest results for maximizing the overall future worth of a concern. However, for the purpose of explaining the basic principles of replacement economic analyses, the simple rate-of-return-on-investment method of analysis is just as effective as those methods involving the time value of money. Thus, to permit the use of direct illustrations which will not detract from
the principles under consideration, the following analysis of methods for economic decisions on replacements uses the annual rate of return on initial investment as the profitability measure. The identical principles can be treated by more complex rate-of-return and net-present-worth solutions by merely applying the methods described earlier in this chapter.

**Typical Example of Replacement Policy**

The following example illustrates the type of economic analysis involved in determining if a replacement should be made: A company is using a piece of equipment which originally cost $30,000. The equipment has been in use for 5 years, and it now has a net realizable value of $6000. At the time of installation, the service life was estimated to be 10 years and the salvage value at the end of the service life was estimated to be zero. Operating costs amount to $22,000/year. At the present time, the remaining service life of the equipment is estimated to be 3 years.

A proposal has been made to replace the present piece of property by one of more advanced design. The proposed equipment would cost $40,000, and the operating costs would be $15,000/year. The service life is estimated to be 10 years with a nonzero salvage value. Each piece of equipment will perform the same service, and all costs other than those for operation and depreciation will remain constant. Depreciation costs are determined by the straight-line method. The company will not make any unnecessary investments in equipment unless it can obtain an annual return on the necessary capital of at least 10 percent.

The two alternatives in this example are to continue the use of the present equipment or to make the suggested replacement. In order to choose the better alternative, it is necessary to consider both the reduction in expenses obtainable by making the change and the amount of new capital necessary. The only variable expenses are those for operation and depreciation. Therefore, the annual variable expenses for the proposed equipment would be $15,000 + $40,000/10 = $19,000.

The net realizable value of the existing equipment is $6000. In order to make a fair comparison between the two alternatives, all costs must be based on conditions at the present time. Therefore, the annual depreciation cost for the existing equipment during the remaining three years of service life would be $6000/3 = $2000. For purposes of comparison, the annual variable expenses if the equipment is retained in service would be $22,000 + $2000 = $24,000.

An annual saving of $24,000 - $19,000 = $5000 can be realized by making the replacement. The cost of the new equipment is $40,000, but the sale of the existing property would provide $6000; therefore, it would be necessary to invest

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*†The use of the sinking-fund method for determining depreciation is sometimes advocated for replacement studies. However, for most practical situations, the straight-line method is satisfactory.*
only $34,000 to bring about an annual saving of $5000. Since this represents a return greater than 10 percent, the existing equipment should be replaced.

**Book Values and Unamortized Values**

In the preceding example, the book value of the existing property was $15,000 at the time of the proposed replacement. However, this fact was given no consideration in determining if the replacement should be made. The book value is based on past conditions, and the correct decision as to the desirability of making a replacement must be based on present conditions.

The difference between the book value and the net realizable value at any time is commonly designated as the *unamortized* value. In the example, the unamortized value was $15,000 – $6000 = $9000. This means that a $9000 loss was incurred because of incorrect estimation of depreciation allowances.

Much of the confusion existing in replacement studies is caused by unamortized values. Some individuals feel that a positive unamortized value represents a loss which would be caused by making a replacement. This is not correct because the loss is a result of past mistakes, and the fact that the error was not apparent until a replacement was considered can have no bearing on the conditions existing at the present time. *When making theoretical replacement studies, unamortized values must be considered as due to past errors, and these values are of no significance in the present decision as to whether or not a replacement should be made.*

Although unamortized values have no part in a replacement study, they must be accounted for in some manner. One method for handling these capital losses or gains is to charge them directly to the profit-and-loss account for the current operating period. When a considerable loss is involved, this method may have an unfavorable and unbalanced effect on the profits for the current period. Many concerns protect themselves against such unfavorable effects by building up a surplus reserve fund to handle unamortized values. This fund is accumulated by setting aside a certain sum each accounting period. When losses due to unamortized values occur, they are charged against the accumulated fund. In this manner, unamortized values can be handled with no serious effects on the periodic profits.

**Investment on which Replacement Comparison is Based**

As indicated in the preceding section, the unamortized value of an existing property is based on past conditions and plays no part in a replacement study.

†As explained in Chap. 8 (Taxes and Insurance) and Chap. 9 (Depreciation) and as illustrated in Example 6 of this Chapter, part of the loss may be recovered by income-tax write-offs if the Internal Revenue Service will agree that the maximum expected life was used.
The advisability of making a replacement is usually determined by the rate of return which can be realized from the necessary investment. It is, therefore, important to consider the amount of the investment. The difference between the total cost of the replacement property and the net realizable value of the misting property equals the necessary investment. Thus, the correct determination of the investment involves only consideration of the present capital outlay required if the replacement is made.

Net Realizable Value

In replacement studies, the net realizable value of an existing property should be assumed to be the market value. Although this may be less than the actual value of the property as far as the owner is concerned, it still represents the amount of capital which can be obtained from the old equipment if the replacement is made. Any attempt to assign an existing property a value greater than the net realizable value tends to favor replacements which are uneconomical.

Analysis of Common Errors Made in Replacement Studies

Most of the errors in replacement studies are caused by failure to realize that a replacement analysis must be based on conditions existing at the present time. Some persons insist on trying to compensate for past mistakes by forcing new ventures to pay off losses incurred in the past. Instead, these losses should be accepted, and the new venture should be considered on its own merit.

INCLUDING UNAMORTIZED VALUE AS AN ADDITION TO THE REPLACEMENT INVESTMENT. This is one of the most common errors. It increases the apparent cost for the replacement and tends to prevent replacements which are really economical. Some persons incorporate this error into the determination of depreciation cost for the replacement equipment, while others include it only in finding the investment on which the rate of return is based. In any case, this method of attempting to account for unamortized values is incorrect. The unamortized value must be considered as a dead loss (or gain) due to incorrect depreciation accounting in the past.

USE OF BOOK VALUE FOR OLD EQUIPMENT IN REPLACEMENT STUDIES. This error is caused by refusal to admit that the depreciation accounting methods used in the past were wrong. Persons who make this error attempt to justify their actions by claiming that continued operation of the present equipment would eventually permit complete depreciation. This viewpoint is completely unrealistic because it gives no consideration to the competitive situation existing in modern business. The concern which can operate with good profit and still offer a given product or service at the lowest price can remain in business and force competitors either to reduce their profits or to cease operation.
When book values are used in replacement studies, the apparent costs for the existing equipment are usually greater than they should be, while the apparent capital outlay for the replacement is reduced. Therefore, this error tends to favor replacements which are really uneconomical.

Example 6 An extreme situation to illustrate result of replacement economic analysis. A new manufacturing unit has just been constructed and put into operation by your company. The basis of the manufacturing process is a special computer for control (designated as OVT computer) as developed by your research department. The plant has now been in operation for less than one week and is performing according to expectations. A new computer (designated as NTR computer) has just become available on the market. This new computer can easily be installed at once in place of your present computer and will do the identical job at far less annual cash expense because of reduced maintenance and personnel costs. However, if the new computer is installed, your present computer is essentially worthless because you have no other use for it.

Following is pertinent economic information relative to the two computers:

<table>
<thead>
<tr>
<th></th>
<th>OVT computer</th>
<th>NTR computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital investment</td>
<td>$2,000,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Estimated economic life</td>
<td>10 years</td>
<td>10 years</td>
</tr>
<tr>
<td>Salvage value</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>at end of economic life</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual cash expenses</td>
<td>$250,000</td>
<td>$50,000</td>
</tr>
</tbody>
</table>

What recommendation would you make relative to replacing the present $2,000,000 computer with the new computer?

Solution. Assuming straight-line depreciation, the annual total expenses with the NTR computer = $50,000 + $1,000,000/10 = $150,000.

For replacement economic comparison, the OVT computer is worth nothing at the present time; therefore, annual total expenses with the OVT computer = $250,000 (i.e., no depreciation charge).

The $2,000,000 investment for the OVT computer is completely lost if the NTR computer is installed; so the total necessary investment to make an annual saving of $250,000 − $150,000 is $1,000,000. Therefore, the return on investment would be (100,000/1,000,000)(100) = 10 percent.

If your company is willing to accept a return on investment of 10 percent before taxes [or (0.66)(10) = 6.6 percent after taxes, assuming a 34 percent tax rate], the replacement should be made.

If income taxes are taken completely into consideration, the result can tend to favor the replacement. For example, if your company can write off the $2,000,000 loss in one lump sum against other capital gains which would normally be taxed at 30 percent, the net saving will be 30 percent of $2,000,000 or $600,000. Assuming a 34 percent tax rate on annual profits and dividing the $600,000 capital-gains tax
saving over the ten years of the new computer life, the percent return after taxes =

\[
\frac{600,000/10 + (0.66)(100,000)}{1,000,000}(100) = 12.6 \text{ percent}
\]

Because of the reduced costs for the NTR computer, profitability evaluation including time value of money will tend to favor the replacement more than does the method of rate of return on investment as used for the solution of this example.

**PRACTICAL FACTORS IN ALTERNATIVE-INVESTMENT AND REPLACEMENT STUDIES**

The previous discussion has presented the theoretical viewpoint of alternative-investment and replacement studies; however, certain practical considerations also influence the final decision. In many cases, the amount of available capital is limited. From a practical viewpoint, therefore, it may be desirable to accept the smallest investment which will give the necessary service and permit the required return. Although a greater investment might be better on a theoretical basis, the additional return would not be worth the extra risks involved when capital must be borrowed or obtained from some other outside source.

A second practical factor which should be considered is the accuracy of the estimations used in determining the rates of return. A theoretically sound investment might not be accepted because the service life used in determining depreciation costs appear to be too long. **All** risk factors should be given careful consideration before making any investment, and the risk factors should receive particular attention before accepting an investment greater than that absolutely necessary.

The economic conditions existing at the particular time have an important practical effect on the final decision. In depression periods or in times when economic conditions are very uncertain, it may be advisable to refrain from investing any more capital than is absolutely necessary. The tax situation for the corporation can also have an effect on the decision.

Many intangible factors enter into the final decision on a proposed investment. Sometimes it may be desirable to impress the general public by the external appearance of a property or by some unnecessary treatment of the final product. These advertising benefits would probably receive no consideration in a theoretical economic analysis, but they certainly would influence management’s final decision in choosing the best investment.

Personal whims or prejudices, desire to better a competitor’s rate of production or standards, the availability of excess capital, and the urge to expand an existing plant are other practical factors which may be involved in determining whether or not a particular investment will be made.

Theoretical analyses of alternative investments and replacements can be used to obtain a dollar-and-cents indication of what should be done about a
proposed investment. The final decision depends on these theoretical results plus practical factors determined by the existing circumstances.

**NOMENCLATURE FOR CHAPTER 10**

- \( C_n \) = annual cash expenses in year \( n \)’, dollars
- \( CP_{\text{zero \ time}} \) = total cash position at zero time, dollars
- \( C_R \) = replacement cost, dollars
- \( C_Y \) = original cost, dollars
- \( d_n \) = discount factor for determining present value
- \( e \) = base of the natural logarithm = 2.71828...
- \( i \) = annual interest rate of return, percent/100
- \( K \) = capitalized cost, dollars
- \( n \) = estimated service life, years
- \( n' \) = year of project life to which cash flow applies
- \( P \) = present value, dollars
- \( R \) = end-of-year (or ordinary) annuity amount, dollars/year
- \( R' \) = total of all ordinary annuity payments occurring regularly throughout the time period of one year, dollars/year
- \( r \) = nominal continuous interest rate, percent/100
- \( S \) = future worth, dollars
- \( V_Y \) = estimated salvage value at end of service life, dollars
- \( X \) = number of evaporator units
- \( Y \) = time period for construction, years
- \( Z \) = time period land is owned before plant startup, years

**PROBLEMS**

1. A proposed chemical plant will require a fixed-capital investment of $10 million. It is estimated that the working capital will amount to 25 percent of the total investment, and annual depreciation costs are estimated to be 10 percent of the fixed-capital investment. If the annual profit will be $3 million, determine the standard percent return on the total investment and the minimum payout period.

2. An investigation of a proposed investment has been made. The following result has been presented to management: The minimum payout period based on capital recovery using a minimum annual return of 10 percent as a fictitious expense is 10 years; annual depreciation costs amount to 8 percent of the total investment. Using this information, determine the standard rate of return on the investment.

3. The information given in Prob. 2 applies to conditions before income taxes. If 34 percent of all profits must be paid out for income taxes, determine the standard rate of return after taxes using the figures given in Prob. 2.

4. A heat exchanger has been designed and insulation is being considered for the unit. The insulation can be obtained in thickness of 1, 2, 3, or 4 in. The following data
have been determined for the different insulation thicknesses:

<table>
<thead>
<tr>
<th>Btu/h saved</th>
<th>1 in.</th>
<th>2 in.</th>
<th>3 in.</th>
<th>4 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost for installed insulation</td>
<td>300,000</td>
<td>350,000</td>
<td>370,000</td>
<td>380,000</td>
</tr>
<tr>
<td>Annual fixed charges</td>
<td>$1200</td>
<td>$1600</td>
<td>$1800</td>
<td>$1870</td>
</tr>
<tr>
<td>Annual return</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

What thickness of insulation should be used? The value of heat is 30 cents/1,000,000 Btu. An annual return of 15 percent on the fixed-capital investment is required for any capital put into this type of investment. The exchanger operates 300 days per year.

5. A company must purchase one reactor to be used in an overall operation. Four reactors have been designed, all of which are equally capable of giving the required service. The following data apply to the four designs:

<table>
<thead>
<tr>
<th>Design</th>
<th>Fixed-capital investment</th>
<th>Sum of operating and fixed costs per year (all other costs are constant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000</td>
<td>3,000</td>
</tr>
<tr>
<td>2</td>
<td>$12,000</td>
<td>2,800</td>
</tr>
<tr>
<td>3</td>
<td>$14,000</td>
<td>2,350</td>
</tr>
<tr>
<td>4</td>
<td>$16,000</td>
<td>2,100</td>
</tr>
</tbody>
</table>

If the company demands a 15 percent return on any unnecessary investment, which of the four designs should be accepted?

6. The capitalized cost for a piece of equipment has been found to be $55,000. This cost is based on the original cost plus the present value of an indefinite number of renewals. An annual interest rate of 12 percent was used in determining the capitalized cost. The salvage value of the equipment at the end of the service life was taken to be zero, and the service life was estimated to be 10 years. Under these conditions, what would be the original cost of the equipment?

7. An existing warehouse is worth $500,000, and the average value of the goods stored in it is $400,000. The annual insurance rate on the warehouse is 1.1 percent, and the insurance rate on the stored goods is 0.95 percent. If a proposed sprinkling system is installed in the warehouse, both insurance rates would be reduced to three-quarters of the original rate. The installed sprinkler system would cost $20,000, and the additional annual cost of maintenance, inspection, and taxes would be $300. The required write-off period for the entire investment in the sprinkler system is 20 years. The capital necessary to make the investment is available. The operation of the warehouse is now giving an 8 percent return on the original investment. Give reasons why you would or would not recommend installing the sprinkler system.

8. A power plant for generating electricity is one part of a plant-design proposal. Two alternative power plants with the necessary capacity have been suggested. One uses a boiler and steam turbine while the other uses a gas turbine. The following informa-
tion applies to the two proposals:

<table>
<thead>
<tr>
<th></th>
<th>Boiler and steam turbine</th>
<th>Gas turbine</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial investment</strong></td>
<td>$600,000</td>
<td>$400,000</td>
</tr>
<tr>
<td><strong>Fuel costs, per year</strong></td>
<td>16,000</td>
<td>23,000</td>
</tr>
<tr>
<td><strong>Maintenance and repairs, per year</strong></td>
<td>12,000</td>
<td>15,000</td>
</tr>
<tr>
<td><strong>Insurance and taxes, per year</strong></td>
<td>18,000</td>
<td>12,000</td>
</tr>
<tr>
<td><strong>Service life, years</strong></td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td><strong>Salvage value at end of service life</strong></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

All other costs are the same for either type of power plant. A 12 percent return is required on any investment. If one of these power plants must be accepted, which one should be recommended?

9. The facilities of an existing chemical company must be increased if the company is to continue in operation. There are two alternatives. One of the alternatives is to expand the present plant. If this is done, the expansion would cost $130,000. Additional labor costs would be $150,000 per year, while additional costs for overhead, depreciation, taxes, and insurance would be $60,000 per year.

A second alternative requires construction and operation of new facilities at a location about 50 miles from the present plant. This alternative is attractive because cheaper labor is available at this location. The new facilities would cost $200,000. Labor costs would be $120,000 per year. Overhead costs would be $70,000 per year. Annual insurance and taxes would amount to 2 percent of the initial cost. All other costs except depreciation would be the same at each location. If the minimum return on any acceptable investment is 9 percent, determine the minimum service life allowable for the facilities at the distant location for this alternative to meet the required incremental return. The salvage value should be assumed to be zero, and straight-line depreciation accounting may be used.

10. A chemical company is considering replacing a batch-wise reactor with a modernized continuous reactor. The old unit cost $40,000 when new 5 years ago, and depreciation has been charged on a straight-line basis using an estimated service life of 15 years and final salvage value of $1000. It is now estimated that the unit has a remaining service life of 10 years and a final salvage value of $1000.

The new unit would cost $70,000 and would result in an increase of $5000 in the gross annual income. It would permit a labor saving of $7000 per year. Additional costs for taxes and insurance would be $1000 per year. The service life is estimated to be 12 years with a final salvage value of $1000. All costs other than those for labor, insurance, taxes, and depreciation may be assumed to be the same for both units. The old unit can now be sold for $5000. If the minimum required return on any investment is 15 percent, should the replacement be made?

11. The owner of a small antifreeze plant has a small canning unit which cost him $5000 when he purchased it 10 years ago. The unit has completely depreciated, but the owner estimates that it will still give him good service for 5 more years. At the end of 5 years the unit will be worth a junk value of $100. The owner now has an
opportunity to buy a more efficient canning unit for $6000 having an estimated service life of 10 years and zero salvage or junk value. This new unit would reduce annual labor and maintenance costs by $1000 and increase annual expenses for taxes and insurance by $100. All other expenses except depreciation would be unchanged. If the old canning unit can be sold for $600, what replacement return on his capital investment will the owner receive if he decides to make the replacement?

12. An engineer in charge of the design of a plant must choose either a batch or a continuous system. The batch system offers a lower initial outlay but, owing to higher labor requirements, exhibits a higher operating cost. The cash flows relevant to this problem have been estimated as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Batch system</th>
<th>Continuous system</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−$20,000</td>
<td>−$30,000</td>
</tr>
<tr>
<td>1-10</td>
<td>$5600</td>
<td>$7650</td>
</tr>
</tbody>
</table>

Discounted-cash-flow rate of return
- Batch system: 25%
- Continuous system: 22%

Net present worth at 10%
- Batch system: $14,400
- Continuous system: $17,000

Check the values given for the discounted-cash-flow rate of return and net present worth. If the company requires a minimum rate of return of 10 percent, which system should be chosen?

13. A company is considering the purchase and installation of a pump which will deliver oil at a faster rate than the pump already in use. The purchase and installation of the larger pump will require an immediate layout of $1600, but it will recover all the cost by the end of one year. The relevant cash flows have been established as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Install larger pump</th>
<th>Operate existing pump</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−$1600</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$20,000</td>
<td>$10,000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$10,000</td>
</tr>
</tbody>
</table>

Discounted-cash-flow rate of return
- Install larger pump: 1250%
- Operate existing pump: ?

Net present worth at 10%
- Install larger pump: $16,580
- Operate existing pump: $17,355

Explain the values given for the discounted-cash-flow rate of return and net present worth. If the company requires a minimum rate of return of 10 percent, which alternative should be chosen?

14. An oil company is offered a lease of a group of oil wells on which the primary reserves are close to exhaustion. The major condition of the purchase is that the oil company must agree to undertake a water-flood project at the end of five years to make possible secondary recovery. No immediate payment by the oil company is
required. The relevant cash flows have been estimated as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Discounted-cash-flow rate of return</th>
<th>Net present worth at 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$50,000</td>
<td>$227,000</td>
</tr>
<tr>
<td>1-4</td>
<td>−$650,000</td>
<td>$100,000</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>?</td>
</tr>
<tr>
<td>6-20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Should the lease-and-flood arrangement be accepted? How should this proposal be presented to the company board of directors who understand and make it a policy to evaluate by discounted-cash-flow rate of return?

15. For Example 3 in this chapter, determine the profitability index using the simplified Eq. (D) in the example instead of Eq. (E) as was used for Table 3. As a first approximation, assume the profitability index is 30 percent.
An optimum design is based on the best or most favorable conditions. In almost every case, these optimum conditions can ultimately be reduced to a consideration of costs or profits. Thus, an optimum economic design could be based on conditions giving the least cost per unit of time or the maximum profit per unit of production. When one design variable is changed, it is often found that some costs increase and others decrease. Under these conditions, the total cost may go through a minimum at one value of the particular design variable, and this value would be considered as an optimum.

An example illustrating the principles of an optimum economic design is presented in Fig. 11-1. In this simple case, the problem is to determine the optimum thickness of insulation for a given steam-pipe installation. As the insulation thickness is increased, the annual fixed costs increase, the cost of heat loss decreases, and all other costs remain constant. Therefore, as shown in Fig. 11-1, the sum of the costs must go through a minimum at the optimum insulation thickness.

Although cost considerations and economic balances are the basis of most optimum designs, there are times when factors other than cost can determine the most favorable conditions. For example, in the operation of a catalytic reactor, an optimum operation temperature may exist for each reactor size because of equilibrium and reaction-rate limitations. This particular temperature could be based on the maximum percentage conversion or on the maximum amount of final product per unit of time. Ultimately, however, cost variables
need to be considered, and the development of an optimum operation design is usually merely one step in the determination of an optimum economic design.

**INCREMENTAL COSTS**

The subject of incremental costs is covered in detail in Chap. 10 (Profitability, Alternative Investments, and Replacements). Consideration of incremental costs shows that a final recommended design does not need to correspond to the optimum economic design, because the incremental return on the added investment may become unacceptable before the optimum point is reached. However, the optimum values can be used as a basis for starting the incremental-cost analyses.

This chapter deals with methods for determining optimum conditions, and it is assumed that the reader understands the role of incremental costs in establishing a final recommended design.

†See Fig. 10-5 and the related discussion in Chap. 10. The material presented in Chap. 11 considers optimum designs based on maximum or minimum values of a specified variable. The same type of approach could be used if the term optimum (referring to an investment) were defined on the basis of a stipulated incremental return.
INTANGIBLE AND PRACTICAL CONSIDERATIONS

The various mathematical methods for determining optimum conditions, as presented in this chapter, represent on a theoretical basis the conditions that best meet the requirements. However, factors that cannot easily be quantitized or practical considerations may change the final recommendation to other than the theoretically correct optimum condition. Thus, a determination of an “optimum condition,” as described in this chapter, serves as a base point for a cost or design analysis, and it can often be quantitized in specific mathematical form. From this point, the engineer must apply judgment to take into account other important practical factors, such as return on investment or the fact that commercial equipment is often available in discrete intervals of size.

As an example, consider the case where an engineer has made an estimation of the optimum pipe diameter necessary to handle a given flow stream based on minimizing the costs due to fixed charges and frictional pumping costs. The mathematical result shows that the optimum inside pipe diameter is 2.54 in. based on costs for standard (schedule 40) steel pipe. Nominal pipe diameters available commercially in this range are 2; in. (ID of 2.469 in.) and 3 in. (ID of 3.069 in.). The practical engineer would probably immediately recommend a nominal pipe diameter of 2; in. without going to the extra effort of calculating return on investment for the various sizes available. This approach would normally be acceptable because of the estimations necessarily involved in the optimum calculation and because of the fact that an investment for pipe represents only a small portion of the total investment.

Intangible factors may have an effect on the degree of faith that can be placed on calculated results for optimum conditions. Perhaps the optimum is based on an assumed selling price for the product from the process, or it might be that a preliminary evaluation is involved in which the location of the plant is not final. Obviously, for cases of this type, an analysis for optimum conditions can give only a general idea of the actual results that will be obtained in the final plant, and it is not reasonable to go to extreme limits of precision and accuracy in making recommendations. Even for the case of a detailed and firm design, intangibles, such as the final bid from various contractors for the construction, may make it impractical to waste a large amount of effort in bringing too many refinements into the estimation of optimum conditions.

GENERAL PROCEDURE FOR DETERMINING OPTIMUM CONDITIONS

The first step in the development of an optimum design is to determine what factor is to be optimized. Typical factors would be total cost per unit of production or per unit of time, profit, amount of final product per unit of time, and percent conversion. Once the basis is determined, it is necessary to develop relationships showing how the different variables involved affect the chosen
factor. Finally, these relationships are combined graphically or analytically to give the desired optimum conditions.

**PROCEDURE WITH ONE VARIABLE**

There are many cases in which the factor being optimized is a function of a single variable. The procedure then becomes very simple. Consider the example presented in Fig. 11-1, where it is necessary to obtain the insulation thickness which gives the least total cost. The primary variable involved is the thickness of the insulation, and relationships can be developed showing how this variable affects all costs.

Cost data for the purchase and installation of the insulation are available, and the length of service life can be estimated. Therefore, a relationship giving the effect of insulation thickness on fixed charges can be developed. Similarly, a relationship showing the cost of heat lost as a function of insulation thickness can be obtained from data on the value of steam, properties of the insulation, and heat-transfer considerations. All other costs, such as maintenance and plant expenses, can be assumed to be independent of the insulation thickness.

The two cost relationships obtained might be expressed in a simplified form similar to the following:

Fixed charges: 
\[ \phi(x) = ax + b \]  
(1)

Cost of heat loss: 
\[ \phi'(x) = \frac{c}{x} + d \]  
(2)

Total variable cost: 
\[ C_v = \phi(x) + \phi'(x) = \phi''(x) = ax + b + \frac{c}{x} + d \]  
(3)

where \( a, b, c, \) and \( d \) are constants and \( x \) is the common variable (insulation thickness).

The graphical method for determining the optimum insulation thickness is shown in Fig. 11-1. The optimum thickness of insulation is found at the minimum point on the curve obtained by plotting total variable cost versus insulation thickness.

The slope of the total-variable-cost curve is zero at the point of optimum insulation thickness. Therefore, if Eq. (3) applies, the optimum value can be found analytically by merely setting the derivative of \( C_v \) with respect to \( x \) equal to zero and solving for \( x \).

\[ \frac{dC_v}{dx} = a - \frac{c}{x^2} = 0 \]  
(4)

\[ x = \left( \frac{c}{a} \right)^{1/2} \]  
(5)

If the factor being optimized \( (C_v) \) does not attain a usable maximum or minimum value, the solution for the dependent variable will indicate this
condition by giving an impossible result, such as infinity, zero, or the square root of a negative number.

The value of $x$ shown in Eq. (5) occurs at an optimum point or a point of inflection. The second derivative of Eq. (3), evaluated at the given point, indicates if the value occurs at a minimum (second derivative greater than zero), maximum (second derivative less than zero), or point of inflection (second derivative equal to zero). An alternative method for determining the type of point involved is to calculate values of the factor being optimized at points slightly greater and slightly smaller than the optimum value of the dependent variable.

The second derivative of Eq. (3) is

$$\frac{d^2C_T}{dx^2} = \frac{2c}{x^3}$$

If $x$ represents a variable such as insulation thickness, its value must be positive; therefore, if $c$ is positive, the second derivative at the optimum point must be greater than zero, and $(c/a)^{1/2}$ represents the value of $x$ at the point where the total variable cost is a minimum.

**PROCEDURE WITH TWO OR MORE VARIABLES**

When two or more independent variables affect the factor being optimized, the procedure for determining the optimum conditions may become rather tedious; however, the general approach is the same as when only one variable is involved.

Consider the case in which the total cost for a given operation is a function of the two independent variables $x$ and $y$, or

$$C_T = \phi^iii(x, y)$$

By analyzing all the costs involved and reducing the resulting relationships to a simple form, the following function might be found for Eq. (7):

$$C_T = ax + \frac{b}{xy} + cy + d$$

where $a$, $b$, $c$, and $d$ are positive constants.

**GRAPHICAL PROCEDURE.** The relationship among $C_T$, $x$, and $y$ could be shown as a curved surface in a three-dimensional plot, with a minimum value of $C_T$ occurring at the optimum values of $x$ and $y$. However, the use of a three-dimensional plot is not practical for most engineering determinations.

The optimum values of $x$ and $y$ in Eq. (8) can be found graphically on a two-dimensional plot by using the method indicated in Fig. 11-2. In this figure, the factor being optimized is plotted against one of the independent variables ($x$), with the second variable ($y$) held at a constant value. A series of such plots is made with each dashed curve representing a different constant value of the
Finol plot for determining optimum values with three independent variables

Each point represents on optimum value of $C_T$, $x$, and $y$

Optimum value of $C_T$, $x$, $y$, and $z$

Third variable ($z$) = constant

$C_T$

$x$

$y''$

$y'''$

$y'$

$C_T'$

$y' = constant$

Gives optimum values of $C_T = C_T'$, $y = y'$, and $x = x'$

**FIGURE 11-2**
Graphical determination of optimum conditions with two or more independent variables.

...second variable. As shown in Fig. 11-2, each of the curves (A, B, C, D, and E) gives one value of the first variable $x$ at the point where the total cost is a minimum. The curve NM represents the locus of all these minimum points, and the optimum value of $x$ and $y$ occurs at the minimum point on curve NM.

Similar graphical procedures can be used when there are more than two independent variables. For example, if a third variable $z$ were included in Eq. (8), the first step would be to make a plot similar to Fig. 11-2 at one constant value of $z$. Similar plots would then be made at other constant values of $z$. Each plot would give an optimum value of $x$, $y$, and $C_T$ for a particular $z$. Finally, as shown in the insert in Fig. 11-2, the overall optimum value of $x$, $y$, $z$, and $C_T$ could be obtained by plotting $z$ versus the individual optimum values of $C_T$.

**ANALYTICAL PROCEDURE.** In Fig. 11-2, the optimum value of $x$ is found at the point where $(\partial C_T/\partial x)_{y = y'}$ is equal to zero. Similarly, the same results would be obtained if $y$ were used as the abscissa instead of $x$. If this were done, the optimum value of $y$ (i.e., $y'$) would be found at the point where $(\partial C_T/\partial y)_{x = x'}$ is
equal to zero. This immediately indicates an analytical procedure for determining optimum values.

Using Eq. (8) as a basis,

\[
\frac{\partial C_T}{\partial x} = a - \frac{b}{x^2 y} \quad (9)
\]

\[
\frac{\partial C_T}{\partial y} = c - \frac{b}{xy^2} \quad (10)
\]

At the optimum conditions, both of these partial derivatives must be equal to zero; thus, Eqs. (9) and (10) can be set equal to zero and the optimum values of \( x = \left(\frac{cb}{a^2}\right)^{1/3} \) and \( y = \left(\frac{ab}{c^2}\right)^{1/3} \) can be obtained by solving the two simultaneous equations. If more than two independent variables were involved, the same procedure would be followed, with the number of simultaneous equations being equal to the number of independent variables.

Example 1 Determination of optimum values with two independent variables. The following equation shows the effect of the variables \( x \) and \( y \) on the total cost for a particular operation:

\[
C_T = 2.33x + \frac{11,900}{xy} + 1.86y + 10
\]

Determine the values of \( x \) and \( y \) which will give the least total cost.

Solution

Analytical method.

\[
\frac{\partial C_T}{\partial x} = 2.33 - \frac{11,900}{x^2 y}
\]

\[
\frac{\partial C_T}{\partial y} = 1.86 - \frac{11,900}{xy^2}
\]

At the optimum point,

\[
2.33 - \frac{11,900}{x^2 y} = 0
\]

\[
1.86 - \frac{11,900}{xy^2} = 0
\]

Solving simultaneously for the optimum values of \( x \) and \( y \),

\[
x = 16
\]

\[
y = 20
\]

\[
C_T = 121.6
\]
A check should be made to make certain the preceding values represent conditions of minimum cost.

\[
\frac{\partial^2 C_T}{\partial x^2} = \frac{(2)(11,900)}{x^3 y} = \frac{(2)(11,900)}{(16)^3(20)} = + \text{ at optimum point}
\]

\[
\frac{\partial^2 C_T}{\partial y^2} = \frac{(2)(11,900)}{xy^3} = \frac{(2)(11,900)}{(16)(20)^3} = + \text{ at optimum point}
\]

Since the second derivatives are positive, the optimum conditions must occur at a point of minimum cost.

**Graphical method.** The following constant values of \( y \) are chosen arbitrarily:

\[
y^{ii} = 32 \quad y^{iii} = 26 \quad y^{i} = 20 \quad y^{iv} = 15 \quad y' = 12
\]

At each constant value of \( y \), a plot is made of \( C_T \) versus \( x \). These plots are presented in Fig. 11-2 as curves A, B, C, D, and E. A summary of the results is presented in the following table:

<table>
<thead>
<tr>
<th>( y )</th>
<th>Optimum ( x )</th>
<th>Optimum ( C_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^{ii} = 32 )</td>
<td>12.7</td>
<td>128.3</td>
</tr>
<tr>
<td>( y^{iii} = 26 )</td>
<td>14.1</td>
<td>123.6</td>
</tr>
<tr>
<td>( y^{i} = 20 )</td>
<td>16.0</td>
<td>121.6</td>
</tr>
<tr>
<td>( y^{iv} = 15 )</td>
<td>18.5</td>
<td>123.9</td>
</tr>
<tr>
<td>( y' = 12 )</td>
<td>20.1</td>
<td>128.5</td>
</tr>
</tbody>
</table>

One curve (\( NM \) in Fig. 11-2) through the various optimum points shows that the overall optimum occurs at

\[
x = 16 \\
y = 20 \\
C_T = 121.6
\]

**Note:** In this case, a value of \( y \) was chosen which corresponded to the optimum value. Usually, it is necessary to interpolate or make further calculations in order to determine the final optimum conditions.

**COMPARISON OF GRAPHICAL AND ANALYTICAL METHODS**

In the determination of optimum conditions, the same final results are obtained with either graphical or analytical methods. Sometimes it is impossible to set up one analytical function for differentiation, and the graphical method must be used. If the development and simplification of the total analytical function require complicated mathematics, it may be simpler to resort to the direct graphical solution; however, each individual problem should be analyzed on the basis of the existing circumstances. For example, if numerous repeated trials are
necessary, the extra time required to develop an analytical solution may be well spent.

The graphical method has one distinct advantage over the analytical method. The shape of the curve indicates the importance of operating at or very close to the optimum conditions. If the maximum or minimum occurs at a point where the curve is flat with only a gradual change in slope, there will be a considerable spread in the choice of final conditions, and incremental cost analyses may be necessary. On the other hand, if the maximum or minimum is sharp, it may be essential to operate at the exact optimum conditions.

THE BREAK-EVEN CHART FOR PRODUCTION SCHEDULE AND ITS SIGNIFICANCE FOR OPTIMUM ANALYSIS

In considering the overall costs or profits in a plant operation, one of the factors that has an important effect on the economic results is the fraction of total available time during which the plant is in operation. If the plant stands idle or operates at low capacity, certain costs, such as those for raw materials and labor, are reduced, but costs for depreciation and maintenance continue at essentially the same rate even though the plant is not in full use.

There is a close relationship among operating time, rate of production, and selling price. It is desirable to operate at a schedule which will permit maximum utilization of fixed costs while simultaneously meeting market sales demand and using the capacity of the plant production to give the best economic results. Figure 11-3 shows graphically how production rate affects costs and profits. The fixed costs remain constant while the total product cost, as

![FIGURE 11-3](image-url)

Break-even chart for operating production plant (based on situation presented in Example 2).
well as the profit, increases with increased rate of production. The point where total product cost equals total income represents the break-even point, and the optimum production schedule must be at a production rate higher than that corresponding to the break-even point.

OPTIMUM PRODUCTION RATES
IN PLANT OPERATION

The same principles used for developing an optimum design can be applied when determining the most favorable conditions in the operation of a manufacturing plant. One of the most important variables in any plant operation is the amount of product produced per unit of time. The production rate depends on many factors, such as the number of hours in operation per day, per week, or per month; the load placed on the equipment; and the sales market available. From an analysis of the costs involved under different situations and consideration of other factors affecting the particular plant, it is possible to determine an optimum rate of production or a so-called economic lot size.

When a design engineer submits a complete plant design, the study ordinarily is based on a given production capacity for the plant. After the plant is put into operation, however, some of the original design factors will have changed, and the optimum rate of production may vary considerably from the “designed capacity.” For example, suppose a plant had been designed originally for the batchwise production of an organic chemical on the basis of one batch every 8 hours. After the plant has been put into operation, cost data on the actual process are obtained, and tests with various operating procedures are conducted. It is found that more total production per month can be obtained if the time per batch is reduced. However, when the shorter batch time is used, more labor is required, the percent conversion of raw materials is reduced, and steam and power costs increase. Here is an obvious case in which an economic balance can be used to find the optimum production rate. Although the design engineer may have based the original recommendations on a similar type of economic balance, price and market conditions do not remain constant, and the operations engineer now has actual results on which to base an economic balance. The following analysis indicates the general method for determining economic production rates or lot sizes.

The total product cost per unit of time may be divided into the two classifications of operating costs and organization costs. Operating costs depend on the rate of production and include expenses for direct labor, raw materials, power, heat, supplies and similar items which are a function of the amount of material produced. Organization costs are due to expenses for directive personnel, physical equipment, and other services or facilities which must be maintained irrespective of the amount of material produced. Organization costs are independent of the rate of production.
It is convenient to consider operating costs on the basis of one unit of production. When this is done, the operating costs can be divided into two types of expenses as follows: (1) minimum expenses for raw materials, labor, power, etc., that remain constant and must be paid for each unit of production as long as any amount of material is produced; and (2) extra expenses due to increasing the rate of production. These extra expenses are known as superproduction costs. They become particularly important at high rates of production. Examples of superproduction costs are extra expenses caused by overload on power facilities, additional labor requirements, or decreased efficiency of conversion. Superproduction costs can often be represented as follows:

\[
\text{Superproduction costs per unit of production} = mP^n
\]  

(11)

where \( P \) = rate of production as total units of production per unit of time  
\( m \) = a constant  
\( n \) = a constant

Designating \( h \) as the operating costs which remain constant per unit of production and \( O_c \) as the organization costs per unit of time, the total product cost \( c_T \) per unit of production is

\[
c_T = h + mP^n + \frac{O_c}{P}
\]  

(12)

The following equations for various types of costs or profits are based on Eq. (12):

\[
C_T = c_T P = \left( h + mP^n + \frac{O_c}{P} \right) P
\]  

(13)

\[
r = s - c_T = s - h - mP^n - \frac{O_c}{P}
\]  

(14)

\[
R' = rP = \left( s - h - mP^n - \frac{O_c}{P} \right) P
\]

where \( C_T \) = total product cost per unit of time  
\( r \) = profit per unit of production  
\( R' \) = profit per unit of time  
\( s \) = selling price per unit of production

OPTIMUM PRODUCTION RATE FOR MINIMUM COST PER UNIT OF PRODUCTION

It is often necessary to know the rate of production which will give the least cost on the basis of one unit of material produced. This information shows the
selling price at which the company would be forced to cease operation or else operate at a loss. At this particular optimum rate, a plot of the total product cost per unit of production versus the production rate shows a minimum product cost; therefore, the optimum production rate must occur where \( dc = 0 \). An analytical solution for this case may be obtained from Eq. (12), and the optimum rate \( P_o \) giving the minimum cost per unit of production is found as follows:

\[
\frac{dc}{dP} = 0 = nmP_0^{n-1} - \frac{\alpha_c}{P_0^2}
\]

The optimum rate shown in Eq. (17) would, of course, give the maximum profit per unit of production if the selling price remains constant.

OPTIMUM PRODUCTION RATE FOR MAXIMUM TOTAL PROFIT PER UNIT OF TIME

In most business concerns, the amount of money earned over a given time period is much more important than the amount of money earned for each unit of product sold. Therefore, it is necessary to recognize that the production rate for maximum profit per unit of time may differ considerably from the production rate for minimum cost per unit of production.

Equation (15) presents the basic relationship between costs and profits. A plot of profit per unit of time versus production rate goes through a maximum. Equation (15), therefore, can be used to find an analytical value of the optimum production rate. When the selling price remains constant, the optimum rate giving the maximum profit per unit of time is

\[
P_o = \left( \frac{\alpha_c}{nm} \right)^{1/(n+1)}
\]

The optimum rate shown in Eq. (17) would, of course, give the maximum profit per unit of production if the selling price remains constant.

Example 2 Determination of profits at optimum production rates. A plant produces refrigerators at the rate of \( P \) units per day. The variable costs per refrigerator have been found to be \( $47.73 + 0.1P^{12} \). The total daily fixed charges are $1750, and all other expenses are constant at $7325 per day. If the selling price per refrigerator is $173, determine:

(a) The daily profit at a production schedule giving the minimum cost per refrigerator.
(b) The daily profit at a production schedule giving the maximum daily profit.
(c) The production schedule at the break-even point.
Solution
(a) Total cost per refrigerator = \( c_T = 47.73 + 0.1P^{1.2} + \frac{(1750 + 7325)}{P} \). At a production schedule for minimum cost per refrigerator,
\[
\frac{dc_T}{dP} = 0 = 0.12P_0^{0.2} - \frac{9075}{P^2}
\]
\( P_o = 165 \) units per day for minimum cost per unit.

Daily profit at a production schedule for minimum cost per refrigerator
\[
= \left[ 173 - 47.73 - 0.1(165)^{1.2} - \frac{9075}{165} \right] 165
\]
\[ = $4040 \]

(b) Daily profit is
\[
R' = \left( 173 - 47.73 - 0.1P^{1.2} - \frac{1750 + 7325}{P} \right) P
\]
At a production schedule for maximum profit per day,
\[
\frac{dR'}{dP} = 0 = 125.27 - 0.22P_0^{1.2}.
\]
\( P_o = 198 \) units per day for maximum daily profit.

Daily profit at a production schedule for maximum daily profit
\[
= \left[ 173 - 47.73 - 0.1(198)^{1.2} - \frac{9075}{198} \right] 198
\]
\[ = $4400 \]

(c) Total profit per day =
\[
\left( 173 - \left[ 47.73 + 0.1P^{1.2} + \frac{(1750 + 7325)}{P} \right] \right) P = 0
\]
at the break-even point.
Solving the preceding equation for \( P \),
\[
P \text{ at break-even point} = 88 \text{ units/day}
\]

OPTIMUM CONDITIONS
IN CYCLIC OPERATIONS
Many processes are carried out by the use of cyclic operations which involve periodic shutdowns for discharging, cleanout, or reactivation. This type of operation occurs when the product is produced by a batch process or when the rate of production decreases with time, as in the operation of a plate-and-frame filtration unit. In a true batch operation, no product is obtained until the unit is shut down for discharging. In semicontinuous cyclic operations, product is delivered continuously while the unit is in operation, but the rate of delivery decreases with time. Thus, in batch or semicontinuous cyclic operations, the
variable of total time required per cycle must be considered when determining optimum conditions.

Analyses of cyclic operations can be carried out conveniently by using the time for one cycle as a basis. When this is done, relationships similar to the following can be developed to express overall factors, such as total annual cost or annual rate of production:

\[
\text{Total annual cost} = \frac{\text{operating and shutdown costs/cycle}}{\text{cycles/year}} \times \text{cycles/year} + \text{annual fixed costs} \tag{19}
\]

\[
\text{Annual production} = \left(\frac{\text{production/cycle}}{\text{cycles/year}}\right) \times \text{cycles/year} \tag{20}
\]

\[
\text{Cycles/year} = \frac{\text{operating + shutdown time used/year}}{\text{operating + shutdown time/cycle}} \tag{21}
\]

The following example illustrates the general method for determining optimum conditions in a batch operation.

**Example 3 Determination of conditions for minimum total cost in a batch operation.** An organic chemical is being produced by a batch operation in which no product is obtained until the batch is finished. Each cycle consists of the operating time necessary to complete the reaction plus a total time of 1.4 h for discharging and charging. The operating time per cycle is equal to \(1.5P_b^{0.25}\) h, where \(P_b\) is the kilograms of product produced per batch. The operating costs during the operating period are $20 per hour, and the costs during the discharge-charge period are $15 per hour. The annual fixed costs for the equipment vary with the size of the batch as follows:

\[
C_F = 340P_b^{0.8} \text{ dollars per batch}
\]

Inventory and storage charges may be neglected. If necessary, the plant can be operated 24 h per day for 300 days per year. The annual production is 1 million kg of product. At this capacity, raw-material and miscellaneous costs, other than those already mentioned, amount to $260,000 per year. Determine the cycle time for conditions of minimum total cost per year.

**Solution**

\[
\text{Cycles/year} = \frac{\text{annual production}}{\text{production/cycle}} = \frac{1,000,000}{P_b}
\]

Cycle time = operating + shutdown time = \(1.5P_b^{0.25} + 1.4\) h

Operating + shutdown costs/cycle = \((20)(1.5P_b^{0.25}) + (15)(1.4)\) dollars

Annual fixed costs = \(340P_b^{0.8} + 260,000\) dollars

Total annual costs = \((30P_b^{0.25} + 21)(1,000,000/P_b) + 340P_b^{0.8}

\]

260,000 dollars

The total annual cost is a minimum where \(d(\text{total annual cost})/dP_b = 0\).
Performing the differentiation, setting the result equal to zero, and solving for $P_b$ gives

$$P_{b, \text{ optimum cost}} = 1630 \text{ kg per batch}$$

This same result could have been obtained by plotting total annual cost versus $P_b$ and determining the value of $P_b$ at the point of minimum annual cost. For conditions of minimum annual cost and 1 million kg/year production,

$$\text{Cycle time} = (1.5)(1630)^{0.25} + 1.4 = 11 \text{ h}$$

$$\text{Total time used per year} = (11)\left(\frac{1,000,000}{1630}\right) = 6750 \text{ h}$$

Total time available per year = (300)(24) = 7200 h

Thus, for conditions of minimum annual cost and a production of 1 million kg/year, not all the available operating and shutdown time would be used.

**SEMICONtinuous CYCLiC OPERATIONS**

Semicontinuous cyclic operations are often encountered in the chemical industry, and the design engineer should understand the methods for determining optimum cycle times in this type of operation. Although product is delivered continuously, the rate of delivery decreases with time owing to scaling, collection of side product, reduction in conversion efficiency, or similar causes. It becomes necessary, therefore, to shut down the operation periodically in order to restore the original conditions for high production rates. The optimum cycle time can be determined for conditions such as maximum amount of production per unit of time or minimum cost per unit of production.

**Scale Formation in Evaporation**

During the time an evaporator is in operation, solids often deposit on the heat-transfer surfaces, forming a scale. The continuous formation of the scale causes a gradual increase in the resistance to the flow of heat and, consequently, a reduction in the rate of heat transfer and rate of evaporation if the same temperature-difference driving forces are maintained. Under these conditions, the evaporation unit must be shut down and cleaned after an optimum operation time, and the cycle is then repeated.

Scale formation occurs to some extent in all types of evaporators, but it is of particular importance when the feed mixture contains a dissolved material that has an inverted solubility. The expression *inverted solubility* means the solubility decreases as the temperature of the solution is increased. For a material of this type, the solubility is least near the heat-transfer surface where the temperature is the greatest. Thus, any solid crystallizing out of the solution does so near the heat-transfer surface and is quite likely to form a scale on this surface. The most common scale-forming substances are calcium sulfate, cal-
When true scale formation occurs, the overall heat-transfer coefficient may be related to the time the evaporator has been in operation by the straight-line equation?

\[ \frac{1}{U^2} = a \theta_b + d \]  

(22)

where \( a \) and \( d \) are constants for any given operation and \( U \) is the overall heat-transfer coefficient at any operating time \( \theta_b \) since the beginning of the operation.

If it is not convenient to determine the heat-transfer coefficients and the related constants as shown in Eq. (22), any quantity that is proportional to the heat-transfer coefficient may be used. Thus, if all conditions except scale formation are constant, feed rate, production rate, and evaporation rate can each be represented in a form similar to Eq. (22). Any of these equations can be used as a basis for finding the optimum conditions. The general method is illustrated by the following treatment, which employs Eq. (22) as a basis.

If \( Q \) represents the total amount of heat transferred in the operating time \( \theta_b \), and \( A \) and \( At \) represent, respectively, the heat-transfer area and temperature-difference driving force, the rate of heat transfer at any instant is

\[ \frac{dQ}{d\theta_b} = UA \Delta t = \frac{A At}{(a \theta_b + d)^{1/2}} \]  

(23)

The instantaneous rate of heat transfer varies during the time of operation, but the heat-transfer area and the temperature-difference driving force remain essentially constant. Therefore, the total amount of heat transferred during an operating time of \( \theta_b \) can be determined by integrating Eq. (23) as follows:

\[ \int_0^Q dQ = A \Delta t \int_0^{\theta_b} \left( \frac{1}{a \theta_b + d} \right)^{1/2} d\theta_b \]

(24)

\[ Q = \frac{2A At}{a} \left[ (a \theta_b + d)^{1/2} - d^{1/2} \right] \]

(25)

**CYCLE TIME FOR MAXIMUM AMOUNT OF HEAT TRANSFER.** Equation (25) can be used as a basis for finding the cycle time which will permit the maximum amount of heat transfer during a given period. Each cycle consists of an operating (or boiling) time of \( \theta_b \) h. If the time per cycle for emptying, cleaning, and recharging is \( \theta_c \), the total time in hours per cycle is \( \theta_t = \theta_b + \theta_c \). Therefore, designating the total time used for actual operation, emptying, cleaning, and refilling as \( H \), the number of cycles during \( H \) h = \( H / (\theta_b + \theta_c) \).

The total amount of heat transferred during

\[ H_h = Q_H = (Q/\text{cycle}) \times (\text{cycles}/H_h) \]

Therefore,

\[ Q_H = \frac{2A}{a} \frac{A t}{a} \left[ (a\theta_b + d)^{1/2} - d^{1/2} \right] \frac{H}{\theta_b + \theta_c} \quad (26) \]

Under ordinary conditions, the only variable in Eq. (26) is the operating time \( \theta_b \). A plot of the total amount of heat transferred versus \( \theta_b \) shows a maximum at the optimum value of \( \theta_b \). Figure 11.4 presents a plot of this type. The optimum cycle time can also be obtained by setting the derivative of Eq. (26) with respect to \( \theta_b \) equal to zero and solving for \( \theta_b \). The result is

\[ \theta_b, \text{ per cycle for maximum amount of heat transfer} = \theta_c + \frac{2}{a} \sqrt{a d \theta_c} \quad (27) \]

The optimum boiling time given by Eq. (27) shows the operating schedule necessary to permit the maximum amount of heat transfer. All the time available for operation, emptying, cleaning, and refilling should be used. For

![FIGURE 11.4](image_url)

Determination of optimum operating time for maximum amount of heat transfer in evaporator with scale formation.
constant operating conditions, this same schedule would also give the maximum amount of feed consumed, product obtained, and liquid evaporated.

A third method for determining the optimum cycle time is known as the **tangential method for finding optimum conditions**, and it is applicable to many types of cyclic operations. This method is illustrated for conditions of constant cleaning time ($\theta_c$) in Fig. 11-5, where a plot of amount of heat transferred versus boiling time is presented. Curve $OB$ is based on Eq. (25). The average amount of heat transferred per unit of time during one complete cycle is $Q/\theta_b + \theta_c$. When the total amount of heat transferred during a number of repeated cycles is a maximum, the average amount of heat transferred per unit of time must also be a maximum. The optimum cycle time, therefore, occurs when $Q/\theta_b + \theta_c$ is a maximum.

The straight line $CD'$ in Fig. 11-5 starts at a distance equivalent to $\theta_c$ on the left of the plot origin. The slope of this straight line is $Q/\theta_b + \theta_c$, with the values of $Q$ and $\theta_b$ determined by the point of intersection between line $CD'$ and curve $OB$. The maximum value of $Q/\theta_b + \theta_c$ occurs when line $CD$ is tangent to the curve $OB$, and the point of tangency indicates the optimum value of the boiling time per cycle for conditions of maximum amount of heat transfer.

**FIGURE 11-5**
Tangential method for finding optimum time for maximum amount of heat transfer in evaporator with scale formation.
CYCLE TIME FOR MINIMUM COST ‘PER UNIT OF HEAT TRANSFER’

There are many different circumstances which may affect the minimum cost per unit of heat transferred in an evaporation operation. One simple and commonly occurring case will be considered. It may be assumed that an evaporation unit of fixed capacity is available, and a definite amount of feed and evaporation must be handled each day. The total cost for one cleaning and inventory charge is assumed to be constant no matter how much boiling time is used. The problem is to determine the cycle time which will permit operation at the least total cost.

The total cost includes (1) fixed charges on the equipment and fixed overhead expenses, (2) steam, materials, and storage costs which are proportional to the amount of feed and evaporation, (3) expenses for direct labor during the actual evaporation operation, and (4) cost of cleaning. Since the size of the equipment and the amounts of feed and evaporation are fixed, the costs included in (1) and (2) are independent of the cycle time. The optimum cycle time, therefore, can be found by minimizing the sum of the costs for cleaning and for direct labor during the evaporation.

If $C_c$ represents the cost for one cleaning and $S_b$ is the direct labor cost per hour during operation, the total variable costs during $H$ h of operating and cleaning time must be

$$C_{T,for \, H \, h} = (C_c + S_b \theta_b) \frac{H}{\theta_b + \theta_c}$$

Equations (26) and (28) may be combined to give

$$C_{T,for \, H \, h} = \frac{aQ_H (C_c + S_b \theta_b)}{2A \Delta t \left[ \left( a \theta_b + d \right)^{1/2} - d^{1/2} \right]}$$

The optimum value of $\theta_b$ for minimum total cost may be obtained by plotting $C_T$ versus $\theta_b$ or by setting the derivative of Eq. (29) with respect to $\theta_b$ equal to zero and solving for $\theta_b$. The result is

$$\theta_{b, \text{per cycle for minimum total cost}} = \frac{C_c}{S_b} + \frac{2}{aS_b} \sqrt{a \, dC_c \, S_b}$$

Equation (30) shows that the optimum cycle time is independent of the required amount of heat transfer $Q_H$. Therefore, a check must be made to make certain the optimum cycle time for minimum cost permits the required amount of heat transfer. This can be done easily by using the following equation, which is based on Eq. (26):

$$\theta_t = \theta_b + \theta_c = \frac{2AH' \Delta t}{aQ_H} \left[ (a \theta_{b,\text{opt}} + d)^{1/2} - d^{1/2} \right]$$

where $H'$ is the total time available for operation, emptying, cleaning, and recharging. If $\theta_t$ is equal to or greater than $\theta_{b,\text{opt}} + \theta_c$, the optimum boiling time indicated by Eq. (30) can be used, and the required production can be obtained at conditions of minimum cost.
The optimum cycle time determined by the preceding methods may not fit into convenient operating schedules. Fortunately, as shown in Figs. 11-4 and 11-5, the optimum points usually occur where a considerable variation in the cycle time has little effect on the factor that is being optimized. It is possible, therefore, to adjust the cycle time to fit a convenient operating schedule without causing much change in the final results.

The approach described in the preceding sections can be applied to many different types of semicontinuous cyclic operations. An illustration showing how the same reasoning is used for determining optimum cycle times in filter-press operations is presented in Example 4.

Example 4 Cycle time for maximum amount of production from a plate-and-frame filter press. Tests with a plate-and-frame filter press, operated at constant pressure, have shown that the relation between the volume of filtrate delivered and the time in operation can be represented as follows:

\[ P_f^2 = 2.25 \times 10^4 (\theta_f + 0.11) \]

where \( P_f \) = cubic feet of filtrate delivered in filtering time \( \theta_f \) h.

The cake formed in each cycle must be washed with an amount of water equal to one-sixteenth times the volume of filtrate delivered per cycle. The washing rate remains constant and is equal to one-fourth of the filtrate delivery rate at the end of the filtration. The time required per cycle for dismantling, dumping, and reassembling is 6 h. Under the conditions where the preceding information applies, determine the total cycle time necessary to permit the maximum output of filtrate during each 24 h.

Solution. Let \( \theta_f \) = hours of filtering time per cycle.

Filtrate delivered per cycle = \( P_{f,cycle} = 150(\theta_f + 0.11)^{1/2} \text{ ft}^3 \). Rate of filtrate delivery at end of cycle is

\[ \frac{dP_f}{d\theta_f} = \frac{150}{2} (\theta_f + 0.11)^{-1/2} \text{ ft}^3 / \text{h} \]

Washing rate \( \times 4 = \frac{\text{volume of wash water}}{\text{washing rate}} = \frac{(4)(2)(150)(\theta_f + 0.11)^{1/2}}{(16)(150)(\theta_f + 0.11)^{-1/2}} = \frac{\theta_f + 0.11}{2} \text{ h} \]

Total time per cycle = \( \theta_f + \frac{\theta_f + 0.11}{2} + 6 = 1.5\theta_f + 6.06 \text{ h} \)

Cycles per 24 h = \( \frac{24}{1.5\theta_f + 6.06} \)

Filtrate in \( \text{ft}^3 \) delivered/24 h is

\[ P_{f,cycle}(\text{cycles per 24 h}) = 150(\theta_f + 0.11)^{1/2} \frac{24}{1.5\theta_f + 6.06} \]
At the optimum cycle time giving the maximum output of filtrate per 24 h,
\[
\frac{d}{d\theta_f} \left( \frac{\text{ft}^3 \text{ filtrate delivered}}{24 \text{ h}} \right) = 0
\]
Performing the differentiation and solving for \( \theta_f \),
\[
\theta_{f, \text{opt}} = 3.8 \text{ h}
\]
Total cycle time necessary to permit the maximum output of filtrate = \((1.5)(3.8) + 6.06 = 11.8 \text{ h.}\)

**ACCURACY AND SENSITIVITY OF RESULTS**

The purpose of the discussion and examples presented in the preceding sections of this chapter has been to give a basis for understanding the significance of optimum conditions plus simplified examples to illustrate the general concepts. Costs due to taxes, time value of money, capital, efficiency or inefficiency of operation, and special maintenance are examples of factors that have not been emphasized in the preceding. Such factors may have a sufficiently important influence on an optimum condition that they need to be taken into account for a final analysis. The engineer must have the practical understanding to recognize when such factors are important and when the added accuracy obtained by including them is not worth the difficulty they cause in the analysis.

A classic example showing how added refinements can come into an analysis for optimum conditions is involved in the development of methods for determining optimum economic pipe diameter for transportation of fluids. The following analysis, dealing with economic pipe diameters, gives a detailed derivation to illustrate how simplified expressions for optimum conditions can be developed. Further discussion showing the effects of other variables on the sensitivity is also presented.

**FLUID DYNAMICS (OPTIMUM ECONOMIC PIPE DIAMETER)**

The investment for piping and pipe fittings can amount to an important part of the total investment for a chemical plant. It is necessary, therefore, to choose pipe sizes which give close to a minimum total cost for pumping and fixed charges. For any given set of flow conditions, the use of an increased pipe diameter will cause an increase in the fixed charges for the piping system and a decrease in the pumping or blowing charges. Therefore, an optimum economic pipe diameter must exist. The value of this optimum diameter can be determined by combining the principles of fluid dynamics with cost considerations. The optimum economic pipe diameter is found at the point at which the sum of pumping or blowing costs and fixed charges based on the cost of the piping system is a minimum.
Pumping or Blowing Costs

For any given operating conditions involving the flow of a noncompressible fluid through a pipe of constant diameter, the total mechanical-energy balance can be reduced to the following form:

\[
\text{Work}' = \frac{2fV^2L(1 + J)}{g_c D} + B
\]  

(32)

where Work' = mechanical work added to system from an external mechanical source, \( \text{ft-lbf/lb} \)

\( f = \) Fanning friction factor, dimensionless?

\( V = \) average linear velocity of fluid, \( \text{ft/s} \)

\( L = \) length of pipe, \( \text{ft} \)

\( J = \) frictional loss due to fittings and bends, expressed as equivalent fractional loss in a straight pipe

\( g_c = \) conversion factor in Newton’s law of motion, 32.17 \( \text{ft-lbf/s}(\text{lbm})(\text{s}) \)

\( D = \) inside diameter of pipe, \( \text{ft} \), subscript \( i \) means in.

\( B = \) a constant taking all other factors of the mechanical-energy balance into consideration

In the region of turbulent flow (Reynolds number greater than 2100), \( f \) may be approximated for new steel pipes by the following equation:

\[
\frac{0.04}{(N_{Re})^{0.16}}
\]

(33)

where \( N_{Re} \) is the Reynolds number or \( DV\rho/\mu \).

If the flow is viscous (Reynolds number less than 2100),

\[
\frac{16}{N_{Re}}
\]

(34)

By combining Eqs. (32) and (33) and applying the necessary conversion factors, the following equation can be obtained representing the annual pumping cost when the flow is turbulent:

\[
C_{\text{pumping}} = \frac{0.273q_f^{2.84}p^{0.84}g_c^{0.16}K(1 + J)H_y + B'}{D_i^{4.84}E}
\]

(35)

where \( C_{\text{pumping}} = \) pumping costs as dollars per year per foot of pipe length when flow is turbulent

\( q_f = \) fluid-flow rate, \( \text{ft}^3/\text{s} \)

\( \rho = \) fluid density, \( \text{lb}/\text{ft}^3 \)

†Based on Fanning equation written as \( \Sigma(\text{friction}) = 2fV^2L/g_c D \).
\( \mu_c = \) fluid viscosity, centipoises  
\( K = \) cost of electrical energy, \$/kWh  
\( H_y = \) hours of operation per year  
\( E = \) efficiency of motor and pump expressed as a fraction  
\( B' = \) a constant independent of \( D_i \)

Similarly, Eqs. (32) and (34) and the necessary conversion factors can be combined to give the annual pumping costs when the flow is viscous:

\[
C'_{\text{pumping}} = \frac{0.024q_f^2 \mu_c K (1 + J) H_y}{D_i^4 E} + B'
\]  

where \( C'_{\text{pumping}} = \) pumping cost as dollars per year per foot of pipe length when flow is viscous.

Equations (35) and (36) apply to noncompressible fluids. In engineering calculations, these equations are also generally accepted for gases if the total pressure drop is less than 10 percent of the initial pressure.

**Fixed Charges for Piping System**

For most types of pipe, a plot of the logarithm of the pipe diameter versus the logarithm of the purchase cost per foot of pipe is essentially a straight line. Therefore, the purchase cost for pipe may be represented by the following equation:

\[
c_{\text{pipe}} = XD_i^n
\]

where \( c_{\text{pipe}} = \) purchase cost of new pipe per foot of pipe length, \$/ft  
\( X = \) purchase cost of new pipe per foot of pipe length if pipe diameter is 1 in., \$/ft  
\( n = \) a constant with value dependent on type of pipe

The annual cost for the installed piping system may be expressed as follows:

\[
C_{\text{pipe}} = (1 + F) XD_i^n K_F
\]

where \( C_{\text{pipe}} = \) cost for installed piping system as dollars per year per foot of pipe length?  
\( F = \) ratio of total costs for fittings and installation to purchase cost for new pipe  
\( K_F = \) annual fixed charges including maintenance, expressed as a fraction of initial cost for completely installed pipe

\("Pump\) cost could be included if desired; however, in this analysis, the cost of the pump is considered as essentially invariant with pipe diameter.
Optimum Economic Pipe Diameter

The total annual cost for the piping system and pumping can be obtained by adding Eqs. (35) and (38) or Eqs. (36) and (38). The only variable in the resulting total-cost expressions is the pipe diameter. The optimum economic pipe diameter can be found by taking the derivative of the total annual cost with respect to pipe diameter, setting the result equal to zero, and solving for \( D_i \). This procedure gives the following results:

For turbulent flow,

\[
D_{i, \text{opt}} = \frac{1.32 q_f^{2.84} \rho^{0.84} \mu_c^{0.16} K (1 + J) H_y^{1/(4.84+n)}}{n (1 + F) XE K_F} \tag{39}
\]

For viscous flow,

\[
D_{i, \text{opt}} = \frac{0.096 q_f^2 \mu_c K (1 + J) H_y^{1/(4.0+n)}}{n (1 + F) XE K_F} \tag{40}
\]

The value of \( n \) for steel pipes is approximately 1.5 if the pipe diameter is 1 in. or larger and 1.0 if the diameter is less than 1 in. Substituting these values in Eqs. (39) and (40) gives:

For turbulent flow in steel pipes,

\[
\begin{align*}
D_i \geq 1 \text{ in.} & : \\
D_{i, \text{opt}} &= q_f^{0.448} \rho^{0.132} \mu_c^{0.025} \left[ \frac{0.88 K (1 + J) H_y^{0.158}}{(1 + F) XE K_F} \right]
\end{align*}
\tag{41}
\]

\[
D_i < 1 \text{ in.} : \\
D_{i, \text{opt}} &= q_f^{0.487} \rho^{0.144} \mu_c^{0.027} \left[ \frac{1.32 K (1 + J) H_y^{0.171}}{(1 + F) XE K_F} \right]
\tag{42}
\]

For viscous flow in steel pipes,

\[
\begin{align*}
D_i \geq 1 \text{ in.} & : \\
D_{i, \text{opt}} &= q_f^{0.364} \mu_c^{0.182} \left[ \frac{0.064 K (1 + J) H_y^{0.182}}{(1 + F) XE K_F} \right]
\end{align*}
\tag{43}
\]

\[
D_i < 1 \text{ in.} : \\
D_{i, \text{opt}} &= q_f^{0.40} \mu_c^{0.20} \left[ \frac{0.096 K (1 + J) H_y^{0.20}}{(1 + F) XE K_F} \right]
\tag{44}
\]

The exponents involved in Eqs. (41) through (44) indicate that the optimum diameter is relatively insensitive to most of the terms involved. Since the exponent of the viscosity term in Eqs. (41) and (42) is very small, the value of \( \mu_c^{0.025} \) and \( \mu_c^{0.027} \) may be taken as unity over a viscosity range of 0.02 to 20 centipoises. It is possible to simplify the equations further by substituting
average numerical values for some of the less critical terms. The following values are applicable under ordinary industrial conditions:

\[ \begin{align*}
K &= \$0.09/kWh \\
J &= 0.35 \text{ or } 35 \text{ percent} \\
H_y &= 8760 \text{ h/year} \\
E &= 0.50 \text{ or } 50 \text{ percent} \\
F &= 1.4 \\
K_F &= 0.20 \text{ or } 20 \text{ percent} \\
X &= \$0.74 \text{ per foot for } 1\text{-in.-diameter steel pipe}
\end{align*} \]

Substituting these values into Eqs. (41) through (44) gives the following simplified results:

For turbulent flow in steel pipes, 

\[ D_i \geq 1 \text{ in.:} \]

\[
D_{i, \text{opt}} = 3.9q_f^{0.45} \rho^{0.13} \frac{2w^{0.45}}{\rho^{0.32}}
\]  

(45)

where \( w_m \) = thousands of pounds mass flowing per hour.

\[ D_i < 1 \text{ in.:} \]

\[
D_{i, \text{opt}} = 4.7q_f^{0.49} \rho^{0.14}
\]  

(46)

For viscous flow in steel pipes, 

\[ D_i \geq 1 \text{ in.:} \]

\[
D_{i, \text{opt}} = 3.0q_f^{0.36} \mu_c^{0.18}
\]  

(47)

\[ D_i < 1 \text{ in.:} \]

\[
D_{i, \text{opt}} = 3.6q_f^{0.40} \mu_c^{0.20}
\]  

(48)

Depending on the accuracy desired and the type of flow, Eqs. (39) through (48) may be used to estimate optimum economic pipe diameters. The simplified Eqs. (45) through (48) are sufficiently accurate for design estimates under ordinary plant conditions, and, as shown in Table 2, the diameter estimates obtained are usually on the safe side in that added refinements in the calculation methods generally tend to result in smaller diameters. A nomograph based on these equations is presented in Chap. 14 (Materials Transfer, Handling, and Treatment Equipment-Design and Costs).

ANALYSIS INCLUDING TAX EFFECTS AND COST OF CAPITAL

The preceding analysis clearly neglects a number of factors that may have an influence on the optimum economic pipe diameter, such as cost of capital or return on investment, cost of pumping equipment, taxes, and the time value of money. If the preceding development of Eq. (39) for turbulent flow is refined to include the effects of taxes and the cost of capital (or return on investment) plus a more accurate expression for the frictional loss due to fittings and bends, the result is:†

For turbulent flow,

\[
\frac{D_{\text{opt}}^{4.84+n}}{I + 0.794L_{e}D_{\text{opt}}} = \frac{0.000189YKW_{s}^{-0.16}X'[1 + (a' + b') M] (1 - \phi) + ZM}{n'(1 + F)X'[Z + (a + b)(1 - \phi)]E\rho^2} \quad (49)
\]

where \(D_{\text{opt}}\) = optimum economic inside diameter, ft
\(X'\) = purchase cost of new pipe per foot of pipe length if pipe diameter is 1 ft, based on \(c_{\text{pipe}} = X'D_{\text{opt}}\), $/ft
\(L_{e}'\) = frictional loss due to fittings and bends, expressed as equivalent pipe length in pipe diameters per unit length of pipe, \(L_{e}' = J/D_{\text{opt}}\)
\(W_{s}\) = pounds mass flowing per second
\(M\) = ratio of total cost for pumping installation to yearly cost of pumping power required
\(Y\) = days of operation per year
\(a\) = fraction of initial cost of installed piping system for annual depreciation
\(a'\) = fraction of initial cost of pumping installation for annual depreciation
\(b\) = fraction of initial cost of installed piping system for annual maintenance
\(b'\) = fraction of initial cost of pumping installation for annual maintenance
\(\phi\) = fractional factor for rate of taxation
\(Z\) = fractional rate of return (or cost of capital before taxes) on incremental investment

TABLE 1

Values of variables used to obtain Eq. (50)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value used</th>
<th>Variable</th>
<th>Value used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>231.5</td>
<td>$\phi$</td>
<td>1.3012</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>328</td>
<td>$X$</td>
<td>31.0</td>
</tr>
<tr>
<td>$K$</td>
<td>0.09</td>
<td>$a + b$</td>
<td>0.18</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>1.0</td>
<td>$a' + b'$</td>
<td>0.40</td>
</tr>
<tr>
<td>$M$</td>
<td>0.8</td>
<td>$E$</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The variable $F$ is a function of diameter and can be approximated by $F \equiv 0.75/D_{\text{opt}} + 3$.

By using the values given in Table 1, for turbulent flow in steel pipes, Eq. (49) simplifies to:

$$D_i \geq 1 \text{ in.}$$

$$D_{\text{opt}} = \frac{(1 + 1.865D_{\text{opt}})^{0.158}(0.32)w_s^{0.45}}{(1 + F)^{0.158} \rho^{0.32}}$$  \hspace{1cm} (50)

SENSITIVITY OF RESULTS

The simplifications made in obtaining Eqs. (45) to (48) and Eq. (50) illustrate an approach that can be used for approximate results when certain variables appear in a form where relatively large changes in them have little effect on the final results. The variables appearing in Table 1 and following Eq. (44) are relatively independent of pipe diameter, and they are raised to a small power for the final determination of diameter. Thus, the final results are not particu-

TABLE 2

Comparison of optimum economic pipe diameter estimated from Eqs. (50) and (45)

<table>
<thead>
<tr>
<th>$D_{i,\text{opt}} \text{ in.}$</th>
<th>$w_s \text{, lb/s}$</th>
<th>$\rho \text{, lb/ft}^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>By Eq. (50)</td>
<td>By Eq. (45)</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>11.2</td>
<td>4.50</td>
</tr>
<tr>
<td>5.0</td>
<td>6.1</td>
<td>4.5</td>
</tr>
<tr>
<td>3.0</td>
<td>3.9</td>
<td>12.5</td>
</tr>
<tr>
<td>1.5</td>
<td>2.2</td>
<td>0.45</td>
</tr>
</tbody>
</table>
larly sensitive to the variables listed in Table 1, and the practical engineer may decide that the simplification obtained by using the approximate equations is worth the slight loss in absolute accuracy.

Table 2 shows the extent of change in optimum economic diameter obtained by using Eq. (50) versus Eq. (45) and illustrates the effect of bringing in added refinements as well as changes in values of some of the variables.

**HEAT TRANSFER (OPTIMUM FLOW RATE OF COOLING WATER IN CONDENSER)**

If a condenser, with water as the cooling medium, is designed to carry out a given duty, the cooling water may be circulated at a high rate with a small change in water temperature or at a low rate with a large change in water temperature. The temperature of the water affects the temperature-difference driving force for heat transfer. Use of an increased amount of water, therefore, will cause a reduction in the necessary amount of heat-transfer area and a resultant decrease in the original investment and fixed charges. On the other hand, the cost for the water will increase if more water is used. An economic balance between conditions of high water rate-low surface area and low water rate-high surface area indicates that the optimum flow rate of cooling water occurs at the point of minimum total cost for cooling water and equipment fixed charges.

Consider the general case in which heat must be removed from a condensing vapor at a given rate designated by $q$ Btu/h. The vapor condenses at a constant temperature of $t_1^\circ F$, and cooling water is supplied at a temperature of $t_2^\circ F$. The following additional notation applies:

- $w =$ flow rate of cooling water, lb/h
- $c_p =$ heat capacity of cooling water, Btu/(lb)$^\circ F$
- $t_2 =$ temperature of cooling water leaving condenser, $^\circ F$
- $U =$ constant overall coefficient of heat transfer determined at optimum conditions, Btu/(h)$^2$ft$^2$$^\circ F$
- $A =$ area of heat transfer, ft$^2$

$A t_{\text{cond}} =$ log-mean temperature-difference driving force over condenser, $^\circ F$
- $H_y =$ hours the condenser is operated per year, h/year
- $C_w =$ cooling-water cost assumed as directly proportional to amount of water supplied, $\?$/lb
- $C_A =$ installed cost of heat exchanger per square foot of heat-transfer area, $\?$/ft$^2$
- $K_F =$ annual fixed charges including maintenance, expressed as a fraction of initial cost for completely installed equipment

†Cooling water is assumed to be available at a pressure sufficient to handle any pressure drop in the condenser; therefore, any cost due to pumping the water is included in $C_c$. 
The rate of heat transfer as Btu per hour can be expressed as

$$q = w c_p (t_2 - t_1) = UA A t_{\text{in}} = \frac{UA(t_2 - t_1)}{\ln[(t' - t_1)/(t' - t_2)]}$$  \hspace{1cm} (51)

Solving for $w$,

$$w = \frac{4}{c_p(t_2 - t_1)}$$  \hspace{1cm} (52)

The design conditions set the values of $q$ and $t_{\text{in}}$, and the heat capacity of water may ordinarily be approximated as 1 Btu/(lb°F). Therefore, Eq. (52) shows that the flow rate of the cooling water is fixed if the temperature of the water leaving the condenser ($t_2$) is fixed. Under these conditions, the optimum flow rate of cooling water can be found directly from the optimum value of $t_2$.

The annual cost for cooling water is $wH_y C_w$. From Eq. (52),

$$wH_y C_w = \frac{qH_y C_w}{c_p(t_2 - t_1)}$$  \hspace{1cm} (53)

The annual fixed charges for the condenser are $AK_F C_A$, and the total annual cost for cooling water plus fixed charges is

$$\text{Total annual variable cost} = \frac{qH_y C_w}{c_p(t_2 - t_1)} + AK_F C_A$$  \hspace{1cm} (54)

Substituting for $A$ from Eq. (51),

$$\text{Total annual variable cost} = \frac{qH_y C_w}{c_p(t_2 - t_1)} + \frac{qK_F C_A}{U(t_2 - t_1)} \ln\left([t' - t_1]/(t' - t_2)\right)$$  \hspace{1cm} (55)

The only variable in Eq. (55) is the temperature of the cooling water leaving the condenser. The optimum cooling-water rate occurs when the total annual cost is a minimum. Thus, the corresponding optimum exit temperature can be found by differentiating Eq. (55) with respect to $t_2$ (or, more simply, with respect to $t' - t_2$) and setting the result equal to zero. When this is done, the following equation is obtained:

$$\frac{t' - t_1}{t' - t_{2,\text{opt}}} - 1 + \frac{n}{t' - t_1} = \frac{UH_y C_w}{K_F c_p C_A}$$  \hspace{1cm} (56)

The optimum value of $t_2$ can be found from Eq. (56) by a trial-and-error solution, and Eq. (52) can then be used to determine the optimum flow rate of cooling water. The trial-and-error solution can be eliminated by use of Fig. 11-6, which is a plot of Eq. (56).†

†See Fig. 15-32 for a similar plot for crossflow coolers.
Example 5 Optimum cooling-water flow rate in condenser. A condenser for a distillation unit must be designed to condense 5000 lb (2268 kg) of vapor per hour. The effective condensation temperature for the vapor is 170°F (350 K). The heat of condensation for the vapor is 200 Btu/lb (4.65 x 10^5 J/kg). Cooling water is available at 70°F (294 K). The cost of the cooling water is $0.097 per 1000 gal ($25.60 per 1000 m^3). The overall heat-transfer coefficient at the optimum conditions may be taken as 50 Btu/(h)(ft^2)(°F) (284 J/m^2 . s . K). The cost for the installed heat exchanger is $34 per square foot of heat-transfer area ($366 per square meter of heat-transfer area) and annual fixed charges including maintenance are 20 percent of the initial investment. The heat capacity of the water may be assumed to be constant at 1.0 Btu/(lb)(°F) (4.2 kJ/kg . K). If the condenser is to operate 6000 h/yr, determine the cooling-water flow rate in pounds per hour and in kilograms per hour for optimum economic conditions.

Solution

\[ U = 50 \text{ Btu/}(\text{h})(\text{ft}^2)(\text{°F}) \]
\[ H_y = 6000 \text{ h/year} \]
\[ K_F = 0.20 \]
\[ c_p = 1.0 \text{ Btu/}(\text{lb})(\text{°F}) \]
\[ C_A = $34/\text{ft}^2 \]
\[ C_w = \frac{0.097}{(1000)(8.33)} = $0.0000116/\text{lb} \]
\[ \frac{UH_y C_w}{K_F c_p C_A} = \frac{(50)(6000)(0.0000116)}{(0.20)(1.0)(34)} = 0.512 \]
The optimum exit temperature may be obtained by a trial-and-error solution of Eq. (56) or by use of Fig. 11-6. From Fig. 11-6, when the abscissa is 0.512,

\[
\frac{t' - t_{2,\text{opt}}}{t' - t_1} = 0.42
\]

where \( t' = 170^\circ\text{F} \)
\( t_1 = 70^\circ\text{F} \)
\( t_{2,\text{opt}} = 128^\circ\text{F} \)

By Eq. (52), at the optimum economic conditions,

\[
w = \frac{q}{c_p(t_2 - t_1)} = \frac{(5000)(200)}{(1.0)(128 - 70)} = 17,200 \text{ lb water/h (7800 kg water/h)}
\]

**MASS TRANSFER (OPTIMUM REFLUX RATIO)**

The design of a distillation unit is ordinarily based on specifications giving the degree of separation required for a feed supplied to the unit at a known composition, temperature, and flow rate. The design engineer must determine the size of column and reflux ratio necessary to meet the specifications. As the reflux ratio is increased, the number of theoretical stages required for the given separation decreases. An increase in reflux ratio, therefore, may result in lower fixed charges for the distillation column and greater costs for the reboiler heat supply and condenser coolant.

![FIGURE 11-7](image)

Optimum reflux ratio in distillation operation.
As indicated in Fig. 11-7, the optimum reflux ratio occurs at the point where the sum of fixed charges and operating costs is a minimum. As a rough approximation, the optimum reflux ratio usually falls in the range of 1.1 to 1.3 times the minimum reflux ratio. The following example illustrates the general method for determining the optimum reflux ratio in distillation operations.

**Example 6 Determination of optimum reflux ratio.** A sieve-plate distillation column is being designed to handle 700 lb mol (318 kg mol) of feed per hour. The unit is to operate continuously at a total pressure of 1 atm. The feed contains 45 mol% benzene and 55 mol% toluene, and the feed enters at its boiling temperature. The overhead product from the distillation tower must contain 92 mol% benzene, and the bottoms must contain 95 mol% toluene. Determine the following:

(a) The optimum reflux ratio as moles liquid returned to tower per mole of distillate product withdrawn.

(b) The ratio of the optimum reflux ratio to the minimum reflux ratio.

(c) The percent of the total variable cost due to steam consumption at the optimum conditions.

The following data apply:

Vapor-liquid equilibrium data for benzene-toluene mixtures at atmospheric pressure are presented in Fig. 11-8.

The **molal** heat capacity for liquid mixtures of benzene and toluene in all proportions may be assumed to be 40 Btu/(lb mol)(°F) (1.67 x 10^5 J/kg mol °K).

The molal heat of vaporization of benzene and toluene may be taken as 13,700 Btu/lb mol (3.19 x 10^7 J/kg mol).

Effects of change in temperature on heat capacity and heats of vaporization are negligible. Heat losses from the column are negligible. Effects of pressure drop over the column may be neglected.

The overall coefficient of heat transfer is 80 Btu/(h)(ft^2)(°F) (454 J/m^2 . s °K) in the reboiler and 100 Btu/(h)(ft^2)(°F) (568 J/m^2 . s °K) in the condenser.

![Equilibrium diagram for benzene-toluene mixtures at total pressure of 760 mm Hg (McCabe-Thiele method for determining number of theoretical plates).](image-url)
The boiling temperature is 201°F (367 K) for the feed, 179°F (367 K) for the distillate, and 227°F (381 K) for the bottoms. The temperature-difference driving force in the reflux condenser may be based on an average cooling-water temperature of 90°F (305 K), and the change in cooling-water temperature is 50°F (27.8 K) for all cases. Saturated steam at 60 psi (413.6 kPa) is used in the reboiler. At this pressure, the temperature of the condensing steam is 292.7°F (418 K) and the heat of condensation is 915.5 Btu/lb (2.13 X 10^6 J/kg). No heat-savings devices are used.

The column diameter is to be based on a maximum allowable vapor velocity of 2.5 ft/s (0.76 m/s) at the top of the column. The overall plate efficiency may be assumed to be 70 percent. The unit is to operate 8500 h per year.

Cost data.

Steam = $1.50/1000 lb ($3.31/1000 kg).
Cooling water = $0.090/1000 gal or $0.108/10,000 lb ($0.238/10,000 kg).

The sum of costs for piping, insulation, and instrumentation can be estimated to be 60 percent of the cost for the installed equipment. Annual fixed charges amount to 15 percent of the total cost for installed equipment, piping, instrumentation, and insulation.

The following costs are for the installed equipment and include delivery and erection costs:

<table>
<thead>
<tr>
<th>Sieve-plate distillation column</th>
<th>Values may be interpolated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td></td>
</tr>
<tr>
<td>in.</td>
<td>(m)</td>
</tr>
<tr>
<td>60</td>
<td>(1.52)</td>
</tr>
<tr>
<td>70</td>
<td>(1.78)</td>
</tr>
<tr>
<td>80</td>
<td>(2.03)</td>
</tr>
<tr>
<td>90</td>
<td>(2.29)</td>
</tr>
<tr>
<td>100</td>
<td>(2.54)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condenser-tube-and-shell heat exchanger</th>
<th>Values may be interpolated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat-transfer area</td>
<td></td>
</tr>
<tr>
<td>ft^2</td>
<td>(m^2)</td>
</tr>
<tr>
<td>800</td>
<td>(74.3)</td>
</tr>
<tr>
<td>1000</td>
<td>(92.9)</td>
</tr>
<tr>
<td>1200</td>
<td>(111.5)</td>
</tr>
<tr>
<td>1400</td>
<td>(130.1)</td>
</tr>
<tr>
<td>1600</td>
<td>(148.6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reboiler-tube-and-shell heat exchanger</th>
<th>Values may be interpolated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat-transfer area</td>
<td></td>
</tr>
<tr>
<td>ft^2</td>
<td>(m^2)</td>
</tr>
<tr>
<td>1000</td>
<td>(92.9)</td>
</tr>
<tr>
<td>1400</td>
<td>(130.1)</td>
</tr>
<tr>
<td>1800</td>
<td>(167.2)</td>
</tr>
<tr>
<td>2200</td>
<td>(204.4)</td>
</tr>
<tr>
<td>2600</td>
<td>(241.5)</td>
</tr>
</tbody>
</table>
Solution. The variable costs involved are cost of column, cost of reboiler, cost of condenser, cost of steam, and cost of cooling water. Each of these costs is a function of the reflux ratio, and the optimum reflux ratio occurs at the point where the sum of the annual variable costs is a minimum. The total variable cost will be determined at various reflux ratios, and the optimum reflux ratio will be found by the graphical method.

Sample calculation for reflux ratio = 1.5:

**Annual cost for distillation column.** The McCabe-Thiele simplifying assumptions apply for this case, and the number of theoretical plates can be determined by the standard graphical method shown in Fig. 11-8. The slope of the enriching operating line is \( 1.5 / (1.5 + 1) = 0.6 \). From Fig. 11-8, the total number of theoretical stages required for the given separation is 12.1.

The actual number of plates = \((12.1 - 1)/0.70 = 16\).

The moles of distillate per hour \( (M_D) \) and the moles of bottoms per hour \( (M_B) \) may be determined by a benzene material balance as follows:

\[
(700)(0.45) = (M_D)(0.92) + (700 - M_D)(0.05)
\]

\[
M_D = 322 \text{ moles distillate/h}
\]

\[
M_B = 700 - 322 = 378 \text{ moles bottoms/h}
\]

Moles vapor per hour at top of column = \( 322(1 + 1.5) = 805 \). Applying the perfect-gas law,

Vapor velocity at top of tower = 2.5 ft/s

\[
= \frac{(805)(359)(460 + 179)(4)}{(3600)(492)(\pi)(\text{diameter})^2}
\]

Diameter = 7.3 ft

Cost per plate for plate and vessel = $4290

Annual cost for distillation column = \( (4290)(16)(1 + 0.60)(0.15) \)

= $16,470

**Annual cost for condenser.** Rate of heat transfer per hour in condenser =

\( \text{moles vapor condensed per hour} \times \text{molar latent heat of condensation} \) = \( (805)(13,700) = 11,000,000 \) Btu/h.

From the basic heat-transfer-rate equation \( q = UA \ At \),

\[
A = \text{heat-transfer area} = \frac{(11,000,000)}{(100)(179 - 90)} = 1240 \text{ sq ft}
\]

Cost per square foot = \( \frac{25,650}{1240} \)

Annual cost for condenser = \( \frac{25,650}{1240}(1240)(1 + 0.60)(0.15) \)

= $6150

**Annual cost for reboiler.** The rate of heat transfer in the reboiler \( (q_r) \) can be determined by a total energy balance around the distillation unit.
Optimum design and design strategy

Base energy level on liquid at 179°F.

Heat input = heat output

\[ q_r + (700)(201 - 179)(40) = 11,000,000 + (378)(227 - 179)(40) \]

\[ q_r = 11,110,000 \text{ Btu/h} \]

At

\[ A = \text{heat-transfer area} = \frac{11,110,000}{(80)(292.7 - 227)} = 2120 \text{ ft}^2 \]

Cost per square foot = \[ \frac{54,300}{2120} \]

Annual cost for reboiler = \[ \frac{54,300}{2120}(2120)(1 + 0.60)(0.15) \]

\[ = 13,020 \text{ dollars} \]

Annual cost for cooling water. The rate of heat transfer in the condenser = 11,000,000 Btu/h. The heat capacity of water may be taken as 1.0 Btu/(lb°F).

Annual cost for cooling water = \[ \frac{(11,000,000)(0.108)(8500)}{(1.0)(50)(10,000)} \]

\[ = 20,220 \text{ dollars} \]

Annual cost for steam. The rate of heat transfer in the reboiler = 11,110,000 Btu/h.

Annual cost for steam = \[ \frac{(11,110,000)(1.50)(8500)}{(915.5)(1000)} \]

\[ = 155,100 \text{ dollars} \]

Total annual variable cost at reflux ratio of 1.5

\[ \$16,470 + \$6150 + \$13,020 + \$20,220 + \$155,100 = \$210,960 \]

By repeating the preceding calculations for different reflux ratios, the following table can be prepared:

<table>
<thead>
<tr>
<th>Reflux ratio</th>
<th>Number of actual plates required</th>
<th>Column diameter, ft</th>
<th>Annual cost, dollars, for Column</th>
<th>Annual cost, dollars, for Condenser</th>
<th>Annual cost, dollars, for Reboiler</th>
<th>Annual cost, dollars, for Cooling water</th>
<th>Annual cost, dollars, for Steam</th>
<th>Total annual cost, dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.14</td>
<td>∞</td>
<td>6.7</td>
<td>∞</td>
<td>5610</td>
<td>11,880</td>
<td>17,340</td>
<td>132,900</td>
<td>∞</td>
</tr>
<tr>
<td>1.2</td>
<td>29</td>
<td>6.8</td>
<td>26,790</td>
<td>5730</td>
<td>12,120</td>
<td>18,600</td>
<td>142,500</td>
<td>198,960</td>
</tr>
<tr>
<td>1.3</td>
<td>21</td>
<td>7.0</td>
<td>19,860</td>
<td>58.50</td>
<td>12,390</td>
<td>18,600</td>
<td>142,500</td>
<td>199,200</td>
</tr>
<tr>
<td>1.4</td>
<td>18</td>
<td>7.1</td>
<td>17,760</td>
<td>60.00</td>
<td>12,720</td>
<td>19,410</td>
<td>148,800</td>
<td>204,690</td>
</tr>
<tr>
<td>1.5</td>
<td>16</td>
<td>7.3</td>
<td>16,470</td>
<td>61.50</td>
<td>13,020</td>
<td>20,220</td>
<td>155,100</td>
<td>210,960</td>
</tr>
<tr>
<td>1.7</td>
<td>14</td>
<td>7.7</td>
<td>15,870</td>
<td>64.50</td>
<td>13,620</td>
<td>21,870</td>
<td>167,100</td>
<td>224,910</td>
</tr>
<tr>
<td>2.0</td>
<td>13</td>
<td>8.0</td>
<td>15,630</td>
<td>68.40</td>
<td>14,400</td>
<td>24,300</td>
<td>185,400</td>
<td>246,570</td>
</tr>
</tbody>
</table>
(a) The data presented in the preceding table are plotted in Fig. 11-7. The minimum total cost per year occurs at a reflux ratio of 1.25.

Optimum reflux ratio = 1.25

(b) For conditions of minimum reflux ratio, the slope of the enriching line in Fig. 11-8 is 0.532

\[
\frac{\text{Minimum reflux ratio}}{\text{Minimum reflux ratio} + 1} = 0.532
\]

Optimum reflux ratio = 1.25

Minimum reflux ratio = \frac{1.25}{1.14} = 1.1

(c) At the optimum conditions,

Annual steam cost = $139,500

Total annual variable cost = $198,000

Percent of variable cost due to steam consumption = \frac{139,500}{198,000} (100) = 70%

THE STRATEGY OF LINEARIZATION FOR OPTIMIZATION ANALYSIS

In the preceding analyses for optimum conditions, the general strategy has been to establish a partial derivative of the dependent variable from which the absolute optimum conditions are determined. This procedure assumes that an absolute maximum or minimum occurs within attainable operating limits and is restricted to relatively simple conditions in which limiting constraints are not exceeded. However, practical industrial problems often involve establishing the best possible program to satisfy existing conditions under circumstances where the optimum may be at a boundary or limiting condition rather than at a true maximum or minimum point. A typical example is that of a manufacturer who must determine how to blend various raw materials into a final mix that will meet basic specifications while simultaneously giving maximum profit or least cost. In this case, the basic limitations or constraints are available raw materials, product specifications, and production schedule, while the overall objective (or objective function) is to maximize profit.

LINEAR PROGRAMMING FOR OBTAINING OPTIMUM CONDITIONS

One strategy for simplifying the approach to a programming problem is based on expressing the constraints and the objective in a linear mathematical form. The “straight-line” or linear expressions are stated mathematically as

\[
a x_1 + b x_2 + \cdots + j x_j + \cdots + n x_n = z
\]

(57)

where the coefficients a, \cdots, n and z are known values and x_1, \cdots x_n are unknown variables. With two variables, the result is a straight line on a two-dimensional plot, while a plane in a three-dimensional plot results for the case of three variables. Similarly, for more than three variables, the geometric result is a hyperplane.
The general procedure mentioned in the preceding paragraph is designated as *linear programming*. It is a mathematical technique for determining optimum conditions for allocation of resources and operating capabilities to attain a definite objective. It is also useful for analysis of alternative uses of resources or alternative objectives.

**EXAMPLE OF APPROACH IN LINEAR PROGRAMMING**

As an example to illustrate the basic methods involved in linear programming for determining optimum conditions, consider the following simplified problem. A brewery has received an order for 100 gal of beer with the special constraints that the beer must contain 4 percent alcohol by volume and it must be supplied immediately. The brewery wishes to fill the order, but no 4 percent beer is now in stock. It is decided to mix two beers now in stock to give the desired final product. One of the beers in stock (Beer A) contains 4.5 percent alcohol by volume and is valued at $0.32 per gallon. The other beer in stock (Beer B) contains 3.7 percent alcohol by volume and is valued at $0.25 per gallon. Water (W) can be added to the blend at no cost. What volume combination of the two beers in stock with water, including at least 10 gal of Beer A, will give the minimum ingredient cost for the 100 gal of 4 percent beer?

This example is greatly simplified because only a few constraints are involved and there are only three independent variables, i.e., amount of Beer A ($V_A$), amount of Beer B ($V_B$), and amount of water ($V_W$). When a large number of possible choices is involved, the optimum set of choices may be far from obvious, and a solution by linear programming may be the best way to approach the problem. A step-by-step rational approach is needed for linear programming. This general rational approach is outlined in the following with application to the blending example cited.

**RATIONAL APPROACH TO PROBLEMS INVOLVING LINEAR PROGRAMMING**

A systematic rationalization of a problem being solved by linear programming can be broken down into the following steps:

1. **A systematic description of the limitations or constraints.** For the brewery example, the constraints are as follows:
   a. Total volume of product is 100 gallons, or
   
   \[ V_W + V_A + V_B = 100 \]  
   (58)

   \[ 0.0V_W + 4.5V_A + 3.7V_B = (4.0)(100) \]  
   (59)

   b. Product must contain 4 percent alcohol, or

   \[ V_W \geq 0 \quad V_B \geq 0 \quad V_A \geq 10 \quad \text{or} \quad V_A \sim S = 10 \]  
   (60)

   where $S$ is the so-called “slack variable.”
2. A systematic description of the objective. In the brewery example, the objective is to minimize the cost of the ingredients; i.e., the objective function is

\[ C = \text{cost} = \text{a minimum} = 0.0V_W + 0.32V_A + 0.25V_B \] (61)

3. Combination of the constraint conditions and the objective function to choose the best result out of many possibilities. One way to do this would be to use an intuitive approach whereby every reasonable possibility would be considered to give ultimately, by trial and error, the best result. This approach would be effective for the brewery example because of its simplicity. However, the intuitive approach is not satisfactory for most practical situations, and linear programming can be used. The computations commonly become so involved that a computer is required for the final solution. If a solution is so simple that a computer is not needed, linear programming would probably not be needed. To illustrate the basic principles, the brewery example is solved in the following by linear programming including intuitive solution, graphical solution, and computer solution.

From Eqs. (58) to (61), the following linearized basic equations can be written:

\[ V_W + V_A + V_B = 100 \] (58)
\[ 4.5V_A + 3.7V_B = 400 \] (59)
\[ 0.32V_A + 0.25V_B = C = \text{minimum} \] (61)

where \( C \) is designated as the objective function.

Combination of Eqs. (58) and (59) gives

\[ V_A = 37.5 + 4.625V_W \] (62)

![Graphical representation of linear-programming solution based on brewery example.](image)
Equation (62) is plotted as line $OE$ in Fig. 11-9, and the optimum must fall on this line.

Equation (61) combined with Eq. (58) gives

$$V_A = \frac{C - 25}{0.07} + 3.57V_w$$  \hspace{1cm} (63)

**INTUITIVE SOLUTION.** It can be seen intuitively, from Eqs. (63) and (62), that the minimum value of the objective function $C$ occurs when $V_w$ is zero. Therefore, the optimum value of $V_A$, from Eq. (62), is 37.5 gal and the optimum value of $V_B$, from Eq. (58), is 62.5 gal.

**LINEAR PROGRAMMING GRAPHICAL SOLUTION.** Figure 11-9 is the graphical representation of this problem. Line $OE$ represents the overall constraint placed on the problem by Eqs. (58), (59), and (60). The parallel dashed lines represent possible conditions of cost. The goal of the program is to minimize cost (that is, $C$) while still remaining within the constraints of the problem. The minimum value of $C$ that still meets the constraints occurs for the line $OD$, and the optimum must be at point 0. Thus, the recommended blend is no water, 37.5 gal of $A$, 62.5 gal of $B$, and a total cost $C$ of $27.63$ for 100 gal of blend.

**LINEAR PROGRAMMING COMPUTER SOLUTION.** Although the simplicity of this problem makes it trivial to use a computer for solution, the following is presented to illustrate the basic type of reasoning that is involved in developing a computer program for the linearized system.

An iterative procedure must be used for the computer solution to permit the computer to make calculations for repeated possibilities until the minimum objective function $C$ is attained. In this case, there are four variables ($V_A, V_B, V_w$, and $S$) and three nonzero constraints (total volume, final alcohol content, and $V_A = 10 + S$). Because the number of real variables cannot exceed the number of nonzero constraints, one of the four variables must be zero.\(^\dagger\) Thus, one approach for a computer solution merely involves solving a four-by-three matrix with each variable alternatively being set equal to zero, followed by instruction that the desired combination is the one giving the least total cost.

The computer logic, from which the computer diagram, program, and solution can be developed directly, is presented in Tables 3 and 4.\(^\ddagger\)


\(^\ddagger\)In Prob. 18 at the end of this chapter, the student is requested to develop the full computer program and solve this problem on a computer.
## Table 3

Computer logic for “linear programming” solution to brewery example

The computer must solve a linearized situation in which there are four variables and three nonzero constraints to meet a specified objective function of minimum cost. Under these conditions, one of the variables must be zero. Thus, one method for computer approach is to set each variable in turn equal to zero, solve the resulting three-by-three matrix, and determine the final solution to make cost a minimum. Instead of one total computer solution with instructions for handling a three-by-four matrix, the approach will be simplified by repeating four times the solution by determinants of a standard case of three equations and three unknowns with each of the four variables alternatingly set equal to zero.

**Basis:** $E(I, J)$ and $X(J)$; where $E(I, J)$ designates the appropriate coefficient, $X(J)$ designates the appropriate variable, $I$ designates the proper row, and $J$ designates the proper column. Thus, $E(I, J)$ and $X(J)$ are such that:

### Equations

**Constraints**

\[
\begin{align*}
E(1, 1)X(1) + E(1, 2)X(2) + E(1, 3)X(3) &= E(1, 4) \\
E(2, 1)X(1) + E(2, 2)X(2) + E(2, 3)X(3) &= E(2, 4) \\
E(3, 1)X(1) + E(3, 2)X(2) + E(3, 3)X(3) &= E(3, 4)
\end{align*}
\]

**Objective function**

\[
C(1)X(1) + C(2)X(2) + C(3)X(3) = F(C)
\]

**Equivalent to**

### Logic and procedure based on arbitrary choice of one variable set equal to zero

**Step 1:** Read in to the computer the constant coefficients for the three variables being retained based on Eqs. (58), (59), and (60). Read in to computer the constant coefficients for the objective function, i.e., Eq. (61).

**Step 2:** Solve the resulting three equations simultaneously by determinants.

A. Evaluate the determinant of the coefficients of the system = $F(E) = D$:

\[
D \equiv \begin{vmatrix}
E(1, 1) & E(1, 2) & E(1, 3) \\ 
E(2, 1) & E(2, 2) & E(2, 3) \\ 
E(3, 1) & E(3, 2) & E(3, 3)
\end{vmatrix}
\]

B. Evaluate $DX(J) = F(W)$:

\[
D \begin{vmatrix}
W(1, 1) & W(1, 2) & W(1, 3) \\ 
W(2, 1) & W(2, 2) & W(2, 3) \\ 
W(3, 1) & W(3, 2) & W(3, 3)
\end{vmatrix}
\]

where $W(I, J)$ designates the appropriate coefficient for the $DX$ working matrix in which the column of equality constraints is substituted in the appropriate determinant column.

**Step 3:** Evaluate the objective function.

**Step 4:** Print out the values of the three variables and the value of the objective function.

**Step 5:** Repeat steps 1 to 4 three more times with each variable set equal to zero.

**Step 6:** After all results have been printed out, choose the set of results giving the minimum value for the objective function with all variables meeting the requirements.

---

† For normal methods used for linear-programming solutions, there are various rules which can be used to determine which variable should be set equal to zero so that not all possible combinations must be tried. See later discussion of the Simpler Algorithm.
Computer diagram based on logic of Table 3 for linear-programmed brewery example

FORTRAN

Step 1

((E(i,j), j=1,4)i=1,3)
(C(i),i=1,3)

Step 2A

Step 2

B

Step 2C

Step 3

Step 4

Step 5

Note: N is the column to be deleted
GENERALIZATION OF STRATEGY FOR LINEAR PROGRAMMING

The basic problem in linear programming is to maximize or minimize a linear function of a form as shown in Eq. (57). There are various strategies that can be developed to simplify the methods of solution, some of which can lead to algorithms which allow rote or pure number-plugging methods of solution that are well adapted for machine solution.

In linear programming, the variables \( x_1, \ldots, x_n \) are usually restricted (or can be transformed) to values of zero or greater. This is known as a nonnegativity restriction on \( x_j \); that is,

\[
x_j \geq 0 \quad j = 1, 2, \ldots, n
\]

Consider a simple two-dimensional problem such as the following: The objective function is to maximize

\[
3x_1 + 4x_2
\]

subject to the linear constraints of

\[
2x_1 + 5x_2 \leq 10 \quad (65)
\]

\[
4x_1 + 3x_2 \leq 12 \quad (66)
\]

\[
x_1 \geq 0
\]

\[
x_2 \geq 0
\]

This problem and its solution is pictured graphically in Fig. 11-10, which shows that the answer is \( 3x_1 + 4x_2 = 11 \). From Fig. 11-10 it can be seen that the linear constraints, in the form of inequalities, restrict the solution region to the cross-hatched area. This solution region is a polygon designated as convex because all points on the line between any two points in the cross-hatched region are in the set of points that satisfy the constraints. The set of the objective function is a family of lines with slope of \(-\frac{3}{4}\). The maximum value of the objective function occurs for the line passing through the polygon vertex \( D \). Thus, the maximum value of the objective function occurs for the case of \( 3x_1 + 4x_2 = 11 \) at \( x_1 = \frac{15}{7} \) and \( x_2 = \frac{9}{7} \).

For the two-dimensional case considered in the preceding, one linear condition defines a line which divides the plane into two half-planes. For a three-dimensional case, one linear condition defines a set plane which divides the volume into two half-volumes. Similarly, for an n-dimensional case, one linear condition defines a hyperplane which divides the space into two half-spaces.

For the n-dimensional case, the region that is defined by the set of hyperplanes resulting from the linear constraints represents a convex set of all points which satisfy the constraints of the problem. If this is a bounded set, the enclosed space is a convex polyhedron, and, for the case of monotonically increasing or decreasing values of the objective function, the maximum or minimum value of the objective function will always be associated with a vertex.
or extreme point of the convex polyhedron. This indicates that the linear-programming solution for the model of inequality or equality constraints combined with the requested value for the objective function will involve determination of the value of the objective function at the extreme points of the set of all points that satisfy the constraints of the problem. The desired objective function can then be established by comparing the values found at the extreme points. If two extremes give the same result, then an infinite number of solutions exist as defined by all points on the line connecting the two extreme points.

SIMULTANEOUS EQUATIONS

Linear programming is concerned with solutions to simultaneous linear equations where the equations are developed on the basis of restrictions on the variables. Because these restrictions are often expressed as inequalities, it is necessary to convert these inequalities to equalities. This can be accomplished by the inclusion of a new variable designated as a slack variable.

For a restriction of the form

$$a_1x_1 + a_2x_2 + a_3x_3 \leq b$$ \hspace{1cm} (69)

the inequality is converted to a linear equation by adding a slack variable $S_4$ to

\[\text{Cases}\] are often encountered in design calculations where a large number of design equations and variables are involved with long and complex simultaneous solution of the equations being called for. The amount of effort involved for the simultaneous solutions can be reduced by using the so-called structural-array algorithm which is a purely mechanical operation involving crossing out rows for equations and columns for variables to give the most efficient order in which the equations should be solved. For details, see D. F. Rudd and C. C. Watson, "Strategy of Process Engineering," pp. 45-49, John Wiley & Sons, Inc., New York, 1968.
The slack variable takes on whatever value is necessary to satisfy the equation and normally is considered as having a nonnegativity restriction. Therefore, the slack variable would be subtracted from the left-hand side for an inequality of the form

$$a_1x_1 + a_2x_2 + a_3x_3 \geq b$$

(71)

to give

$$a_1x_1 + a_2x_2 + a_3x_3 - S_4 = b$$

(72)

After the inequality constraints have been converted to equalities, the complete set of restrictions becomes a set of linear equations with \( n \) unknowns. The linear-programming problem then will involve, in general, maximizing or minimizing a linear objective function for which the variables must satisfy the set of simultaneous restrictive equations with the variables constrained to be nonnegative. Because there will be more unknowns in the set of simultaneous equations than there are equations, there will be a large number of possible solutions, and the final solution must be chosen from the set of possible solutions.

If there are \( m \) independent equations and \( n \) unknowns with \( m < n \), one approach is to choose arbitrarily \( n - m \) variables and set them equal to zero. This gives \( m \) equations with \( m \) unknowns so that a solution for the \( m \) variables can be obtained. Various combinations of this type can be obtained so that the total possible number of solutions by this process becomes

$$\binom{n}{m} = \frac{n!}{m!(n - m)!}$$

(73)

representing the total number of possible combinations obtainable by taking \( n \) variables \( m \) at a time. Another approach is to let \( n - m \) combinations of variables assume any zero or nonzero value which results in an infinite number of possible solutions. Linear programming deals only with the situation where the excess variables are set equal to zero.

TWO EXAMPLES TO SHOW APPROACH BY SIMULTANEOUS EQUATIONS

To illustrate the introductory ideas presented for a linear-programming problem, consider the following example which is solved by using a step-by-step simultaneous-equation approach:

A production facility is being used to produce three different products, \( x_1 \), \( x_2 \), and \( x_3 \). Each of these products requires a known number of employee-hours and machine-hours for production such that

- product \( x_1 \) requires 10 employee-hours and 15 machine-hours per unit,
- product \( x_2 \) requires 25 employee-hours and 10 machine-hours per unit,
- product \( x_3 \) requires 20 employee-hours and 10 machine-hours per unit.
The profit per unit is $5 for \( x_1 \), $10 for \( x_2 \), and $12 for \( x_3 \). Over the base production period under consideration, a total of 300 employee-hours and a total of 200 machine-hours are available. With the special restriction that all employee-hours are to be used, what mix of products will maximize profits?

For this problem, the linear constraints are:

for machine hours,

\[
15x_1 + 10x_2 + 10x_3 \leq 200
\]

for employee-hours,

\[
10x_1 + 25x_2 + 20x_3 = 300
\]

The objective function is to maximize profits, or

\[
\text{Maximize } 5x_1 + 10x_2 + 12x_3
\]

By including a slack variable for Eq. (74), the constraining equalities become

\[
15x_1 + 10x_2 + 10x_3 + s_4 = 200
\]

\[
10x_1 + 25x_2 + 20x_3 = 300
\]

For this case, \( n = 4 \) and \( m = 2 \). Setting any two of the variables equal to zero and solving the result gives

\[
\binom{n}{m} = \frac{n!}{m!(n-m)!} = \frac{4!}{2!2!} = 6
\]

possible solutions. These six solutions are shown in Table 5.

Solutions 3 and 4 are infeasible because the nonnegativity restriction has been violated, while Solution 6 is a feasible solution which maximizes the objective function. Thus, Solution 6 is the desired solution for this example and represents the optimal solution.

**TABLE 5**

<table>
<thead>
<tr>
<th>Solution number</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_4 )</th>
<th>Objective function ( 5x_1 + 10x_2 + 12x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2121</td>
<td>9.0909</td>
<td>0</td>
<td>0</td>
<td>121.27</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0</td>
<td>12.5</td>
<td>0</td>
<td>175</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Infeasible (negative) ( -250 )</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>0</td>
<td>Infeasible (negative)</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>50</td>
<td>180</td>
</tr>
</tbody>
</table>
For the example presented graphically in Fig. 11-10, the two linear constraining equations made into equalities by the slack variables $S_3$ and $S_4$ are

$$2x_1 + 5x_2 + S_3 + OS, = 10 \tag{79}$$
$$4x_1 + 3x_2 + OS, + S_4 = 12 \tag{80}$$

with the objective function being

$$3x_1 + 4x_2 + OS, + OS, = z = a \text{ maximum} \tag{81}$$

In this case, there are two equations ($m = 2$) and four variables ($n = 4$). Thus, the approach with $n - m = 4 - 2 = 2$ of the variables being zero for each solution will involve having

$$\binom{n}{m} = \frac{n!}{m!(n - m)!} = \frac{4!}{2!2!} = 6$$

possible solutions. These solutions are shown in Fig. 11-10 as points A to F and in Table 6 as obtained by simultaneous-equation solution with the optimum result being the D solution.

### TABLE 6

<table>
<thead>
<tr>
<th>Solution designation</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>Objective function $3x_1 + 4x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>$a$</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>D</td>
<td>15/7</td>
<td>8/7</td>
<td>0</td>
<td>0</td>
<td>11$^\dagger$</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>- $a$</td>
<td>Infeasible (negative)</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>4</td>
<td>-10</td>
<td>0</td>
<td>Infeasible (negative)</td>
</tr>
</tbody>
</table>

$^\dagger$ Maximum feasible value; so the optimum solution is D.

The preceding examples, although they represent an approach for solving linear-programming problems, are very inefficient because of the large number of useless solutions that may be generated if many variables are involved. More efficient procedures are available, and these are discussed in the following sections.

### GENERALIZATION OF LINEAR PROGRAMMING APPROACH FOR ALGORITHM SOLUTION

To permit efficient solutions for linear-programming problems, an algorithm can be developed. An algorithm, basically, is simply an objective mathematical method for solving a problem and is purely mechanical so that it can be taught
to a nonprofessional or programmed for a computer. The algorithm may consist of a series of repeated steps or *iterations*. To develop this form of approach for linear-programming solutions, the set of linear inequalities which form the constraints, written in the form of “equal to or less than” equations is

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \leq b_1 \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \leq b_2 \\
    \vdots & \vdots \\
    a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \leq b_m
\end{align*}
\] (82)

or, in general summation form,

\[
\sum_{j=1}^{n} a_{ij}x_j \leq b_i \quad i = 1, 2, \ldots, m
\] (82a)

for

\[
x_j \geq 0 \quad j = 1, 2, \ldots, n
\]

where \(i\) refers to columns (or number of equations, \(m\)) in the set of inequalities and \(j\) refers to rows (or number of variables, \(n\)).

As has been indicated earlier, these inequalities can be changed to equalities by adding a set of slack variables, \(x_{n+1} \cdots x_{n+m}\) (here \(X\) is used in place of \(S\) to simplify the generalized expressions), so that

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + x_{n+1} & = b_1 \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + x_{n+2} & = b_2 \\
    \vdots & \vdots \\
    a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + x_{n+m} & = b_m
\end{align*}
\] (83)

†This can be written in standard matrix form and notation as

\[
AX = B
\]

where

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & 0 & \cdots & 0 \\
    a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & 0 & \cdots & 1 & 0 \\
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n \\
    x_{n+1} \\
    x_{n+2} \\
    \vdots \\
    x_{n+m}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
    b_1 \\
    b_2 \\
    \vdots \\
    b_m
\end{bmatrix}
\]

and standard matrix operations of multiplying, addition, etc., can be applied.
or, in general summation form,
\[ \sum_{j=1}^{n} (a_{ij}x_j + x_{n+i}) = b_i \quad i = 1, 2, \ldots, m \]  
\tag{83a}

for
\[ x_j \geq 0 \quad j = 1, 2, \ldots, n + m \]

In addition to the constraining equations, there is an objective function for the linear program which is expressed in the form of
\[ z = \text{maximum (or minimum)} \ of \ c_1x_1 + c_2x_2 + \cdots + c_jx_j + \cdots + c_nx_n \]  
\tag{84}†

where the variables \( x_j \) are subject to \( x_j \geq 0 \) \((j = 1, 2, \ldots, n + m)\). Note that, in this case, all variables above \( x_n \) are slack variables and make no direct contribution to the value of the objective function.

Within the constraints as indicated by Eqs. (82) and (83), a solution for values of the variables, \( x_j \), must be found which meets the maximum or minimum requirement of the objective function, Eq. (84). As has been demonstrated in the preceding examples, the solution to a problem of this sort must lie on an extreme point of the set of possible feasible solutions. For any given solution, the number of equations to be solved simultaneously must be set equal to the number of variables, and this is accomplished by setting \( n \) (number of variables) minus \( m \) (number of equations) equal to zero and then proceeding to obtain a solution.

While the preceding generalization is sufficient to allow for reaching a final solution ultimately, it can be very inefficient unless some sort of special method is used to permit generation of extreme-point solutions in an efficient manner to allow rapid and effective approach to the optimum condition. This is what the simplex method does.‡

THE SIMPLEX ALGORITHM

The basis for the simplex method is the generation of extreme-point solutions by starting at any one extreme point for which a feasible solution is known and then proceeding to a neighboring extreme point. Special rules are followed which cause the generation of each new extreme point to be an improvement toward the desired objective function. When the extreme point is reached where no further improvement is possible, this will represent the desired optimum feasible solution. Thus, the simplex algorithm is an iterative process that starts at one extreme-point feasible solution, tests this point for optimality, and

†In a more compact form using matrix notation, the problem is to find the solution to \( AX = B \) which maximizes or minimizes \( z = cX \) where \( X \geq 0 \).

‡The simplex method and algorithm were first made generally available when published by G. B. Dantzig in “Activity Analysis of Production and Allocations,” edited by T. C. Koopmans, Chap. XXI, Wiley, New York, 1951.
proceeds toward an improved solution. If an optimal solution exists, this algorithm can be shown to lead ultimately and efficiently to the optimal solution.

The stepwise procedure for the simplex algorithm is as follows (based on the optimum being a maximum):

1. State the linear-programming problem in standard equality form.
2. Establish the initial feasible solution from which further iterations can proceed. A common method to establish this initial solution is to base it on the values of the slack variables where all other variables are assumed to be zero. With this assumption, the initial matrix for the simplex algorithm can be set up with a column showing those variables which will be involved in the first solution. The coefficient for these variables appearing in the matrix table should be 1 with the rest of the column being 0.
3. Test the initial feasible solution for optimality. The optimality test is accomplished by the addition of rows to the matrix which give (a) a value of \( z_j \) for each column where \( z_j \) is defined as the sum of the objective-function coefficient for each solution variable (\( c_j \) corresponding to solution \( x_i \) in that row) times the coefficient of the constraining-equation variable for that column \( [a_{ij} \text{ in Eq. (83a)}] \); (that is, \( z_j = \sum_{i=1}^{m} c_i a_{ij} (j = 1, 2, \ldots, n) \)), (b) \( c_j \) [see Eq. (84)], and (c) \( c_j - z_j \). If \( c_j - z_j \) is positive for at least one column, then a better program is possible.
4. Iteration toward the optimal program is accomplished as follows: Assuming that the optimality test indicates that the optimal program has not been found, the following iteration procedure can be used:
   a. Find the column in the matrix with the maximum value of \( c_j - z_j \) and designate this column as \( k \). The incoming variable for the new test will be the variable at the head of this column.
   b. For the matrix applying to the initial feasible solution, add a column showing the ratio of \( b_i / a_{ik} \). Find the minimum positive value of this ratio and designate the variable in the corresponding row as the outgoing variable.
   c. Set up a new matrix with the incoming variable, as determined under (a), substituted for the outgoing variable, as determined under (b). The modification of the table is accomplished by matrix operations so that the entering variable will have a 1 in the row of the departing variable and zeros in the rest of that column. The matrix operations involve row manipulations of multiplying rows by constants and subtracting from or adding to other rows until the necessary 1 and 0 values are reached. This new matrix should have added to it the additional rows and column as explained under parts 3, 4a, and 4b.
   d. Apply the optimality test to the new matrix.
   e. Continue the iterations until the optimality test indicates that the optimum objective function has been attained.
5. Special cases:
   a. If the initial solution obtained by use of the method given in the preceding
      is not feasible, a feasible solution can be obtained by adding more
      artificial variables which must then be forced out of the final solution.
   b. Degeneracy may occur in the simplex method when the outgoing variable
      is selected. If there are two or more minimal values of the same size, the
      problem is degenerate, and a poor choice of the outgoing variable may
      result in cycling, although cycling almost never occurs in real problems.
      This can be eliminated by a method of ratioing each element in the rows
      in question by the positive coefficients of the kth column and choosing the
      row for the outgoing variable as the one first containing the smallest
      algebraic ratio.

6. The preceding method for obtaining a maximum as the objective function
   can be applied for the case when the objective function is a minimum by
   recognizing that maximizing the negative of a function is equivalent to
   minimizing the function.

THE SIMPLEX ALGORITHM APPLIED TO THE EXAMPLE
SHOWN IN FIGURE 11-10

In the example used previously, and whose graphical solution is shown in Fig.
11-10, the problem in standard linear-programming form is: Find the values of
the variables which represent a solution to

\[
\begin{align*}
2x_1 + 5x_2 + x_3 &= 10 \\
4x_1 + 3x_2 + x_4 &= 12
\end{align*}
\]

which maximizes

\[
3x_1 + 4x_2 \quad \text{(express as } 3x_1 + 4x_2 + 0x_3 + 0x_4 = z)\]

where \( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, \) and \( x_4 \geq 0. \)

The next step after the appropriate statement of the linear-programming
problem is to establish an initial feasible solution from which further iterations
can proceed. For this case, let \( x_1 = x_2 = 0, x_3 = 10, x_4 = 12, z = 0. \)
(Solution A in Fig. 11-10 or Table 6.) The corresponding matrix in a standard
tableau form is shown in Table 7.

The top row, \( c_i^{t} \) in the tableau permits a convenient recording of the
coefficients on the variables in the objective function, with these values listed at
the head of the appropriate columns.

The first column, \( c_i \), gives the coefficients of the variables in the objective
function for this first solution. In this case, both are zero because \( x_3 \) and \( x_4 \) do
not appear in the objective function.

The second column, Solution, gives the variables involved in the current
solution and shows the row for which the variables involved apply.
TABLE 7
Tableau form of matrix for initial feasible solution \((x_1 = x_2 = 0)\)

<table>
<thead>
<tr>
<th>(c_j)</th>
<th>(c_i)</th>
<th>Solution</th>
<th>(b)</th>
<th>(x_1)</th>
<th>(x_4)</th>
<th>(x_3)</th>
<th>(x_2)</th>
<th>(b_i/a_{ik})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(x_3)</td>
<td>10</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td>0</td>
<td>(x_4)</td>
<td>12</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td></td>
<td>(-\frac{2}{3})</td>
</tr>
<tr>
<td>(z_j)</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_j)</td>
<td>(z_j)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td></td>
<td>(k)</td>
</tr>
</tbody>
</table>

The third column, \(b\), gives the list of condition constants for the limiting equations.

The columns following \(b\) have \(x\) headings and represent variables. The slack variables are \(x_3\) and \(x_4\), designated as unity for the appropriate row, while the structural variables are \(x_1\) and \(x_2\) with normal matrix form based on the coefficients for \(x_1\) and \(x_2\) in the limiting equations.

The final column on the right, \(b_i/a_{ik}\), is used to record the indicated ratios for each row during the iteration process.

The bottom two rows are included to give a convenient method for recording the objective-function row component \(z_j\) and the values of \(c_j - z_j\) for each column. By definition of \(z_j\) as \(\sum_{i=1}^{n} c_i a_{ij}\) for \(j = 1, 2, \ldots, n\), the value of \(z_j\) is 0 for all columns because both \(c_i\)'s are 0.

Because row \(c_j - z_j\) in Table 7 has at least one positive value in it, a better optimal program is available. The variable at the head of the column \((k)\) with the maximum value of \(c_j - z_j\) is \(x_2\). Therefore, \(x_2\) will be the incoming variable. The minimum value of \(b_i/a_{ik}\) occurs for the \(x_3\) row; so \(x_3\) will be the outgoing variable and the encircled 5 becomes the so-called pivotal point.

To eliminate \(x_3\) from the basis, the use of the indicated pivotal point gives, as a first step, the matrix tableau shown in the top part of Table 8 where the corresponding element for the pivotal point has been reduced to 1 by dividing the \(x_3\) row by 5. The bottom portion of Table 8 is the matrix tableau for the next iteration with \(x_3 = x_1 = 0\). This is established by a matrix row operation to reduce the other elements in the \((k)\) column to zero (i.e., for this case, the multiplying factor for the \(x_3\) row is -3, and the \(x_3\) result is added to the \(x_4\) row). The values of \(z_j\) are \((4)(2) + (0)(6) = 8, (4)(\frac{1}{2}) + (0X - \frac{3}{5}) = \frac{4}{5}, (4)(0) + (0)(1) = 0, (4)(\frac{2}{5}) + (0)(\frac{14}{5}) = \frac{8}{5}\), and \((4)(1) + (0)(0) = 4\) for the five columns from left to right, respectively.

Therefore, from Table 8, another extreme-point solution is \(x_2 = 2, x_4 = 6, x_1 = x_3 = 0\). (Point B in Fig. 11-10.) This is still not the optimal solution because row \(c_j - z_j\) has a positive value in it. The encircled \(\frac{14}{5}\) is the pivotal point for the next iteration which will have \(x_4\) as the outgoing variable and \(x_1\) as the incoming variable. The same procedure is followed for this second iteration as for the first iteration. The steps are shown in Table 9, where the pivotal-point
### TABLE 8
Tableau form of matrix for first iteration \((x_1 = x_3 = 0)\)

<table>
<thead>
<tr>
<th>(c_j)</th>
<th>Solution</th>
<th>(b_i)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(b_i/a_{ik})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(x_3)</td>
<td>2</td>
<td>(\frac{1}{3})</td>
<td>0</td>
<td>(\frac{2}{5})</td>
<td>1</td>
<td>(\frac{5}{3}) = 2</td>
</tr>
<tr>
<td>0</td>
<td>(x_4)</td>
<td>12</td>
<td>(\frac{6}{5})</td>
<td>1</td>
<td>(\frac{4}{3})</td>
<td>3</td>
<td>(\frac{12}{3}) = 4</td>
</tr>
<tr>
<td>(z_j)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(c_j - z_j)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 9
Tableau form of matrix for second iteration \((x_3 = x_4 = 0)\)

<table>
<thead>
<tr>
<th>(c_j)</th>
<th>Solution</th>
<th>(b_i)</th>
<th>(x_3)</th>
<th>(x_4)</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(b_i/a_{ik})</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(x_2)</td>
<td>2</td>
<td>(\frac{5}{3})</td>
<td>0</td>
<td>(\frac{2}{7})</td>
<td>1</td>
<td>(\frac{22}{14}) = 5</td>
</tr>
<tr>
<td>0</td>
<td>(x_4)</td>
<td>(\frac{44}{14})</td>
<td>(-\frac{3}{14})</td>
<td>(\frac{5}{14})</td>
<td>(\frac{1}{14})</td>
<td>0</td>
<td>(\frac{30}{14}/1 = \frac{30}{14})</td>
</tr>
<tr>
<td>(z_j)</td>
<td>8</td>
<td>(\frac{4}{3})</td>
<td>0</td>
<td>(\frac{8}{3})</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_j - z_j)</td>
<td>-8</td>
<td>(-\frac{4}{3})</td>
<td>0</td>
<td>(\frac{7}{3})</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(k)
element is first reduced to 1 by dividing the $x_4$ row by $\frac{14}{5}$, and the other elements in the $(k)$ column are then reduced to zero by a matrix row operation involving a multiplication factor of $-\frac{2}{5}$ for the $x_4$ row and adding the $x_4$ row to the $x_2$ row.

The results shown in Table 9 give another extreme point of $x_1 = \frac{15}{7}$, $x_2 = \frac{6}{7}$, $x_3 = x_4 = 0$. The $z_j$ value for the $b$ column is $(4)(\frac{6}{7}) + (3)(\frac{15}{7}) = 11$, and the values for the other four columns from left to right are obtained by a similar addition as $(4)(\frac{6}{7}) + (3)(-\frac{1}{7}) + (3)(\frac{2}{7}) = \frac{1}{2}$, $(4)(0) + (3)(1) = 3$, and $(4)(1) + (3)(0) = 4$.

Because row $c_j - z_j$ has only negative or zero values in it, this is the optimal solution, and the objective function is a maximum of $z = (3)(\frac{15}{7}) + (4)(\frac{6}{7}) = 11$ at $x_1 = \frac{15}{7}$ and $x_2 = \frac{6}{7}$, which is the same solution (point D in Fig. 11-10) that was obtained by the graphical analysis in Fig. 11-10. Note that, in each basic, initial, table matrix where the column for that variable has all zeros except for the variable row which is 1, the $b$ column gives the values of the variables and the objective function for that solution.

The preceding information can serve as an introduction to the methods of linear programming including the step-by-step rule approach used for a simplex algorithm. The reader is referred to any of the many standard texts on linear programming for proof of the theorems and rules used in this treatment and further extensions of the methods of linear programming.

THE STRATEGY OF DYNAMIC PROGRAMMING FOR OPTIMIZATION ANALYSIS

The concept of dynamic programming is based on converting an overall decision situation involving many variables into a series of simpler individual problems with each of these involving a small number of total variables. In its extreme, an optimization problem involving a large number of variables, all of which may be subject to constraints, is broken down into a sequence of problems with each of these involving only one variable. A characteristic of the process is that the determination of one variable leaves a problem remaining with one less variable. The computational approach is based on the principle of optimality, which states that an optimal policy has the property that, no matter what the initial state

---

and initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

The use of dynamic programming is pertinent for design in the chemical industry where the objective function for a complicated system can often be obtained by dividing the overall system into a series of stages. Optimizing the resulting simple stages can lead to the optimal solution for the original complex problem.

The general formulation for a dynamic-programming problem, presented in a simplified form, is shown in Fig. 11-11. On the basis of the definitions of terms given in Fig. 11-lla, each of the variables, \( x_{i+1}, x_i, \) and \( d_i, \) may be replaced by vectors because there may be several components or streams involved in the input and output, and several decision variables may be involved. The profit or return \( P_i \) is a scalar which gives a measure of contribution of stage \( i \) to the objective function.

For the operation of a single stage, the output is a function of the input and the decisions, or

\[
x_i = h_i(x_{i+1}, d_i)
\]

Similarly, for the individual-stage objective function \( P_i \)

\[
P_i = g_i(x_{i+1}, x_i, d_i)
\]

or, on the basis of the relation shown as Eq. (88),

\[
P_i = g_i(x_{i+1}, d_i)
\]

For the simple multistage process shown in Fig. 11-llb, the process design strategy to optimize the overall operation can be expressed as

\[
\tilde{f}_i(x_{i+1}) = \max_{d_i} [g_i(x_{i+1}, d_i) + \tilde{f}_{i-1}(x_i)] = \max_{d_i} [Q_i(x_{i+1}, d_i)]
\]

for \( x_i = h_i(x_{i+1}, d_i) \quad i = 1, 2, \ldots, n \)-subject to \( \tilde{f}_0 = 0 \)

The symbolism \( \tilde{f}_i(x_{i+1}) \) indicates that the maximum (or optimum) return or profit from a process depends on the input to that process, and the terms in the square brackets of Eq. (91) refer to the function that is being optimized. Thus, the expression \( Q_i(x_{i+1}, d_i) \) represents the combined return from all stages and must equal the return from stage \( i, \) or \( g_i(x_{i+1}, d_i), \) plus the maximum return from the preceding stages 1 through \( i-1, \) or \( \tilde{f}_{i-1}(x_i). \)

In carrying out the procedure for applying dynamic programming for the solution of appropriate plant-design problems, each input \( x_{i+1} \) is considered as a parameter. Thus, at each stage, the problem is to find the optimum value of the decision variable \( d_i \) for all feasible values of the input variable. By using the dynamic-programming approach involving \( n \) stages, a total of \( n \) optimizations must be carried out. This approach can be compared to the conventional approach in which optimum values of all the stages and decisions would be
Decision input, \( d_i \) (decisions which set the design or operating condition for the \( i \)th stage)

Feed input, \( x_{i+1} \) (depends on decisions made for other stages or on fixed terminal condition)

Output, \( x_i \) (depends on \( d_i \), and \( x_i \))

Profit or return, \( P_i \) (depends on \( x_{i+1}, d_i \), and \( x_i \))

---

**FIGURE 11-11**
Illustration of stages involved in dynamic programming.

made by a basic probability combination analysis. Thus, the conventional method would have a computational effort that would increase approximately exponentially with the number of stages, while the dynamic-programming approach can give a great reduction in necessary computational effort because this effort would only increase about linearly with the number of stages. However, this advantage of dynamic programming is based on a low number of components in the input vector \( x_{i+1} \), and dynamic programming rapidly loses its effectiveness for practical computational feasibility if the number of these components increases above two.
A SIMPLIFIED EXAMPLE OF DYNAMIC PROGRAMMING?

As an illustration of the general procedure and analytical methods used in dynamic programming, consider the example design problem presented in Table 10. The general procedure consists of the following steps:

1. Establish the sequence of single stages into which the process will be divided. These are shown as stages 1 to 5 in Table 10.
2. Decide on the units to be used for expressing the profit function for each individual stage and the overall process. For this case, the problem statement makes it clear that an appropriate unit for this purpose is the profit over a

**TABLE 10**

A dynamic-programming model for production of a new chemical with no recycle including specific data for an example

<table>
<thead>
<tr>
<th>Feed (raw materials)</th>
<th>5 Timer</th>
<th>4 Heater</th>
<th>3 Reactor</th>
<th>2 Reactor</th>
<th>1 Separator</th>
<th>Output (product)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waste</td>
<td>--------</td>
<td>----------</td>
<td>-----------</td>
<td>-----------</td>
<td>-------------</td>
<td>------------------</td>
</tr>
</tbody>
</table>

Feed: 50,000 lb/yr of raw material are fed to stage 5 of the above model of the process at a cost of $1 per pound.

The output from the 5-stage process must be at least 15,000 lb of product per year.

The overall objective function of the entire process is to optimize for a maximum profit over a five-year period. Assume equipment life period is 3 years.

a. **Anticipated selling price of the product vs. annual production**

<table>
<thead>
<tr>
<th>Production, 1000 lb/yr</th>
<th>47.5</th>
<th>45.0</th>
<th>42.5</th>
<th>40.0</th>
<th>37.5</th>
<th>30.0</th>
<th>25.0</th>
<th>22.5</th>
<th>20.0</th>
<th>15.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected selling price, $/lb</td>
<td>3.2</td>
<td>3.3</td>
<td>3.4</td>
<td>3.6</td>
<td>3.8</td>
<td>4.6</td>
<td>5.0</td>
<td>5.2</td>
<td>5.3</td>
<td>5.5</td>
</tr>
</tbody>
</table>

b. **Operating costs for the mixing operation, $1000/yr**

<table>
<thead>
<tr>
<th>Mixing efficiency</th>
<th>1.0</th>
<th>0.8</th>
<th>0.6</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixer A</td>
<td>12.0</td>
<td>6.0</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>B</td>
<td>8.0</td>
<td>4.0</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>C</td>
<td>5.0</td>
<td>3.0</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

c. **Heater operating costs, $1000/yr**

<table>
<thead>
<tr>
<th>Mixing efficiency</th>
<th>Temperature, °F</th>
<th>650</th>
<th>700</th>
<th>750</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>650</td>
<td>1.0</td>
<td>1.0</td>
<td>6.0</td>
<td>10.0</td>
</tr>
<tr>
<td>0.8</td>
<td>700</td>
<td>1.0</td>
<td>1.5</td>
<td>8.0</td>
<td>12.0</td>
</tr>
<tr>
<td>0.6</td>
<td>750</td>
<td>1.5</td>
<td>2.5</td>
<td>10.0</td>
<td>16.0</td>
</tr>
<tr>
<td>0.5</td>
<td>800</td>
<td>2.0</td>
<td>3.0</td>
<td>12.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

TABLE 10
A dynamic-programming model for production of a new chemical with no recycle including specific data for an example (Continued)

d. Reactor, I and catalyst costs

<table>
<thead>
<tr>
<th>Reactor</th>
<th>Initial cost, $1000</th>
<th>Operating cost, $1000/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>IA</td>
<td>40.0</td>
<td>4.0</td>
</tr>
<tr>
<td>IB</td>
<td>20.0</td>
<td>2.0</td>
</tr>
<tr>
<td>IC</td>
<td>5.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Catalyst</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>. . . . 10.0</td>
<td>. . . 4.0</td>
</tr>
<tr>
<td>2</td>
<td>. . . 10.0</td>
<td>. . . 4.0</td>
</tr>
</tbody>
</table>

e. Percent conversion in reactor I

<table>
<thead>
<tr>
<th>Temp., °F</th>
<th>650</th>
<th>700</th>
<th>750</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catalyst</td>
<td>I</td>
<td>2</td>
<td>I</td>
<td>2</td>
</tr>
<tr>
<td>IA</td>
<td>30</td>
<td>25</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>IB</td>
<td>25</td>
<td>20</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>IC</td>
<td>20</td>
<td>15</td>
<td>25</td>
<td>20</td>
</tr>
</tbody>
</table>

f. Total conversion from reactor I plus reactor II

<table>
<thead>
<tr>
<th>Conversion in reactor</th>
<th>I</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second reactor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IIA</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>80</td>
<td>85</td>
<td>90</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>IIb</td>
<td>45</td>
<td>60</td>
<td>75</td>
<td>85</td>
<td>90</td>
<td>95</td>
<td>95</td>
<td>95</td>
<td></td>
</tr>
</tbody>
</table>

g. Reactor II costs

<table>
<thead>
<tr>
<th>Reactor</th>
<th>Initial cost, $1000</th>
<th>Operating cost, $1000/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>II A</td>
<td>60.0</td>
<td>10.0</td>
</tr>
<tr>
<td>II B</td>
<td>80.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

h. Costs for the separation unit

<table>
<thead>
<tr>
<th>One large separator</th>
<th>Two small separators (cost per separator)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost, $1000</td>
<td>Operating cost, $1000/yr</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>40</td>
<td>12</td>
</tr>
<tr>
<td>45</td>
<td>12</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>60</td>
<td>15</td>
</tr>
<tr>
<td>75</td>
<td>20</td>
</tr>
<tr>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>85</td>
<td>20</td>
</tr>
<tr>
<td>90</td>
<td>20</td>
</tr>
<tr>
<td>95</td>
<td>20</td>
</tr>
</tbody>
</table>

i. Initial investment (4)

| Mixer A. | 10,000 |
| Mixer B. | 15,000 |
| Mixer C. | 25,000 |
| Heater:  | 700°F or less, 5,000 |
| More than 700°F. | 20,000 |
TABLE 11
Possible decisions, inputs, and outputs for the example presented in Table 10

<table>
<thead>
<tr>
<th>For stage number</th>
<th>Decisions</th>
<th>Output from stage = input to next stage (expressed as relevant variable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Type of mixer (that is, A, B, or C)</td>
<td>Mixing efficiency</td>
</tr>
<tr>
<td></td>
<td>Mixing efficiency (that is, $\eta = 1.0, 0.8, 0.6, \text{ or } 0.5$)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Temperature level at which heater operates (that is, 650, 700, 750, or 800°F)</td>
<td>Temperature</td>
</tr>
<tr>
<td>3</td>
<td>Type of reactor (that is, $I_A, I_B$, or $I_C$)</td>
<td>Percent conversion</td>
</tr>
<tr>
<td></td>
<td>Type of catalyst (that is, 1 or 2)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Type of reactor (that is, $II_A, II_B$, or none)</td>
<td>Percent conversion</td>
</tr>
<tr>
<td>1</td>
<td>Choice of separators (that is, one large or two small)</td>
<td></td>
</tr>
</tbody>
</table>

A five-year period of operation, and this unit will be used in the solution of the problem.

3. For each stage, determine the possible inputs, decisions, and outputs. These are shown in Table 11.

4. For each stage and for each combination of input decisions, establish the stage output.

5. Establish the optimal return from the overall process and from each stage by the application of the principle shown in Eq. (91).

Steps (1), (2), and (3) are completed, for the indicated example, in Tables 10 and 11. To carry out steps (4) and (5), it is necessary to assume a number of discrete levels for each of the decision variables. The size of the subdivisions for each decision variable, of course, represents an imposed constraint on the system solution, but these constraints are very useful for narrowing down the region which must receive the most careful attention for optimization.

The stage outputs are established in sequence for the subprocesses of stage 1, stage 1-stage 2, stage 1-stage 2-stage 3, stage 1-stage 2-stage 3-stage 4, and finally stage 1-stage 2-stage 3-stage 4-stage 5. The optimum is determined for each subprocess employing all the discrete levels chosen for the variables involved in that subprocess.
### Table 12
**One-stage profits,** $Q_1(x_2, d_1)$, $1000$

<table>
<thead>
<tr>
<th>$x_2$</th>
<th>Stage 1 input,</th>
<th>Stage 1 decision, $d_1$</th>
<th>One separator</th>
<th>Two separators</th>
</tr>
</thead>
<tbody>
<tr>
<td>% conversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>700.0</td>
<td>711.0*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>685.0</td>
<td>693.5*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>667.5</td>
<td>678.5*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>667.5</td>
<td>676.0*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>662.5</td>
<td>668.5*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>650.0</td>
<td>652.0*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>590.0</td>
<td>592.0*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>553.0</td>
<td>555.0*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>503.0*</td>
<td>500.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>388.0*</td>
<td>382.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 13
**Two-stage profits,** $Q_2(x_3, d_2)$, $1000$

<table>
<thead>
<tr>
<th>$x_3$</th>
<th>Stage 2 input,</th>
<th>Stage 2 decision, $d_2$</th>
<th>Stage 2 decisions, $d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% conversion</td>
<td></td>
<td>Reactor $\Pi_A$</td>
<td>Reactor $\Pi_B$</td>
</tr>
<tr>
<td>65</td>
<td>601.0</td>
<td>531.0</td>
<td>652.0*</td>
</tr>
<tr>
<td>50</td>
<td>583.5</td>
<td>531.0</td>
<td>592.0*</td>
</tr>
<tr>
<td>45</td>
<td>568.5*</td>
<td>531.0</td>
<td>555.0</td>
</tr>
<tr>
<td>40</td>
<td>566.0*</td>
<td>513.5</td>
<td>503.0</td>
</tr>
<tr>
<td>30</td>
<td>542.0*</td>
<td>498.5</td>
<td>388.0</td>
</tr>
<tr>
<td>25</td>
<td>482.0</td>
<td>488.5*</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>393.0</td>
<td>472.0*</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>278.0</td>
<td>375.0*</td>
<td></td>
</tr>
</tbody>
</table>

### Table 14
**Three-stage profits,** $Q_3(x_4, d_3)$, $1000$

<table>
<thead>
<tr>
<th>$x_4$, Stage 3 input,</th>
<th>Stage 3 decisions, $d_3$</th>
<th>Reactor $I_A$</th>
<th>Reactor $I_B$</th>
<th>Reactor $I_C$</th>
<th>Catalyst</th>
</tr>
</thead>
<tbody>
<tr>
<td>% conversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>542.0*</td>
<td>512.0</td>
<td>508.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>512.0</td>
<td>518.5</td>
<td>536.0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>482.0</td>
<td>488.5</td>
<td>506.0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>750</td>
<td>488.5</td>
<td>516.0*</td>
<td>512.0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>456.0</td>
<td>462.0*</td>
<td>428.5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>462.0*</td>
<td>438.5</td>
<td>442.0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>650</td>
<td>432.0*</td>
<td>408.5</td>
<td>412.0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>650</td>
<td>432.0*</td>
<td>422.0</td>
<td>345.0</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

### Table 15
**Four-stage profits,** $Q_4(x_5, d_4)$, $1000$

<table>
<thead>
<tr>
<th>$x_5$, Stage 4 input,</th>
<th>Stage 4 decisions, $d_4$</th>
<th>Mixing efficiency</th>
<th>Reactor $C$</th>
<th>Reactor $D$</th>
<th>Reactor $E$</th>
<th>Reactor $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% conversion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>750</td>
<td>700</td>
<td>650</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>472.0*</td>
<td>466.0</td>
<td>452.0</td>
<td>424.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>462.0*</td>
<td>456.0</td>
<td>449.5</td>
<td>422.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>442.0</td>
<td>446.0*</td>
<td>444.5</td>
<td>419.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>422.0</td>
<td>436.0</td>
<td>442.0*</td>
<td>417.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 16
**Five-stage profits,** $Q_5(x_F, d_5)$, $1000$

<table>
<thead>
<tr>
<th>Stage 5 decisions, $d_5$</th>
<th>Mixing efficiency</th>
<th>1.0</th>
<th>0.8</th>
<th>0.6</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixer</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>422.0</td>
<td>422.0</td>
<td>411.0</td>
<td>412.0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>417.0</td>
<td>427.0*</td>
<td>418.5</td>
<td>419.5</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>402.0</td>
<td>422.0</td>
<td>421.0</td>
<td>422.0</td>
<td></td>
</tr>
</tbody>
</table>
The subdivisions for the possible decisions in this example are shown by the data given in Table 10 and are summarized in Table 11. Thus, in stage 5, there are three possible decisions on the choice of mixer, and each of these has four possible efficiency decisions. In stage 4, there are four possible decisions on temperature level for the heater. In stage 3, there are three possible reactors and two possible catalysts. Stage 2 has three possible decisions of reactor II, II, or no reactor. In stage 1, there are two possible decisions of one large separator or two small separators. On an overall basis, therefore, the total possible modes of operation by a completely random approach would be

\[3 \times 4 \times 4 \times 3 \times 2 \times 3 \times 2 = 1728\]

By applying the technique of dynamic programming, the final optimum condition can be established by a stage-by-stage operation so that only about 15 modes of operation must be considered.

**SUBPROCESS OF STAGE 1**

For the dynamic-programming procedure involving only stage 1, it is desirable to base the analysis on final product sales with consideration of only the first stage. With this basis, all possible conversions of the entering stream must be considered. The data given in Table 10 show that at least 30 percent of the feed must be converted. This immediately indicates that the possibility of not including reactor II at stage 2 can only be considered if the conversion leaving reactor I is 30 percent or higher. Therefore, only those conversions of 30 percent or higher, as shown in Table 10f, need to be considered.

For each conversion (for example, for 50 percent conversion), the five-year profit can be evaluated for the cases of one large separator and two small separators. Therefore, using the data given in Table 10 and neglecting the cost of feed which is a constant,

Five-year profit using one large separator

\[= (5)(50,000)(0.5)(5.0) - 15,000 - (5)(4,000) = 590,000\]

Five-year profit using two small separators

\[= (5)(50,000)(0.5)(5.0) - (2)(9,000) - (2)(5)(1,500) = 592,000\]

This indicates that the optimal operation of stage 1 with a 50 percent conversion requires the use of two small separators. These calculations are repeated for all feasible conversions, and the results [i.e., the one-stage profits \(Q_1(x_2, d_1)\)] are presented in Table 12 with the optimum condition for each conversion indicated by an asterisk.

**SUBPROCESS OF STAGE 1-STAGE 2**

This subprocess involves making a decision on the type of reactor II (II, II, or none). Possible conversions for the feed entering stage 2 can be established from Table 10f or Table 10e as 15, 20, 25, 30, 40, 45, 50, or 60 percent. All of
these possibilities, including the decisions on conversion and reactor type, must be evaluated. Each result will give an exit conversion which represents the feed to stage 1, but the optimum condition for stage 1 has already been generated for the various feeds. Therefore, the sum of the optimum cost for stage 1 and the developed cost for stage 2 can be tabulated so that the optimum system for stage 1-stage 2 can be chosen for any appropriate feed to stage 2.

For example, if the stage-2 input conversion is chosen as 40 percent, the following data and calculations apply (neglecting cost of feed which remains constant):

Five-year profit using reactor II,

\[
\text{Optimum for stage 1 with 80\% conversion (Table 12)}
\]

\[
= \$676,000 - \$60,000 - (5)(\$10,000) = \$566,000
\]

Five-year profit using reactor II,

\[
\text{Optimum for stage 1 with 90\% conversion (Table 12)}
\]

\[
= \$693,500 - \$80,000 - (5)(\$20,000) = \$513,500
\]

Five-year profit using no reactor II = \$503,000

The preceding procedure can be repeated for all feasible combinations for the stage l-stage 2 process, and the results [i.e., the one-stage-two-stage profits \(Q_2(x_3, d_2)\)] are tabulated in Table 13.

REMAINING SUBPROCESSES AND FINAL SOLUTION

The same type of optimizing procedure can now be followed for each of the remaining three subsystems, and the results are presented in Tables 14, 15, and 16 with asterisks being used to indicate the optimum sets. The final optimum for the full process can now be established directly from Table 16 as giving a five-year profit of \$427,000. The stage-wise operations should be as follows:

Stage 5: From Table 16, a type B mixer with an efficiency of 80 percent should be used.

Stage 4: From Table 15, the heater should be operated at 800°F.

Stage 3: From Tables 14 and 10, reactor I, with catalyst 1 should be used giving a 60 percent conversion.

Stage 2: From Table 13, no reactor II should be used.

Stage 1: From Table 12, two small separators should be used.

The preceding example illustrates the technique used in dynamic programming. This technique permits a great saving in the amount of computational effort involved as is illustrated by the fact that the stage-by-stage optimization
approach used in the example involved consideration of only about 15 possible modes of operation. This can be compared to the total possible modes of operation of 1728 which would have had to be considered by a totally random approach.

OTHER MATHEMATICAL TECHNIQUES AND STRATEGIES FOR ESTABLISHING OPTIMUM CONDITIONS

Many mathematical techniques, in addition to the basic approaches already discussed, have been developed for application in various situations that require determination of optimum conditions. A summary of some of the other common and more advanced mathematical techniques, along with selected references for additional information, is presented in the following:

APPLICATION OF LAGRANGE MULTIPLIERS

When equality constraints or restrictions on certain variables exist in an optimization situation, a powerful analytical technique is the use of Lagrange multipliers. In many cases, the normal optimization procedure of setting the partial of the objective function with respect to each variable equal to zero and solving the resulting equations simultaneously becomes difficult or impossible mathematically. It may be much simpler to optimize by developing a Lagrange expression, which is then optimized in place of the real objective function.

In applying this technique, the Lagrange expression is defined as the real function to be optimized (i.e., the objective function) plus the product of the Lagrangian multiplier ($\lambda$) and the constraint. The number of Lagrangian multipliers must equal the number of constraints, and the constraint is in the form of an equation set equal to zero. To illustrate the application, consider the situation in which the aim is to find the positive value of variables $x$ and $y$ which make the product $xy$ a maximum under the constraint that $x^2 + y^2 = 10$.

For this simple case, the objective function is $xy$ and the constraining equation, set equal to zero, is $x^2 + y^2 - 10 = 0$. Thus, the Lagrange expression is

$$L.E. \ (x, y) = xy + \lambda (x^2 + y^2 - 10) \quad (92)$$


Method of steepest descent applied to unimodal surface.

Taking the partial of Eq. (92) with respect to \( x \), \( y \), and \( \lambda \), and setting each result equal to zero gives

\[
\begin{align*}
y + 2\lambda x &= 0 \quad (93) \\
x + 2\lambda y &= 0 \quad (94) \\
x^2 + y^2 - 10 &= 0 \quad (95)
\end{align*}
\]

Simultaneous solution of the preceding three equations for \( x \), \( y \), and \( \lambda \) gives, for the case where both \( x \) and \( y \) are positive, the optimum values of \( x \) equal to 2.24 and \( y \) equal to 2.24.

METHOD OF STEEPEST ASCENT OR DESCENT?

For the optimization situation in which two or more independent variables are involved, response surfaces can often be prepared to show the relationship among the variables. Figure 11-12 is an example of a unimodal response surface with a single minimum point. Many methods have been proposed for exploring such response surfaces to determine optimum conditions.

One of the early methods proposed for establishing optimum conditions from response surfaces is known as the method of steepest ascent or descent. The basis of this method is the establishment of a straight line or a two-dimensional plane which represents a restricted region of the curved surface. The gradient at the restricted region is then determined from the linearized approximation,

---

and the desired direction of the gradient is established as that linear direction giving the greatest change in the function being optimized relative to the change in one or more of the independent variables. If the objective function is to be maximized, the line of steepest ascent toward the maximum is sought. For the case of a minimum as the desired objective, the approach would be by means of the steepest descent.

To illustrate the basic ideas involved, consider the case where the objective function to be minimized \( C \) is represented by

\[
C = 2x^2 + y^2 + xy
\]  

(96)

where \( x \) and \( y \) are the independent variables. Equation (96) is plotted as a contour surface in Fig. 11-12, and the objective is to determine, by the method of steepest descent, the values of \( x \) and \( y \) which make \( C \) a minimum. Arbitrarily, a starting point of \( x = 2, y = 2, \) and \( C = 16 \) is chosen and designated as point \( S \) in Fig. 11-12. The gradient at point \( S \) is determined by taking the partial of \( C \) with respect to each of the independent variables to give

\[
\frac{\partial C}{\partial x} = 4x + y = (4)(2) + 2 = 10
\]

(97)

\[
\frac{\partial C}{\partial y} = 2y + x = (2)(2) + 2 = 6
\]

(98)

Both of these partials are positive which means that both \( x \) and \( y \) must change in the negative direction to head toward a minimum for \( C \). The direction to be taken is established by recognizing that \( C \) must change more rapidly in the \( x \) direction than in the \( y \) direction in direct ratio to the partial derivatives. Thus, \( x \) should decrease faster than \( y \) in the ratio of (decrease in \( x \))/(decrease in \( y \)) = 10/6. Assume, arbitrarily, to decrease \( x \) linearly from point \( S \) in increments of 0.5. Then \( y \) must decrease in increments of \((0.5) \times \frac{6}{10} = 0.3\). Under these conditions, the first line of steepest descent is found as follows and is shown as line \( SD \) in Fig. 11-12.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>2.00</td>
<td>16.00</td>
</tr>
<tr>
<td>1.50</td>
<td>1.70</td>
<td>9.94</td>
</tr>
<tr>
<td>1.00</td>
<td>1.40</td>
<td>5.36</td>
</tr>
<tr>
<td>0.50</td>
<td>1.10</td>
<td>2.26</td>
</tr>
<tr>
<td>0.00</td>
<td>0.80</td>
<td>0.64</td>
</tr>
<tr>
<td>-0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>-1.00</td>
<td>0.20</td>
<td>1.84</td>
</tr>
</tbody>
</table>

The minimum for line \( SD \) occurs at \( x_5, y_5 \); so a new line is now established using point \( x_5, y_5 \) as the starting point. Using the same procedure as was followed for finding line \( SD \), the line \( PQ \) is found with a minimum at \( L \). Thus, point \( L \) now becomes the new starting point. This same linearization procedure
is repeated with each line getting closer to the true minimum of $C = 0, x = 0, y = 0$.

The method outlined in the preceding obviously can become very tedious mathematically, and a computer solution is normally necessary. The method also has limitations based on choice of scale and incremental steps for the variables, extrapolation past the region where the straight line approximates the surface, and inability to handle surfaces that are not unimodal.

EXPLORATION OF RESPONSE SURFACES BY GROUP EXPERIMENTS?

In addition to the method of steepest ascent and descent, many other strategies for exploring response surfaces which represent objective functions have been proposed. Many of these are based on making group experiments or calculations in such a way that the results allow a planned search of the surface to approach quickly a unimodal optimum point.

A typical example of an efficient search technique by group experiments is known as the Five-Point Method and is explained in the following. The basis of this method is first to select the overall range of the surface to be examined and then to determine the values of the objective function at both extremes of the surface and at three other points at equally spaced intervals across the surface. Figure 11-13 shows a typical result for these initial five points for a simplified two-dimensional case in which only one maximum or minimum is involved.

From these first five calculations, it can be seen that, by keeping the optimum point and the point on each side of it, the search area can be cut in half with assurance that the remaining area still contains the optimum value. In Fig. 11-13, the optimum is represented by the maximum profit, so the middle half of the search area is retained.

Two more calculations or experiments are then made in the remaining search area with these points again being equally spaced so that the remaining search area is again divided into four equal portions. As before, the optimum (highest profit) point is kept along with the points on each side of it, so the search area is again cut in half.

This procedure can be repeated to reduce the search area by a large amount with a relatively few calculations. For example, as shown in the following, 99.9 percent of the search area can be eliminated by a total of only 23 calculations.

---

FIGURE 11-13
Illustration of Five-Point Method for group-experiment exploration of response surface.

calculations or experiments:

<table>
<thead>
<tr>
<th>For step number</th>
<th>Number of new calculations</th>
<th>Total calculations</th>
<th>Fraction of region isolated = A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>7</td>
<td>$(\frac{1}{2})^2$</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>9</td>
<td>$(\frac{1}{2})^3$</td>
</tr>
</tbody>
</table>

$m_s = \frac{2}{(\frac{1}{2})^{m_s}} = \frac{\log \Delta}{-\log 2} = \frac{n_c - 3}{2}$, and $n_c = 3 - \frac{2 \log \Delta}{\log 2}$

For the case where $A$ is 0.001, or 99.9 percent of the surface has been eliminated, Eq. (99) gives the number of calculations needed ($n_c$) as 23.

A similar approach is used in the **Golden Section Search Technique** which uses as its basis a symmetrical placement of search points located at an arbitrary distance from each side of the search area.† This method can eliminate 99.9

percent of the search area by a total of 17 search points as compared to 23 search points for the simple Five-Point Method.

A so-called dichotomous search for the optimum on a surface representing an objective function is conducted by performing the experiments or calculations in pairs. By locating the pairs at appropriate intervals over the surface, inappropriate regions can be eliminated quickly, and a sequential technique can be developed to permit rapid elimination of major portions of the surface. Similarly, the simplex method, based on a triangulation of experimental or calculated points, can be used to indicate the desired direction of a search. A highly effective sequential search technique, known as the Fibonacci search because the search sequence is based on Fibonacci numbers, can be employed when the objective function has only one optimum and is based on a single independent variable. Experimental errors involved in analyzing response surfaces can be eliminated partially by a so-called evolutionary operations (EVOP) technique based on measuring the response to the operating conditions a sufficient number of times so that the mean of the sample response approaches the true mean.

GEOMETRIC PROGRAMMING?

A technique for optimization, based on the inequality relating the arithmetic mean to the geometric mean for a set of numbers, has been called geometric programming. With this method, the basic idea is to start by finding the optimum way to distribute the total cost among the various factors of the objective function. This is then followed by an analysis of the optimal distribution to establish the final optimum for the objective function. Although this approach can become very involved mathematically and may involve nonlinear equations, it can handle equality and inequality constraints and can often be simpler than a direct nonlinear-programming approach.

OPTIMUM CONDITIONS FOR PRODUCTION, PLANNING, SCHEDULING, AND CONTROLS

A number of special numerical techniques have been developed for effective planning, scheduling, and control of projects. Two of these methods, critical

---


path method (CPM) and program evaluation and review technique (PERT) have received particular attention and have shown the desirability of applying mathematical and graphical analyses to the planning and control of production processes.

The basis of both the critical path method and program evaluation and review technique is a graphical portrait, or network, showing the interdependencies of the various activities in the program leading from the initial input, or startup, to the end objective. PERT is of primary use for organizing and planning projects that involve research and development wherein the activities are usually being attempted for the first time. As a result, estimates of time, cost, and results cannot be made with accuracy, and probability and statistical concepts must be used to develop the predictions. In comparison, CPM is usually applied to projects for which relatively accurate estimations of time, cost, and results can be made, such as for construction projects.

For both CPM and PERT, the overall project is viewed as a series of activities or operations performed in an optimum sequence to reach a desired objective. Each activity is considered as having a beginning and an end so that the overall project consists of a series of these “events.” The general technique, then, is to develop a mathematical model to give the best program or interrelated series of events to achieve a desired goal. The major difference in concept between CPM and PERT is that involved in estimating the time duration of activities. Thus, CPM may be relatively specific on time items, while PERT includes measures of the uncertainties involved.

When the series of activities is diagrammed, it can be seen that many possible paths exist between the “start” and the “end.” The “critical path” is defined as that path involving the desired (usually shortest) duration for completion of the project. The mathematical concepts of both PERT and CPM are normally of sufficient complexity that a digital computer must be used for the solution. By the appropriate network computations, a final sequential procedure is developed which gives the “critical path” that must be followed from the “start” to the “end” to complete the job in the most efficient manner in a given duration of time.

THE STRATEGY OF ACCOUNTING FOR INFLATION IN DESIGN ESTIMATES

The method of correcting for price changes that have occurred in the past when estimating costs for design purposes has been discussed in Chap. 6 (Cost Estimation). As this discussion showed, the history of cost changes in the United States in the recent past has been strongly inflationary. For example, the Marshall and Swift All-Industry Installed-Equipment Cost Index doubled from 273 in 1968 to 545 in 1978. In the ten-year period from 1978 to 1988, the index
increased by about 60 percent to 852. Other price indexes showed about the same factors of increase over these time intervals.

An effective interest rate of 7.18 percent will cause a doubling of value when compounded for 10 years while a 5 percent rate will give a 63 percent increase in 10 years and a 4 percent rate will give a 48 percent increase in 10 years. Consequently, past history of price changes in the United States would indicate that a rate of inflation of at least 3 percent and perhaps as high as 7 percent can be expected for at least the near future, and this factor should be taken into account in presenting design estimates of cost.

The critical element of the strategy for accounting for inflation in design estimates is to present the results in the form of present worth (present value, profitability index, discounted cash flow) with all future dollars discounted to the value of the present dollar at zero time. The discount factor must include both the interest required by the company as minimum return and the estimated interest rate of inflation. If profits on which income taxes are charged are involved, then the present worth based on the after-tax situation should be used.

In order to understand which form of discount factor to use with inflation (or with deflation), the two specific cases for constant annual income in the future and constant annual productivity in the future will be considered. In all cases, effective interest and instantaneous end-of-year cash flow will be assumed.

**CASE OF CONSTANT ANNUAL INCOME IN THE FUTURE**

Assume that a firm wishes to make an investment now to provide $100,000 in cash at the end of each year for the next ten years. The firm expects to receive a 10 percent return \((i = 0.10)\) on its investment irrespective of inflation effects. However, the firm also wishes to account for an assumed annual inflation of 7 percent \((i_{\text{inflation}} = 0.07)\) so that the dollars its invest now are corrected for the fact that these dollars will be worth less in the future. Under these conditions, the question is how to establish the correct discount factor to determine the investment the firm needs to make at this time. In other words, what is the total present value of the future annual incomes of $100,000 for 10 years discounted for both return on investment and inflation?

Consider the case of the first $100,000 coming in at the end of the first year. The present value at zero time of this $100,000 based only on the need to keep the purchasing power of the dollar constant by correcting for inflation is \((\$100,000)(1 + 0.07)^{-1}\) or, in general, \((\$100,000)(1 + i_{\text{inflation}})^{-n'}\) where \(n'\) is the year referred to. In addition, the firm demands a 10 percent direct return on the investment; so an additional discount factor of \((1 + 0.10)^{-1}\) or, in general, \((1 + i)^{-n'}\) must be applied to the annual income value to give its present value at zero time. Thus, the zero-time present value of the first $100,000 is \((\$100,000) X (1 + 0.07)^{-1}(1 + 0.10)^{-1}\). The total present value at
zero time of all the annual incomes is merely the following sum:

\[
\begin{align*}
\text{For first year} & \quad (100,000)(1 + 0.07)^{-1}(1 + 0.10)^{-1} \\
\text{For second year} & \quad (100,000)(1 + 0.07)^{-2}(1 + 0.10)^{-2} \\
\text{For third year} & \quad (100,000)(1 + 0.07)^{-3}(1 + 0.10)^{-3} \\
\vdots & \quad \vdots \\
\text{For tenth year} & \quad (100,000)(1 + 0.07)^{-10}(1 + 0.10)^{-10}
\end{align*}
\]

or, in general

The total present value = \( \sum_{n=1}^{10} (100,000)(1 + i_{\text{inflation}})^{-n}(1 + i)^{-n'} \)

The effective discount factor including both inflation and required return on investment is \([(1 + i_{\text{inflation}})(1 + i)]^{-n'}\) or \([1 + i + i_{\text{inflation}} + (i_{\text{inflation}})(i)]^{-n'}\). Consequently, the effective combined interest \(i_{\text{comb}}\) including both inflation interest and required return on investment is

\[ i_{\text{comb}} = i + i_{\text{inflation}} + (i_{\text{inflation}})(i) \quad (100) \]

The preceding situation, of course, is merely a case of an ordinary annuity \((R = 100,000\) each year\) at an interest rate of \(i_{\text{comb}}\) so that Eq. (24) of Chap. 7 (Interest and Investment Costs) applies as follows:

\[ \text{Present value} = R \frac{(1 + i_{\text{comb}})^n - 1}{i_{\text{comb}}(1 + i_{\text{comb}})^n} \quad (101) \]

For the example under consideration, \(R = \text{annual periodic payment} = 100,000\), \(n = \text{total life period} = 10 \text{ years}\), \(i_{\text{comb}} = 0.10 + 0.07 + (0.07)(0.10) = 0.177\), and present value (or necessary investment now) = \(\frac{100,000[(1 + 0.177)^{-10} - 1]}{0.1770 + 0.177} = 452,240\).

**CASE OF CONSTANT ANNUAL PRODUCTIVITY IN THE FUTURE**

For the typical situation of an industrial operation which has been designed to produce a set number of units per year which will be sold at the prevailing price, there would be no special problem with handling inflation except for the influence of income taxes. If the inflationary costs are considered as having the same effects on the selling price of the product as on the costs for the operation, then return on investment before taxes is the same whether or not inflation is
Example 7 Return on investment before and after taxes with and without inflation. An investment of $1,000,000 will give annual returns as shown in the following over a life of five years. Assume straight-line depreciation, negligible salvage value, and 34 percent income taxes. What is the discounted-cash-flow rate of return on the investment (Profitability Index) before and after taxes with

(a) No inflation and annual returns of $300,000 each year (i.e., cash flow to the company of $300,000) before taxes?

(b) Inflation rate of 7 percent ($i_{\text{inflation}} = 0.07$) and a situation where the increase in profits due to inflation is also at an annual rate of 7 percent so the annual returns remain at the equivalent of $300,000 in zero-time dollars before taxes?

Solution

(a) For the case of no inflation, Eq. (24) of Chap. 7 [or Eq. (101) of this chapter with $i_{\text{comb}} = i$] applies as follows:

$$\text{Present value} = \frac{R (1 + i)^n - 1}{i (1 + i)^n}$$

For return on investment ($i$) before taxes,

$1,000,000 = \frac{300,000 (1 + i)^5 - 1}{i (1 + i)^5}$

By trial and error, or by use of tables of $[(1 + i)^n - 1]/i(1 + i)^n$,

$i = 0.152$ or 15.2% return

For return on investment ($i$) after taxes,

Depreciation = $\frac{1,000,000 - 0}{5} = $200,000 per year

Taxable income = $300,000 - $200,000 = $100,000 per year

Taxes at 34% rate = $34,000 per year

Annual cash flow = $300,000 - $34,000 = $266,000

$1,000,000 = \frac{266,000 (1 + i')^5 - 1}{i (1 + i')^5}$

$i = 0.1033$ or 10.33% return
(b) For the case of 7% inflation

For return on investment \((i)\) before taxes, the actual annual return based on zero-time dollars is $300,000; so the return on the investment is exactly the same as for the case of no inflation, and \(i = 0.152\) or 15.2% return.

For return on investment \((i)\) after taxes, the annual cash flows based on zero-time dollars must have a total present value of $1,000,000. As is shown in the following tabulation, this occurs for a value of \(i = 0.0859\):

<table>
<thead>
<tr>
<th>Year (n')</th>
<th>Before-tax annual return based on zero-time dollars (= 300,000)</th>
<th>Actual dollars received at 7% inflation ((1 + 0.07)^{n'})</th>
<th>Depreciation income</th>
<th>Taxable tax at 34%</th>
<th>Income tax flow</th>
<th>After-tax annual cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$300,000</td>
<td>$321,000</td>
<td>$200,000</td>
<td>$121,000</td>
<td>$41,140</td>
<td>$279,860</td>
</tr>
<tr>
<td>2</td>
<td>$300,000</td>
<td>$343,470</td>
<td>$200,000</td>
<td>$143,470</td>
<td>$48,780</td>
<td>$294,690</td>
</tr>
<tr>
<td>3</td>
<td>$300,000</td>
<td>$367,513</td>
<td>$200,000</td>
<td>$167,513</td>
<td>$56,954</td>
<td>$310,559</td>
</tr>
<tr>
<td>4</td>
<td>$300,000</td>
<td>$393,239</td>
<td>$200,000</td>
<td>$193,239</td>
<td>$65,701</td>
<td>$327,538</td>
</tr>
<tr>
<td>5</td>
<td>$300,000</td>
<td>$420,766</td>
<td>$200,000</td>
<td>$220,766</td>
<td>$75,060</td>
<td>$345,706</td>
</tr>
</tbody>
</table>

Inflation adjustment: 
\[(1 + 0.07)^{n'}\]

Inflation plus return adjustment: 
\[= A (1 + 0.07)^{-n'} \]

Total present value = $1,000,000

Under these conditions, the return on investment after taxes with a 7% inflation rate is \(i = 0.0859\) or 8.59% return.

Thus, as would be expected because profits for the inflation case increased at the same rate as the inflation, the before-tax return on the investment was the same for the cases with or without inflation at 15.2%. However, due to the depreciation costs remaining constant in the case of inflation, the after-tax return on the investment was different for the no-inflation case (10.33%) and the inflation case (8.59%).

The preceding example clearly shows that inflation effects can be important in determining returns on investment. The best strategy for handling such effects is to use the discounted-cash-flow or present-worth method for reporting returns on investment with the results based on the after-tax situation. This method of reporting can be handled easily and effectively by use of an appropriate...
NOMENCLATURE FOR CHAPTER 11

\( a = \) constant, or depreciation factor for installed piping system [See Eq. (82) for definition of \( a_i \)]

\( a' = \) depreciation factor for pumping installation

\( A = \) heat-transfer area, \( \text{ft}^2 \)

\( b = \) constant, or maintenance factor for installed piping system [See Eq. (82) for definition of \( b_i \)]

\( b' = \) maintenance factor for pumping installation

\( B = \) constant

\( B' = \) constant

\( c = \) constant

\( c_i = \) objective-function coefficient for solution variable

\( c_j = \) objective-function coefficient for row in simplex algorithm matrix

\( c_p = \) heat capacity, \( \text{Btu} / (\text{lb} \times ^\circ \text{F}) \)

\( c_{\text{pipe}} = \) purchase cost of new pipe per foot of pipe length, \$/ft

\( C_T = \) total cost per unit of production, \$/unit of production

\( C = \) cost, or objective function

\( C_A = \) installed cost of heat exchanger per square foot of heat-transfer area, \$/ft^2

\( C_c = \) cost for one cleaning, dollars

\( C_F = \) fixed costs, \$/year

\( C_{\text{pipe}} = \) installed cost for piping system expressed as dollars per year per foot of pipe length, \$/year*ft

\( C_{\text{pumping}} = \) pumping cost as dollars per year per foot of pipe length when flow is turbulent, \$/year*ft

\( C'_{\text{pumping}} = \) pumping cost as dollars per year per foot of pipe length when flow is viscous, \$/year*ft

\( C_T = \) total cost for a given unit of time, dollars

\( C_w = \) cost of cooling water, \$/lb

\( d = \) constant, or derivative, or design decision for dynamic programming

\( D = \) inside diameter of pipe, ft, or determinant

\( D_i = \) inside diameter of pipe, in.

\( E = \) efficiency of motor and pump expressed as a fraction

\( f = \) Fanning friction factor, dimensionless, or function for dynamic programming in Eq. (91) indicating optimum return depends on that input

\( f^\dagger = \) Fanning friction factor, dimensionless, or function for dynamic programming in Eq. (91) indicating optimum return depends on that input

\( F \) = ratio of total cost for fittings and installation to purchase cost for new pipe

\( g \) = function

\( g_c \) = conversion factor in Newton’s law of motion, 32.17 \( \text{ft} \) \( \text{lbm} / (s)^2 \text{lbf} \)

\( h \) = operating costs which remain constant per unit of production, \$/unit of production, or function

\( i \) = annual effective interest rate of return, percent/100

\( i_{\text{comb}} \) = annual effective interest rate of change combining regular return and inflation estimate, percent/100

\( i_{\text{inflation}} \) = annual effective interest rate of change based on inflation estimate, percent/100

\( H \) = total time used for actual operation, emptying, cleaning, and recharging, \( h \)

\( H' \) = total time available for operation, emptying, cleaning, and recharging, \( h \)

\( H_y \) = total time of operation per year, \( h/\text{year} \)

\( I \) = row

\( j \) = constant

\( J \) = frictional loss due to fittings and bends, expressed as equivalent fractional loss in a straight pipe, or column

\( k \) = designation for column in simplex algorithm matrix with maximum value of \( c_j - z_j \)

\( K \) = cost of electrical energy, \$/kWh

\( K_F \) = annual fixed charges including maintenance, expressed as a fraction of the initial cost for the completely installed equipment

\( L \) = length of pipe, \( \text{ft} \)

\( L' \) = frictional loss due to fittings and bends, expressed as equivalent pipe length in pipe diameters per unit length of pipe

\( m \) = constant, or number of independent equations

\( m_s \) = number of steps

\( n^4 \) = ratio of total cost for pumping installation to yearly cost of pumping power required, \$/$

\( M_B \) = bottoms flow rate, \( \text{mol/h} \)

\( M_D \) = distillate flow rate, \( \text{mol/h} \)

\( n \) = constant, estimated service life, or number of unknowns or stages

\( n' \) = year of project life to which cash flow applies

\( n_c \) = number of calculations

\( N_{Re} \) = Reynolds number = \( DV_p / \mu \), dimensionless

\( O \) = organization costs per unit of time, \$/day

\( \hat{P} \) = rate of production, units of production/day, or return as objective function in dynamic programming

\( P_b \) = amount of production per batch, \( \text{lb/batch} \)

\( P_f \) = filtrate delivered in, filtering time \( \theta_f \) \( \text{h} \), \( \text{ft}^3 \)

\( P_o \) = optimum rate of production, units of production/day
\[ q = \text{rate of heat transfer, Btu/h} \]
\[ q_f = \text{rate of fluid flow, ft}^3/\text{s} \]
\[ q_r = \text{rate of heat transfer in reboiler, Btu/h} \]
\[ Q = \text{total amount of heat transferred in a given time, Btu} \]
\[ Q_H = \text{total amount of heat transferred in } H \text{ h, Btu} \]
\[ Q_i = \text{function for dynamic programming indicating combined return} \]
\[ \text{from all stages} \]
\[ r = \text{profit per unit of production, } \$/\text{unit of production} \]
\[ R = \text{annual periodic payment in ordinary annuity, } \$/\text{year} \]
\[ R' = \text{profit per unit of time, } \$/\text{day} \]
\[ s = \text{selling price per unit of production, } \$/\text{unit of production} \]
\[ S = \text{slack variable} \]
\[ S_b = \text{direct labor cost per hour during operation, } \$/\text{h} \]
\[ t = \text{temperature, } ^\circ\text{F} \]
\[ t_e = \text{temperature of cooling water entering condenser, } ^\circ\text{F} \]
\[ t_i = \text{temperature of cooling water leaving condenser, } ^\circ\text{F} \]
\[ t' = \text{condensation temperature, } ^\circ\text{F} \]
\[ U = \text{overall coefficient of heat transfer, Btu}/(\text{h})(\text{ft}^2)(^\circ\text{F}) \]
\[ V = \text{average linear velocity, } \text{ft/s} \]
\[ V_A = \text{volume of } A, \text{ gal} \]
\[ V_B = \text{volume of } B, \text{ gal} \]
\[ V_W = \text{volume of water, gal} \]
\[ w = \text{flow rate, lb/h} \]
\[ w_m = \text{thousands of pounds mass flowing per hour, } 1000 \text{ lb/h} \]
\[ w_s = \text{pounds mass flowing per second, lb/s} \]
\[ x = \text{a variable} \]
\[ X = \text{purchase cost for new pipe per foot of pipe length if pipe} \]
\[ \text{diameter is 1 in., } \$/\text{ft} \]
\[ X' = \text{purchase cost for new pipe per foot of pipe length if pipe} \]
\[ \text{diameter is 1 ft, } \$/\text{ft} \]
\[ y = \text{a variable} \]
\[ Y = \text{days of operation per year, days/yr} \]
\[ z = \text{a variable} \]
\[ z_j = \text{objective-function row coefficient component for simplex algo-} \]
\[ \text{rithm matrix defined as } \sum_{j=1}^m c_i a_{ij} (j = 1, 2, \ldots, n) \]
\[ Z = \text{fractional rate of return on incremental investment} \]

**Greek symbols**

\[ \alpha = \text{symbol meaning go to next starting point} \]
\[ A = \text{fraction of search area eliminated} \]
\[ At = \text{temperature-difference driving force (subscript } lm \text{ designates log mean), } ^\circ\text{F} \]
\[ \theta_b = \text{time in operation, h or h/cycle} \]
\[ \theta_c = \text{time for emptying, cleaning, and recharging per cycle, h/cycle} \]
\[ \theta_f = \text{filtering time, h or h/cycle} \]
\[ \theta_i = \text{total time per complete cycle, h/cycle} \]
\[ A = \text{Lagrangian multiplier} \]
\[ \mu = \text{absolute viscosity, lbm/(s)(ft)} \]
\[ \mu_e = \text{absolute viscosity, cP} \]
\[ \rho = \text{density, lbm/ft}^3 \]
\[ \phi, \phi^i, \phi^{ii}, \phi^{iii} = \text{function of the indicated variables, or fractional factor for rate of taxation} \]

**PROBLEMS**

1. A multiple-effect evaporator is to be used for evaporating 400,000 lb of water per day from a salt solution. The total initial cost for the first effect is $18,000, and each additional effect costs $15,000. The life period is estimated to be 10 years, and the salvage or scrap value at the end of the life period may be assumed to be zero. The straight-line depreciation method is used. Fixed charges minus depreciation are 15 percent yearly based on the first cost of the equipment. Steam costs $1.50 per 1000 lb. Annual maintenance charges are 5 percent of the initial equipment cost. All other costs are independent of the number of effects. The unit will operate 300 days per year. If the pounds of water evaporated per pound of steam equals 0.85 \( x \) number of effects, determine the optimum number of effects for minimum annual cost.

2. Determine the optimum economic thickness of insulation that should be used under the following conditions: Saturated steam is being passed continuously through a steel pipe with an outside diameter of 10.75 in. The temperature of the steam is 400°F, and the steam is valued at $1.80 per 1000 lb. The pipe is to be insulated with a material that has a thermal conductivity of 0.03 Btu/(h)(ft²)(°F/ft). The cost of the installed insulation per foot of pipe length is $4.5 \( x \) \( I_o \), where \( I_o \) is the thickness of the insulation in inches. Annual fixed charges including maintenance amount to 20 percent of the initial installed cost. The total length of the pipe is 1000 ft, and the average temperature of the surroundings may be taken as 70°F. Heat-transfer resistances due to the steam film, scale, and pipe wall are negligible. The air-film coefficient at the outside of the insulation may be assumed constant at 2.0 Btu/(h)(ft²)(°F) for all insulation thicknesses.

3. An absorption tower containing wooden grids is to be used for absorbing SO₂ in a sodium sulfite solution. A mixture of air and SO₂ will enter the tower at a rate of 70,000 ft³/min, temperature of 250°F, and pressure of 1.1 atm. The concentration of SO₂ in the entering gas is specified, and a given fraction of the entering SO₂ must be removed in the absorption tower. The molecular weight of the entering gas mixture may be assumed to be 29.1. Under the specified design conditions, the number of transfer units necessary varies with the superficial gas velocity as follows:

\[ \text{Number of transfer units} = 0.32G_x^{0.18} \]

where \( G_x \) is the entering gas velocity as lb/(h)(ft²) based on the cross-sectional area of the empty tower. The height of a transfer unit is constant at 15 ft. The cost for the installed tower is $1 per cubic foot of inside volume, and annual tied charges amount to 20 percent of the initial cost. Variable operating charges for the absorbent, blower power, and pumping power are represented by the following
equation:

\[
\text{Total variable operating costs as } \$/h = 1.8G_x^2 \times 10^{-8} + \frac{81}{G_s} + \frac{4.8}{G_s^{0.8}}
\]

The unit is to operate 8000 h/year. Determine the height and diameter of the absorption tower at conditions of minimum annual cost.

4. Derive an expression for the optimum economic thickness of insulation to put on a flat surface if the annual fixed charges per square foot of insulation are directly proportional to the thickness, (a) neglecting the air film, (b) including the air film. The air-film coefficient of heat transfer may be assumed as constant for all insulation thicknesses.

5. A continuous evaporator is operated with a given feed material under conditions in which the concentration of the product remains constant. The feed rate at the start of a cycle after the tubes have been cleaned has been found to be 5000 kg/h. After 48 h of continuous operation, tests have shown that the feed rate decreases to 2500 kg/h. The reduction in capacity is due to true scale formation. If the down time per cycle for emptying, cleaning, and recharging is 6 h, how long should the evaporator be operated between cleanings in order to obtain the maximum amount of product per 30 days?

6. A solvent-extraction operation is carried out continuously in a plate column with gravity flow. The unit is operated 24 h/day. A feed rate of 1500 ft\(^3\)/day must be handled 300 days per year. The allowable velocity per square foot of cross-sectional tower area is 40 ft\(^3\)/h. Operating and other variable costs depend on the amount of solvent that must be recovered, and these costs are $0.04 for each cubic foot of solvent passing through the tower. What tower diameter should be used for optimum conditions of minimum total cost per year?

7. Prepare a plot of optimum economic pipe diameter versus the flow rate of fluid in the pipe under the following conditions:

Costs and operating conditions ordinarily applicable in industry may be used.

The flow of the fluid may be considered as in the turbulent range.

The viscosity of the fluid may range from 0.1 to 20 centipoises.

The plot is to apply for steel pipe.

Express the diameters in inches and use inside diameters.

The plot should cover a diameter range of 0.1 to 100 in.

Express the flow rate in 1000 lb/h.

The plot should cover a flow-rate range of 10 to 100,000 lb/h.

The plot should be presented on log-log coordinates.

One line on the plot should be presented for each of the following fluid densities: 100, 50, 10, 1, 0.1, 0.01, and 0.001 lb/ft\(^3\).

8. For the conditions indicated in Prob. 7, prepare a log-log plot of fluid velocity in feet per second versus optimum economic pipe diameter in inches. The plot should cover a fluid-velocity range of 1 to 100 ft/s and a pipe-diameter range of 1 to 10 in.
9. A continuous evaporator is being used to concentrate a scale-forming solution of sodium sulfate in water. The overall coefficient of heat transfer decreases according to the following expression:

\[ \frac{1}{U} = 8 \times 10^{-6} \theta_b + 6 \times 10^{-6} \]

where \( U \) = overall coefficient of heat transfer, \( \text{Btu/(hXft^2)(^\circ\text{F})} \), and \( \theta_b \) = time in operation, h.

The only factor which affects the overall coefficient is the scale formation. The liquid enters the evaporator at the boiling point, and the temperature and heat of vaporization are constant. At the operating conditions, 990 Btu are required to vaporize 1 lb of water, the heat-transfer area is 400 \( \text{ft}^2 \), and the temperature-difference driving force is 70°F. The time required to shut down, clean, and get back on stream is 4 h for each shutdown, and the total cost for this cleaning operation is $100 per cycle. The labor costs during operation of the evaporator are $20 per hour. Determine the total time per cycle for minimum total cost under the following conditions:

(a) An overall average of 65,000 lb of water per 24-h day must be evaporated during each 30-day period.

(b) An overall average of 81,000 lb of water per 24-h day must be evaporated during each 30-day period.

10. An organic chemical is produced by a batch process. In this process, chemicals \( X \) and \( Y \) react to form chemical \( Z \). Since the reaction rate is very high, the total time required per batch has been found to be independent of the amounts of the materials, and each batch requires 2 h, including time for charging, heating, and dumping. The following equation shows the relation between the pounds of \( Z \) produced (lb.) and the pounds of \( X \) (lb.) and \( Y \) (lb.) supplied:

\[ \text{lb. } Z = 1.5 (1.1 \text{ lb. } X + 1.3 \text{ lb. } Y - \text{ lb. } Y) \]

Chemical \( X \) costs $0.09 per pound. Chemical \( Y \) costs $0.04 per pound. Chemical \( Z \) sells for $0.80 per pound. If one-half of the selling price for chemical \( Z \) is due to costs other than for raw materials, what is the maximum profit obtainable per pound of chemical \( Z \)?

11. Derive an expression similar to Eq. (56) for finding the optimum exit temperature of cooling water from a heat exchanger when the temperature of the material being cooled is not constant. Designate the true temperature-difference driving force by \( F_G \Delta t_{im} \), where \( F_G \) is a correction factor with value dependent on the geometrical arrangement of the passes in the exchanger. Use primes to designate the temperature of the material that is being cooled.

12. Under the following conditions, determine the optimum economic thickness of insulation for a 1\( \frac{1}{4} \)-in. standard pipe carrying saturated steam at 100 psig. The line is in use continuously. The covering specified is light carbonate magnesia, which is marketed in whole-number thicknesses only (i.e., 1 in., 2 in., 3 in., etc.). The cost of the installed insulation may be approximated as $20 per cubic foot of insulation. Annual fixed charges are 20 percent of the initial investment, and the heat of the steam is valued at $1.50 per 1 million Btu. The temperature of the surroundings may be assumed to be 80°F.
L. B. McMillan, Trans. ASME, 48:1269 (1926), has presented approximate values of optimum economic insulation thickness versus the group 
\[(kb_cH_y \Delta t/a_c)^{0.5}\]
with pipe size as a parameter.

\[k = \text{thermal conductivity of insulation, } \text{Btu}/(\text{h})(\text{ft}^2)(\text{°F}/\text{ft})\]

\[b_c = \text{cost of heat, } \$/\text{Btu}\]

\[H_y = \text{hours of operation per year, h/year}\]

\[\Delta t = \text{overall temperature-difference driving force, } \text{°F}\]

\[a_c = \text{cost of insulation, } \$/\text{(ft}^3)\text{(year)}\]

The following data are based on the results of McMillan, and these data are applicable to the conditions of this problem:

<table>
<thead>
<tr>
<th>( (kb_cH_y \Delta t/a_c)^{0.5} )</th>
<th>( \frac{1}{4} \text{ in.} )</th>
<th>1 in.</th>
<th>2 in.</th>
<th>4 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>\ldots</td>
<td>0.40</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>0.2</td>
<td>0.80</td>
<td>0.95</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>0.3</td>
<td>1.20</td>
<td>1.4</td>
<td>1.6</td>
<td>1.9</td>
</tr>
<tr>
<td>0.5</td>
<td>1.8</td>
<td>2.1</td>
<td>2.45</td>
<td>2.9</td>
</tr>
<tr>
<td>0.8</td>
<td>2.75</td>
<td>3.1</td>
<td>3.6</td>
<td>4.3</td>
</tr>
<tr>
<td>1.2</td>
<td>3.80</td>
<td>4.3</td>
<td>4.9</td>
<td></td>
</tr>
</tbody>
</table>

13. A catalytic process uses a catalyst which must be regenerated periodically because of reduction in conversion efficiency. The cost for one regeneration is constant at $800. This figure includes all shutdown and startup costs, as well as the cost for the actual regeneration. The feed rate to the reactor is maintained constant at 150 lb/day, and the cost for the feed material is $2.50 per pound. The daily costs for operation are $300, and fixed charges plus general overhead costs are $100,090 per year. Tests on the catalyst show that the yield of product as pounds of product per pound of feed during the first day of regeneration with the regenerated catalyst is 0.87, and the yield decreases as \(0.87/(\theta_D)^{0.25}\), where \(\theta_D\) is the time in operation expressed in days. The time necessary to shut down the unit, replace the catalyst, and start up the unit is negligible. The value of the product is $14.08 per pound, and the plant operates 300 days per year. Assuming no costs are involved other than those mentioned, what is the maximum annual profit that can be obtained under these conditions?

14. Derive the following equation for the optimum outside diameter of insulation on a wire for maximum heat loss:

\[D_{\text{opt}} = \frac{2k_m}{(h_c + h_i)}\]

where \(k_m\) is the mean thermal conductivity of the insulation and \((h_c + h_i)_c\) is the combined and constant surface heat-transfer coefficient. The values of \(k_m\) and \((h_c + h_i)_c\) can be considered as constants independent of temperature level and insulation thickness.

16. Using a direct partial derivative approach for the objective function, instead of the Lagrangian multiplier as was used in Eqs. (92) to (95), determine the optimum values of x and y involved in Eqs. (92) to (95).

17. Find the values of x, y, and z that minimize the function \( x + 2y^2 + z^2 \) subject to the constraint that \( x + y + z = 1 \), making use of the Lagrangian multiplier.

18. For the mixing problem referred to in Tables 3 and 4 of this chapter, present the computer solution as
   (a) The computer diagram (similar to Table 4) based on the logic given in Table 3.
   (b) The computer program (Fortran language preferred).
   (c) The printout of the computer solution giving the minimum value of the objective function and the corresponding values of the variables.
   (d) The interpretation of the computer solution.

19. For the linear-programming example problem presented in this chapter where the simultaneous-equation solution is presented in Table 5, solve the problem using the simplex algorithm as was done in the text for the example solved in Fig. 11-10. Use as the initial feasible starting solution the case of solution 2 in Table 5 where \( x_2 = S_4 = 0 \). Note that this starting point should send the solution directly to the optimum point (solution 6) for the second trial.

20. From the data given for the dynamic-programming problem in Table 10 and the appropriate data from Table 13, show how the value of 462 was obtained in Table 14 for 700°F, Reactor IB, and Catalyst 1.

21. Using the method outlined for steepest descent in Eqs. (96) to (98) and presented in Fig. 11-12, what would be the minimum value of C along the first line of steepest descent if the initial point had been chosen arbitrarily as \( x = 2 \) and \( y = 3 \) with \( x \) decreasing in increments of 0.5?

22. In order to continue the operation of a small chemical plant at the same capacity, it will be necessary to make some changes on one of the reactors in the system. The decision has been made by management that the unit must continue in service for the next 12 years and the company policy is that no unnecessary investments are made unless at least an 8 percent rate of return (end-of-year compounding) can be obtained. Two possible ways for making a satisfactory change in the reactor are as follows:
   (1) Make all the critical changes now at a cost of $5800 so the reactor will be satisfactory to use for 12 years.
   (2) Make some of the changes now at a cost of $5000 which will permit operation for 8 years and then make changes costing $2500 to permit operation for the last 4 years.
   (a) Which alternative should be selected if no inflation is anticipated over the next 12 years?
   (b) Which alternative should be selected if inflation at a rate of 7 percent (end-of-year compounding) is assumed for all future costs?
Any engineering design, particularly for a chemical process plant, is only useful when it can be translated into reality by using available materials and fabrication methods. Thus, selection of materials of construction combined with the appropriate techniques of fabrication can play a vital role in the success or failure of a new chemical plant or in the improvement of an existing facility.

**MATERIALS OF CONSTRUCTION**

As chemical process plants turn to higher temperatures and flow rates to boost yields and throughputs, selection of construction materials takes on added importance. This trend to more severe operating conditions forces the chemical engineer to search for more dependable, more corrosion-resistant materials of construction for these process plants, because all these severe conditions intensify corrosive action. Fortunately, a broad range of materials is now available for corrosive service. However, this apparent abundance of materials also complicates the task of choosing the “best” material because, in many cases, a number of alloys and plastics will have sufficient corrosion resistance for a particular application. Final choice cannot be based simply on choosing a suitable material from a corrosion table but must be based on a sound economic analysis of competing materials.

The chemical engineer would hardly expect a metallurgist to handle the design and operation of a complex chemical plant. Similarly, the chemical engineer cannot become a materials specialist overnight. But a good metallur-
TABLE 1
Comparison of purchased cost for metal plate

<table>
<thead>
<tr>
<th>Material</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flange quality steel†</td>
<td>1</td>
</tr>
<tr>
<td>304 stainless-steel-clad steel</td>
<td>5</td>
</tr>
<tr>
<td>316 stainless-steel-clad steel</td>
<td>6</td>
</tr>
<tr>
<td>Aluminum (99 plus)</td>
<td>6</td>
</tr>
<tr>
<td>304 stainless steel</td>
<td>7</td>
</tr>
<tr>
<td>Copper (99.9 plus)</td>
<td>7</td>
</tr>
<tr>
<td>Nickel-clad steel</td>
<td>8</td>
</tr>
<tr>
<td>Monel-clad steel</td>
<td>8</td>
</tr>
<tr>
<td>Inconel-clad steel</td>
<td>9</td>
</tr>
<tr>
<td>316 stainless steel</td>
<td>10</td>
</tr>
<tr>
<td>Monel</td>
<td>10</td>
</tr>
<tr>
<td>Nickel</td>
<td>12</td>
</tr>
<tr>
<td>Inconel</td>
<td>13</td>
</tr>
<tr>
<td>Hastelloy C</td>
<td>15</td>
</tr>
</tbody>
</table>

†Purchased cost for steel plate (January, 1990) can be approximated as 36 to 70 cents per pound, depending on the amount purchased.

gist must have a working knowledge of the chemical plant environment in which the recommendations will be applied. In the same manner, the chemical engineer should also understand something of the materials that make the equipment and processes possible.

The purpose of this chapter is to provide the process designer with a working knowledge of some of the major forms and types of materials available, what they offer, and how they are specified. With this background, the engineer can consult a materials specialist at the beginning of the design, not when the mistakes already have been made.

METALS

Materials of construction may be divided into the two general classifications of metals and nonmetals. Pure metals and metallic alloys are included under the first classification. Table 1 presents data showing the comparison of purchased cost for various types of metals in plate form.

Iron and Steel

Although many materials have greater corrosion resistance than iron and steel, cost aspects favor the use of iron and steel. As a result, they are often used as materials of construction when it is known that some corrosion will occur. If this is done, the presence of iron salts and discoloration in the product can be expected, and periodic replacement of the equipment should be anticipated.
In general, cast iron and carbon steel exhibit about the same corrosion resistance. They are not suitable for use with dilute acids, but can be used with many strong acids, since a protective coating composed of corrosion products forms on the metal surface.

Because of the many types of rolled and forged steel products used in industry, basic specifications are needed to designate the various types. The American Iron and Steel Institute (AISI) has set up a series of standards for steel products. However, even the relatively simple product descriptions provided by AISI and shown in Table 2 must be used carefully. For instance, the AISI 1020 carbon steel does not refer to all 0.20 percent carbon steels. AISI 1020 is part of the numerical designation system defining the chemical composition of certain “standard steels” used primarily in bar, wire, and some tubular steel products. The system almost never applies to sheets, strip, plates, or structural material. One reason is that the chemical composition ranges of standard steels are unnecessarily restrictive for many applications.

Carbon steel plates for reactor vessels are a good example. This application generally requires a minimum level of mechanical properties, weldability, formability, and toughness as well as some assurance that these properties will be uniform throughout. A knowledge of the detailed composition of the steel alone will not assure that these requirements are met. Even welding requirements for plate can be met with far less restrictive chemical compositions than would be needed for the same type of steel used in bar stock suitable for heat treating to a minimum hardness or tensile strength.

**Stainless Steels**

There are more than 100 different types of stainless steels. These materials are high chromium or high nickel-chromium alloys of iron containing small amounts of other essential constituents. They have excellent corrosion-resistance and heat-resistance properties. The most common stainless steels, such as type 302 or type 304, contain approximately 18 percent chromium and 8 percent nickel, and are designated as 18-8 stainless steels.

The addition of molybdenum to the alloy, as in type 316, increases the corrosion resistance and high-temperature strength. If nickel is not included, the low-temperature brittleness of the material is increased and the ductility and pit-type corrosion resistance are reduced. The presence of chromium in the alloy gives resistance to oxidizing agents. Thus, type 430, which contains chromium but no nickel or molybdenum, exhibits excellent corrosion resistance to nitric acid and other oxidizing agents.

Specifications and codes on materials have also been established by the Society of Automotive Engineers (SAE), the American Society of Mechanical Engineers (ASME), and the American Society for Testing Materials (ASTM).
### Table 2: AISI Standard Steels?

(XX’s indicate nominal carbon content within range)

<table>
<thead>
<tr>
<th>AISI Series Designations</th>
<th>Nominal Composition or Range†</th>
</tr>
</thead>
<tbody>
<tr>
<td>10XX</td>
<td><strong>Non-refurburized</strong> carbon steels with 44 compositions ranging from 1008 to 1095. Manganese ranges from 0.30 to 1.65%; if specified, silicon is 0.10 max. to 0.30 max., each depending on grade. Phosphorus is 0.040 max., sulfur is 0.050 max.</td>
</tr>
<tr>
<td>11XX</td>
<td>Resulfurized carbon steels with 15 standard compositions. Sulfur may range up to 0.33%, depending on grade.</td>
</tr>
<tr>
<td>B11XX</td>
<td>Acid Bessemer resulfurized carbon steels with 3 compositions. Phosphorus generally is higher than 11XX series.</td>
</tr>
<tr>
<td>12XX</td>
<td>Rephosphorized and resulfurized carbon steels with 5 standard compositions. Phosphorus may range up to 0.12% and sulfur up to 0.35%, depending on grade.</td>
</tr>
<tr>
<td>13XX</td>
<td>Manganese, 1.75%. Four compositions from 1330 to 1345.</td>
</tr>
<tr>
<td>40XX</td>
<td>Molybdenum, 0.20 or 0.25%. Seven compositions from 4012 to 4047.</td>
</tr>
<tr>
<td>41XX</td>
<td>Chromium, to 0.95%, molybdenum to 0.30%. Nine compositions from 4118 to 4161.</td>
</tr>
<tr>
<td>43XX</td>
<td>Nickel, 1.83%, chromium to 0.80%, molybdenum, 0.25%. Three compositions from 4320 to E4340.</td>
</tr>
<tr>
<td>44XX</td>
<td>Molybdenum, 0.53%. One composition, 4419.</td>
</tr>
<tr>
<td>46XX</td>
<td>Nickel to 1.83%, molybdenum to 0.25%. Four compositions from 4615 to 4626.</td>
</tr>
<tr>
<td>47XX</td>
<td>Nickel, 1.05%, chromium, 0.45%, molybdenum to 0.35%. Two compositions, 4718 and 4720.</td>
</tr>
<tr>
<td>48XX</td>
<td>Nickel, 3.50%, molybdenum, 0.25%. Three compositions from 4815 to 4820.</td>
</tr>
<tr>
<td>50XX</td>
<td>Chromium, 0.40%. One composition, 5015.</td>
</tr>
<tr>
<td>51XX</td>
<td>Chromium to 1.00%. Ten compositions from 5120 to 5160.</td>
</tr>
<tr>
<td>52XX</td>
<td>Carbon, 1.04%, chromium to 1.45%. Two compositions, 51100 and 52100.</td>
</tr>
<tr>
<td>61XX</td>
<td>Chromium to 0.95%, vanadium to 0.15% min. Two compositions, 6118 and 6150.</td>
</tr>
<tr>
<td>86XX</td>
<td>Nickel, 0.55%, chromium, 0.50%, molybdenum, 0.20%. Twelve compositions from 8615 to 8655.</td>
</tr>
<tr>
<td>87XX</td>
<td>Nickel, 0.55%, chromium, 0.50%, molybdenum, 0.25%. Two compositions, 8720 and 8740.</td>
</tr>
<tr>
<td>88XX</td>
<td>Nickel, 0.55%, chromium, 0.50%, molybdenum, 0.35%. One composition, 8822.</td>
</tr>
<tr>
<td>92XX</td>
<td>Silicon, 2.00%. Two compositions, 9255 and 9260.</td>
</tr>
<tr>
<td>50BXX</td>
<td>Chromium to 0.50%, also containing boron. Four compositions from 50B44 to 50B60.</td>
</tr>
<tr>
<td>51BXX</td>
<td>Chromium to 0.80%, also containing boron. One composition, 51B60.</td>
</tr>
<tr>
<td>81BXX</td>
<td>Nickel, 0.30%, chromium, 0.45%, molybdenum, 0.12%, also containing boron. One composition, 81B45.</td>
</tr>
<tr>
<td>94BXX</td>
<td>Nickel, 0.45%, chromium, 0.40%, molybdenum, 0.12%, also containing boron. Two compositions, 94B17 and 94B30.</td>
</tr>
</tbody>
</table>

†When a carbon or alloy steel also contains the letter L in the code, it contains from 0.15 to 0.35 percent lead as a free-machining additive, i.e., 12L14 or 41L40. The prefix E before an alloy steel, such as E4340, indicates the steel is made only by electric furnace. The suffix H indicates an alloy steel made to more restrictive chemical composition than that of standard steels and produced to a measured and known hardenability requirement, e.g., 86308 or 94B30H.

TABLE 3
Classification of stainless steels by alloy content and microstructure

<table>
<thead>
<tr>
<th>Stainless steel types</th>
<th>Chromium types</th>
<th>Hardenable</th>
<th>Nonhardenable</th>
<th>Nonhardenable, except by cold working</th>
<th>Strengthened by aging or precipitation hardening</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>types</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Martensitic -</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ferritic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austenitic -</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semiaustenitic -</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Martensitic -</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although fabricating operations on stainless steels are more difficult than on standard carbon steels, all types of stainless steel can be fabricated successfully. The properties of types 430F, 416, 410, 310, 309, and 303 make these materials particularly well suited for machining or other fabricating operations. In general, machinability is improved if small quantities of phosphorus, selenium, or sulfur are present in the alloy.

†For a detailed discussion of machining and fabrication of stainless steels, see Selection of Stainless Steels, Bulletin OLE 11366, Armco Steel Corporation, Middletown, Ohio 45042; and Fabrication of Stainless Steel, Bulletin 031478, United States Steel Corporation, Pittsburgh, Pa. 15230.
TABLE 4
Stainless steels most commonly used in the chemical process industries?

<p>| Type $|$ Cr    | Ni    | C max | Other significant elements $|$ Major characteristics                                      | Properties                                   | Applications                                      |
|--------|---------|-------|-------|---------------------------------|---------------------------------------------|------------------------------------------------|
| 301    | 16.00-  | 6.00- | 0.15  |                                 | High work-hardening rate combines          | Good structural qualities.                     | Structural applications, bins and containers   |
|        | 18.00   | 8.00  |       |                                 | cold-worked high strength with good ductility. |                                                 | Heat exchangers, towers, tanks, pipes, heaters, general chemical equipment |
| 302    | 17.00-  | 8.00- | 0.15  | s 0.15 min                      | Basic, general purpose austenitic type      | General purpose.                               | Pumps, valves, instruments, fittings          |
|        | 19.00   | 10.00 |       |                                 | with good corrosion resistance and           |                                                 | Perforated blow-pit screens,                 |
|        |         |       |       |                                 | mechanical properties.                      |                                                 | heat-exchanger tubing, preheater tubes        |
| 303    | 17.00-  | 8.00- | 0.15  |                                 | Free machining modification of               | Good corrosion resistance.                     | Funnels, utensils, hoods                     |
|        | 19.00   | 10.00 |       |                                 | type 302; contains extra sulfur.             |                                                 | Welding rod, more ductile welds for type 430 |
| 304    | 18.00-  | 8.00- | 0.08  |                                 | Low carbon variation of type                | In order of their numbers, these alloys show   | Welding rod for type 304, heat exchangers, pump parts |
|        | 20.00   | 12.00 |       |                                 | 302, minimizes carbide precipitation during  | increased resistance to high temperature      | Jacketed high-temperature,                   |
|        |         |       |       |                                 | welding.                                    | corrosion.                                    | high-pressure reactors,                     |
| 305    | 17.00-  | 10.00 | 0.12  |                                 | Higher heat and corrosion resistance        | Type 308S, 309S and 310S are also available   | oil-refining still tubes                     |
|        | 19.00   | 13.00 |       |                                 | than type 304.                              | for welded construction.                      |                                                 |</p>
<table>
<thead>
<tr>
<th>314</th>
<th>23.00-26.00</th>
<th>19.00-22.00</th>
<th>0.25</th>
<th>Si</th>
<th>1.5-3.0</th>
<th>High silicon content.</th>
</tr>
</thead>
<tbody>
<tr>
<td>316</td>
<td>16.00-18.00</td>
<td>10.00-14.00</td>
<td>0.08</td>
<td>Mo</td>
<td>2.00-3.00</td>
<td>Mo improves general corrosion and pitting resistance and high temperature strength over that of type 302.</td>
</tr>
<tr>
<td></td>
<td>18.00-20.00</td>
<td>11.00-15.00</td>
<td>0.08</td>
<td>Mo</td>
<td>3.00-4.00</td>
<td></td>
</tr>
<tr>
<td>321</td>
<td>17.00-19.00</td>
<td>9.00-12.00</td>
<td>0.08</td>
<td>Ti</td>
<td>5 x c, 5 x c, min.</td>
<td>Stabilized to permit use in 420°F-870°F C range without harmful carbide precipitation.</td>
</tr>
<tr>
<td>347</td>
<td>17.00-19.00</td>
<td>9.00-13.00</td>
<td>0.08</td>
<td>Cb-Ta</td>
<td>10 x c, 10 x c, min.</td>
<td>Characteristics similar to type 321. Stabilized by Cb and Ta.</td>
</tr>
<tr>
<td>403</td>
<td>11.50-13.50</td>
<td>0.15</td>
<td>Si</td>
<td>0.50</td>
<td>Version of type 410 with limited hardenability but improved fabricability.</td>
<td></td>
</tr>
<tr>
<td>405</td>
<td>11.50-14.50</td>
<td>0.08</td>
<td>Al</td>
<td>0.10-0.30</td>
<td>Version of type 410 with limited hardenability but improved weldability.</td>
<td></td>
</tr>
<tr>
<td>410</td>
<td>11.50-13.50</td>
<td>0.15</td>
<td></td>
<td></td>
<td>Lowest cost general purpose stainless steel.</td>
<td></td>
</tr>
</tbody>
</table>

Resistant to oxidation in air to 2000°F. Resistant to high pitting corrosion. Also available as 316L for welded construction.

Type 317 has the highest aqueous corrosion resistance of all AISI stainless steels.

Stabilized with titanium and columbium-tantalum, respectively, to permit their use for large welded structures which cannot be annealed after welding.

Not highly resistant to high temperature oxidation in air.

Good weldability and cladding properties.

Wide use where corrosion is not severe.

Radiant tubes, carburizing boxes, annealing boxes
Distillation equipment for producing fatty acids, sulfate paper processing equipment
Process equipment involving strong acids or chlorinated solvents
Furnace parts in presence of corrosive fumes
Like 302 but used where car-bide precipitation during fabrication or service may be harmful, welding rod for type 321
Steam turbine blades
Tower linings, baffles, separator towers, heat exchanger tubing
Bubble-tower parts for petroleum refining, pump rods and valves, machine parts, turbine blades

(continued)
### TABLE 4
Stainless steels most commonly used in the chemical process industries? (Continued)

<table>
<thead>
<tr>
<th>Type</th>
<th>Composition, %</th>
<th>Other significant elements</th>
<th>Major characteristics</th>
<th>Properties</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>416</td>
<td>Cr</td>
<td>Ni</td>
<td>C max</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.00-14.00</td>
<td>0.15 ( S )</td>
<td>0.15 ( \text{min} )</td>
<td></td>
<td>Sulfur added for free machining version of type 410. Type 416Se also available.</td>
<td>The freest machining type of martensitic stainless.</td>
</tr>
<tr>
<td>420</td>
<td></td>
<td></td>
<td></td>
<td>Similar to type 410 but higher carbon produces higher strength and hardness.</td>
<td>High-spring temper.</td>
</tr>
<tr>
<td>430</td>
<td>14.00-18.00</td>
<td>0.12</td>
<td></td>
<td>Most popular of nonhardenable chromium types. Combines good corrosion resistance (to nitric acid and other oxidizing media).</td>
<td>Good heat resistance and good mechanical properties. Also available in type 430F.</td>
</tr>
<tr>
<td>431</td>
<td>15.00-17.00</td>
<td>1.25-2.50</td>
<td>1.20</td>
<td>High yield point</td>
<td>Very resistant to shock.</td>
</tr>
<tr>
<td>442</td>
<td>18.00-23.00</td>
<td>3.25</td>
<td></td>
<td>High chromium nonhardenable type</td>
<td>High temperature uses where high sulfur atmospheres make presence of nickel undesirable.</td>
</tr>
<tr>
<td>446</td>
<td>23.00-27.00</td>
<td>0.20</td>
<td>Similar to type 442 but Cr increased to provide maximum resistance to scaling. Especially suited to intermittent high temperatures.</td>
<td>Excellent corrosion resistance to many liquid solutions, fabrication difficulties limit its use primarily to high temperature applications. Useful in high sulfur atmospheres.</td>
<td>Burner nozzles, stack dampers, boiler baffles, furnace linings, glass molds</td>
</tr>
</tbody>
</table>


$ For a detailed listing of nominal composition or range, see the latest issue of “Data on Physical and Mechanical Properties of Stainless and Heat-Resisting Steels.” Carpenter Steel Company, Reading, PA 19603.

$ In general, stainless steels in the 300 series contain large amounts of chromium and nickel; those in the 400 series contain large amounts of chromium and little or no nickel; those in the 500 series contain low amounts of chromium and little or no nickel; in the 300 series, except for type 309, the nickel content can be 10 percent or less if the second number is zero and greater than 10 percent if the second number is one; in the 400 series, an increase in the number represented by the last two digits indicates an increase in the chromium content.
The types of stainless steel included in the 300 series are hardenable only by cold-working; those included in the 400 series are either nonhardenable or hardenable by heat-treating. As an example, type 410, containing 12 percent chromium and no nickel, can be heat-treated for hardening and has good mechanical properties when heat-treated. It is often used as a material of construction for bubble caps, turbine blades, or other items that require special fabrication.

Stainless steels exhibit the best resistance to corrosion when the surface is oxidized to a passive state. This condition can be obtained, at least temporarily, by a so-called “passivation” operation in which the surface is treated with nitric acid and then rinsed with water. Localized corrosion can occur at places where foreign material collects, such as in scratches, crevices, or corners. Consequently, mars or scratches should be avoided, and the equipment design should specify a minimum of sharp corners, seams, and joints. Stainless steels show great susceptibility to stress corrosion cracking. As one example, stress plus contact with small concentrations of halides can result in failure of the metal wall.

The high temperatures involved in welding stainless steel may cause precipitation of chromium carbide at the grain boundary, resulting in decreased corrosion resistance along the weld. The chances of this occurring can be minimized by using low-carbon stainless steels or by controlled annealing.

A preliminary approach to the selection of the stainless steel for a specific application is to classify the various types according to the alloy content, microstructure, and major characteristic. Table 3 outlines the information according to the classes of stainless steels-austenitic, martensitic, and ferritic. Table 4 presents characteristics and typical applications of various types of stainless steel while Table 5 indicates resistance of stainless steels to oxidation in air.

<table>
<thead>
<tr>
<th>Maximum temperature, °C</th>
<th>Stainless steel type</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>416</td>
</tr>
<tr>
<td>700</td>
<td>403, 405, 410, 414</td>
</tr>
<tr>
<td>800</td>
<td>430F</td>
</tr>
<tr>
<td>850</td>
<td>430, 431</td>
</tr>
<tr>
<td>900</td>
<td>302, 303, 304, 316, 317, 321, 347, 348, 17-14 CuMo</td>
</tr>
<tr>
<td>1000</td>
<td>302B, 308, 442</td>
</tr>
<tr>
<td>1100</td>
<td>309, 310, 314, 329, 446</td>
</tr>
</tbody>
</table>
Hastelloy

The beneficial effects of nickel, chromium, and molybdenum are combined in Hastelloy C to give an expensive but highly corrosion-resistant material. A typical analysis of this alloy shows 56 percent nickel, 17 percent molybdenum, 16 percent chromium, 5 percent iron, and 4 percent tungsten, with manganese, silicon, carbon, phosphorus, and sulfur making up the balance. Hastelloy C is used where structural strength and good corrosion resistance are necessary under conditions of high temperatures. The material can be machined and is readily fabricated. It is used in the form of valves, piping, heat exchangers, and various types of vessels. Other types of Hastelloy are also available for use under special corrosive conditions.

Copper and its Alloys

Copper is relatively inexpensive, possesses fair mechanical strength, and can be fabricated easily into a wide variety of shapes. Although it shows little tendency to dissolve in nonoxidizing acids, it is readily susceptible to oxidation. Copper is resistant to atmospheric moisture or oxygen because a protective coating composed primarily of copper oxide is formed on the surface. The oxide, however, is soluble in most acids, and thus copper is not a suitable material of construction when it must contact any acid in the presence of oxygen or oxidizing agents. Copper exhibits good corrosion resistance to strong alkalies, with the exception of ammonium hydroxide. At room temperature it can handle sodium and potassium hydroxide of all concentrations. It resists most organic solvents and aqueous solutions of organic acids.

Copper alloys, such as brass, bronze, admiralty, and Muntz metals, can exhibit better corrosion resistance and better mechanical properties than pure copper. In general, high-zinc alloys should not be used with acids or alkalies owing to the possibility of dezincification. Most of the low-zinc alloys are resistant to hot dilute alkalies.

Nickel and its Alloys

Nickel exhibits high corrosion resistance to most alkalies. Nickel-clad steel is used extensively for equipment in the production of caustic soda and alkalies. The strength and hardness of nickel is almost as great as that of carbon steel, and the metal can be fabricated easily. In general, oxidizing conditions promote the corrosion of nickel, and reducing conditions retard it.

Monel, an alloy of nickel containing 67 percent nickel and 30 percent copper, is often used in the food industries. This alloy is stronger than nickel and has better corrosion-resistance properties than either copper or nickel. Another important nickel alloy is Inconel (77 percent nickel and 15 percent chromium). The presence of chromium in this alloy increases its resistance to oxidizing conditions.
Aluminum

The lightness and relative ease of fabrication of aluminum and its alloys are factors favoring the use of these materials. Aluminum resists attack by acids because a surface film of inert hydrated aluminum oxide is formed. This film adheres to the surface and offers good protection unless materials which can remove the oxide, such as halogen acids or alkalies, are present.

Lead

Pure lead has low creep and fatigue resistance, but its physical properties can be improved by the addition of small amounts of silver, copper, antimony, or tellurium. Lead-clad equipment is in common use in many chemical plants. The excellent corrosion-resistance properties of lead are caused by the formation of protective surface coatings. If the coating is one of the highly insoluble lead salts, such as sulfate, carbonate, or phosphate, good corrosion resistance is obtained. Little protection is offered, however, if the coating is a soluble salt, such as nitrate, acetate, or chloride. As a result, lead shows good resistance to sulfuric acid and phosphoric acid, but it is susceptible to attack by acetic acid and nitric acid.

Tantalum

The physical properties of tantalum are similar to those of mild steel, with the exception that its melting point (2996°C) is much higher. It is ordinarily used in the pure form, and it is readily fabricated into many different shapes. The corrosion-resistance properties of tantalum resemble those of glass. The metal is attacked by hydrofluoric acid, by hot concentrated alkalies, and by materials containing free sulfur trioxide. It is resistant to all other acids and is often used for equipment involving contact with hydrochloric acid.

Silver

Because of its low mechanical strength and high cost, silver is generally used only in the form of linings. Silver is resistant to alkalies and many hot organic acids. It also shows fair resistance to aqueous solutions of the halogen acids.

Galvanic Action between Two Dissimilar Metals

When two dissimilar metals are used in the construction of equipment containing a conducting fluid in contact with both metals, an electric potential may be set up between the two metals. The resulting galvanic action can cause one of the metals to dissolve into the conducting fluid and deposit on the other metal. As an example, if a piece of copper equipment containing a solution of sodium chloride in water is connected to an iron pipe, electrolysis can occur between
TABLE 6

**Electromotive series of metals**
List of metals arranged in decreasing order of their tendencies to pass into ionic form by losing electrons

<table>
<thead>
<tr>
<th>Metal</th>
<th>Ion</th>
<th>Standard electrode potential 1125°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium</td>
<td>Li⁺</td>
<td>+2.96</td>
</tr>
<tr>
<td>Potassium</td>
<td>K⁺</td>
<td>2.92</td>
</tr>
<tr>
<td>Calcium</td>
<td>Ca²⁺</td>
<td>2.87</td>
</tr>
<tr>
<td>Sodium</td>
<td>Na⁺</td>
<td>2.71</td>
</tr>
<tr>
<td>Magnesium</td>
<td>Mg²⁺</td>
<td>2.40</td>
</tr>
<tr>
<td>Aluminum</td>
<td>Al³⁺</td>
<td>1.70</td>
</tr>
<tr>
<td>Manganese</td>
<td>Mn⁴⁺</td>
<td>1.10</td>
</tr>
<tr>
<td>Zinc</td>
<td>Zn⁺²</td>
<td>0.76</td>
</tr>
<tr>
<td>Chromium</td>
<td>Cr³⁺</td>
<td>0.56</td>
</tr>
<tr>
<td>Gallium</td>
<td>Ga¹⁺</td>
<td>0.50</td>
</tr>
<tr>
<td>Iron</td>
<td>Fe⁺²</td>
<td>0.44</td>
</tr>
<tr>
<td>Cadmium</td>
<td>Cd⁺²</td>
<td>0.40</td>
</tr>
<tr>
<td>Cobalt</td>
<td>Co⁺²</td>
<td>0.28</td>
</tr>
<tr>
<td>Nickel</td>
<td>Ni⁺²</td>
<td>0.23</td>
</tr>
<tr>
<td>Tin</td>
<td>Sn⁺²</td>
<td>0.14</td>
</tr>
<tr>
<td>Lead</td>
<td>Pb⁺²</td>
<td>0.12</td>
</tr>
<tr>
<td>Iron</td>
<td>Fe⁺²⁺</td>
<td>0.045</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>H⁺</td>
<td>0.0000</td>
</tr>
<tr>
<td>Antimony</td>
<td>Sb⁺²⁺</td>
<td>-0.10</td>
</tr>
<tr>
<td>Bismuth</td>
<td>Bi⁺²⁺</td>
<td>-0.23</td>
</tr>
<tr>
<td>Arsenic</td>
<td>As⁺²⁺</td>
<td>-0.30</td>
</tr>
<tr>
<td>Copper</td>
<td>Cu⁺²⁺</td>
<td>-0.34</td>
</tr>
<tr>
<td>Copper</td>
<td>Cu⁺</td>
<td>-0.47</td>
</tr>
<tr>
<td>Silver</td>
<td>Ag⁺</td>
<td>-0.80</td>
</tr>
<tr>
<td>Lead</td>
<td>Pb⁺²⁺</td>
<td>-0.80</td>
</tr>
<tr>
<td>Platinum</td>
<td>Pt⁺²⁺</td>
<td>-0.86</td>
</tr>
<tr>
<td>Gold</td>
<td>Au⁺²⁺</td>
<td>-1.36</td>
</tr>
<tr>
<td>Gold</td>
<td>Au⁺</td>
<td>-1.50</td>
</tr>
</tbody>
</table>

As indicated in Table 6, iron is higher in the electromotive series than copper, and the iron pipe will gradually dissolve and deposit on the copper. The farther apart the two metals are in the electromotive series, the greater is the possible extent of corrosion due to electrolysis.

**NONMETALS**

Glass, carbon, stoneware, brick, rubber, plastics, and wood are common examples of nonmetals used as materials of construction. Many of the nonmetals
have low structural strength. Consequently, they are often used in the form of linings or coatings bonded to metal supports. For example, glass-lined or rubber-lined equipment has many applications in the chemical industries.

**Glass and Glassed Steel**

Glass has excellent resistance and is subject to attack only by hydrofluoric acid and hot alkaline solutions. It is particularly suitable for processes which have critical contamination levels. A chief drawback is its brittleness and damage by thermal shock. On the other hand, glassed steel combines the corrosion resistance of glass with the working strength of steel. Nucerite is a ceramic-metal composite made in a similar manner to glassed steel and resists corrosive hydrogen-chloride gas, chlorine, or sulfur dioxide at 650°C. Its impact strength is 18 times that of safety glass and the abrasion resistance is superior to porcelain enamel.

**Carbon and Graphite**

Generally, impervious graphite is completely inert to all but the most severe oxidizing conditions. This property, combined with excellent heat transfer, has made impervious carbon and graphite very popular in heat exchangers, as brick lining, and in pipe and pumps. One limitation of these materials is low tensile strength. Threshold oxidation temperatures are 350°C for carbon and 400°C for graphite.

**Stoneware and Porcelain**

Materials of stoneware and porcelain are about as resistant to acids and chemicals as glass, but with the advantage of greater strength. This is offset somewhat by poor thermal conductivity and susceptibility to damage by thermal shock. Porcelain enamels are used to coat steel, but the enamel has slightly inferior chemical resistance.

**Brick and Cement Materials**

Brick-lined construction can be used for many severely corrosive conditions, where high alloys would fail. Acidproof refractories can be used up to 900°C.

A number of cement materials are used with brick. Standard are phenolic and furane resins, polyesters, sulfur, silicate, and epoxy-based materials. Carbon-filled polyesters and furanes are good against nonoxidizing acids, salts, and solvents. Silica-filled resins should not be used against hydrofluoric or fluoric acids. Sulfur-based cements are limited to 95°C, while resins can be used to about 175°C. The sodium silicate based cements are good against acids to 400°C.
Rubber and Elastomers

Natural and synthetic rubbers are used as linings or as structural components for equipment in the chemical industries. By adding the proper ingredients, natural rubbers with varying degrees of hardness and chemical resistance can be produced. Hard rubbers are chemically saturated with sulfur. The vulcanized products are rigid and exhibit excellent resistance to chemical attack by dilute sulfuric acid and dilute hydrochloric acid.

Natural rubber is resistant to dilute mineral acids, alkalies, and salts, but oxidizing media, oils, benzene, and ketones will attack it. Chloroprene or neoprene rubber is resistant to attack by ozone, sunlight, oils, gasoline, and aromatic or halogenated solvents. Styrene rubber has chemical resistance similar to natural. Nitrile rubber is known for resistance to oils and solvents. Butyl rubber’s resistance to dilute mineral acids and alkalies is exceptional; resistance to concentrated acids, except nitric and sulfuric, is good. Silicone rubbers, also known as polysiloxanes, have outstanding resistance to high and low temperatures as well as against aliphatic solvents, oils, and greases. Chlorosulfonated polyethylene, known as hypalon, has outstanding resistance to ozone and oxidizing agents except fuming nitric and sulfuric acids. Oil resistance is good. Fluoroelastomers (Viton A, Kel-F) combine excellent chemical and high-temperature resistance. Polyvinyl chloride elastomer (Koroseal) was developed to overcome some of the limitations of natural and synthetic rubbers. It has excellent resistance to mineral acids and petroleum oils.

Plastics

In comparison with metallic materials, the use of plastics is limited to relatively moderate temperatures and pressures (230°C is considered high for plastics). Plastics are also less resistant to mechanical abuse and have high expansion rates, low strengths (thermoplastics), and only fair resistance to solvents. However, they are lightweight, are good thermal and electrical insulators, are easy to fabricate and install, and have low friction factors.

Generally, plastics have excellent resistance to weak mineral acids and are unaffected by inorganic salt solutions-areas where metals are not entirely suitable. Since plastics do not corrode in the electrochemical sense, they offer another advantage over metals: most metals are affected by slight changes in pH, or minor impurities, or oxygen content, while plastics will remain resistant to these same changes.

One of the most chemical-resistant plastics commercially available today is tetrafluoroethylene or TFE (Teflon). This thermoplastic is practically unaffected by all alkalies and acids except fluorine and chlorine gas at elevated temperatures and molten metals. It retains its properties up to 260°C. Chlorotrifluoroethylene or CFE (Kel-F) also possesses excellent corrosion resistance to almost all acids and alkalies up to 175°C. FEP, a copolymer of tetrafluoroethylene and hexafluoropropylene, has similar properties to TFE except that it is
not recommended for continuous exposures at temperatures above 200°C. Also, FEP can be extruded on conventional extrusion equipment, while TFE parts must be made by complicated “powdered-metallurgy” techniques.

Polyethylene is the lowest-cost plastic commercially available. Mechanical properties are generally poor, particularly above 50°C, and pipe must be fully supported. Carbon-filled grades are resistant to sunlight and weathering.

Unplasticized polyvinyl chlorides (type I) have excellent resistance to oxidizing acids other than concentrated, and to most nonoxidizing acids. Resistance is good to weak and strong alkaline materials. Resistance to chlorinated hydrocarbons is not good.

Acrylonitrile butadiene styrene polymers (ABS) have good resistance to nonoxidizing and weak acids but are not satisfactory with oxidizing acids. Upper temperature limit is about 65°C. Resistance to weak alkaline solutions is excellent. They are not satisfactory with aromatic or chlorinated hydrocarbons but have good resistance to aliphatic hydrocarbons.

Chlorinated polyether can be used continuously up to 125°C, intermittently up to 150°C. Chemical resistance is between polyvinyl chloride and the fluorocarbons. Dilute acids, alkalies, and salts have no effect. Hydrochloric, hydrofluoric, and phosphoric acids can be handled at all concentrations up to 105°C. Sulfuric acid over 60 percent and nitric over 25 percent cause degradation, as do aromatics and ketones.

Acetals have excellent resistance to most organic solvents but are not satisfactory for use with strong acids and alkalies.

Cellulose acetate butyrate is not affected by dilute acids and alkalies or gasoline but chlorinated solvents cause some swelling. Nylons resist many organic solvents but are attacked by phenols, strong oxidizing agents, and mineral acids.

Polypropylene’s chemical resistance is about the same as polyethylene, but it can be used at 120°C. Polycarbonate is a relatively high-temperature plastic. It can be used up to 150°C. Resistance to mineral acids is good. Strong alkalis slowly decompose it, but mild alkalis do not. It is partially soluble in aromatic solvents and soluble in chlorinated hydrocarbons.

Among the thermosetting materials are phenolic plastics filled with asbestos, carbon or graphite, and silica. Relatively low cost, good mechanical properties, and chemical resistance (except against strong alkalis) make phenolics popular for chemical equipment. Furane plastics, filled with asbestos, have much better alkali resistance than phenolic asbestos. They are more expensive than the phenolics but also offer somewhat higher strengths.

General-purpose polyester resins, reinforced with fiberglass, have good strength and good chemical resistance, except to alkalies. Some special materials in this class, based on bisphenol, are more alkali resistant. Temperature limit for polyesters is about 95°C. The general area of fiberglass reinforced plastics (FRP) represents a rapidly expanding application of plastics for processing equipment, and it is necessary to solve the problem of development of fabrication standards.
Epoxies reinforced with fiberglass have very high strengths and resistance to heat. Chemical resistance of the epoxy resin is excellent in nonoxidizing and weak acids but not good against strong acids. Alkaline resistance is excellent in weak solutions. Chemical resistance of epoxy-glass laminates may be affected by any exposed glass in the laminate.

Phenolic asbestos, general-purpose polyester glass, saran, and CAB (cellulose acetate butyrate) are adversely affected by alkalies, while thermoplastics generally show poor resistance to organics.

Wood

This material of construction, while fairly inert chemically, is readily dehydrated by concentrated solutions and consequently shrinks badly when subjected to the action of such solutions. It also has a tendency to slowly hydrolyze when in contact with hot acids and alkalies.

LOW- AND HIGH-TEMPERATURE MATERIALS

The extremes of low and high temperatures used in many recent chemical processes has created some unusual problems in fabrication of equipment. For example, some metals lose their ductility and impact strength at low temperatures, although in many cases yield and tensile strengths increase as the temperature is decreased. It is important in low temperature applications to

<table>
<thead>
<tr>
<th>TABLE 7</th>
<th>Metals and alloys for low-temperature process use†</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM specification and grade</td>
<td>Recommended minimum service temp, °C</td>
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<tr>
<td>Carbon and alloy steels:</td>
<td></td>
</tr>
<tr>
<td>T-1</td>
<td>-45</td>
</tr>
<tr>
<td>A 201, A 212, flange or fiefbox quality</td>
<td></td>
</tr>
<tr>
<td>A 203, grades A and B (2 1/2 Ni)</td>
<td>-60</td>
</tr>
<tr>
<td>A 203, grades D and E (3 1/2 Ni)</td>
<td>-100</td>
</tr>
<tr>
<td>A 353 (9% Ni)</td>
<td>-195</td>
</tr>
<tr>
<td>Copper alloys, silicon bronze, 70-30 brass, copper</td>
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</tr>
<tr>
<td>Stainless steels type 302, 304L, 304, 310, 347</td>
<td>-255</td>
</tr>
<tr>
<td>Aluminum alloys 5052, 5083, 5086, 5154, 5356, 5454, 5456</td>
<td></td>
</tr>
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</table>

## TABLE 8
Alloys for high-temperature process use†

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<tr>
<th></th>
<th>Nominal composition, %</th>
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<tr>
<td></td>
<td>Cr</td>
<td>Ni</td>
<td>Fe</td>
<td>Other</td>
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<tr>
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<td>16</td>
<td>bal.</td>
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</tr>
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<td>Austenitic steels:</td>
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<tr>
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</tr>
<tr>
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<td>Type 316</td>
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<td>18</td>
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<tr>
<td>Type 309</td>
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<td>24</td>
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</tr>
<tr>
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</tr>
<tr>
<td>Type 330</td>
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<tr>
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<tr>
<td>Nickel</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Incoloy</td>
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<td>34</td>
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<tr>
<td>Hastelloy B</td>
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<td></td>
<td></td>
<td>Mo</td>
</tr>
<tr>
<td>Hastelloy c</td>
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<td>16</td>
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</tr>
<tr>
<td>60/15</td>
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<td>bal.</td>
<td>7</td>
</tr>
<tr>
<td>80/20</td>
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<td>bal.</td>
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<td>Hastelloy X</td>
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<td>22</td>
<td>bal.</td>
<td>19</td>
</tr>
<tr>
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<td>20</td>
<td>bal.</td>
</tr>
<tr>
<td>Rene 41</td>
<td></td>
<td>19</td>
<td>bal.</td>
<td>5</td>
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<tr>
<td>Cast irons:</td>
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</tr>
<tr>
<td>Ductile iron</td>
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<td></td>
<td></td>
<td>bal.</td>
</tr>
<tr>
<td>Ni-Resist, D-2</td>
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<td>2</td>
<td>20</td>
<td>bal.</td>
</tr>
<tr>
<td>Ni-Resist, D-4</td>
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<td>30</td>
<td>bal.</td>
</tr>
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<td>Cast stainless (ACI types):</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>HC</td>
<td></td>
<td>28</td>
<td>4</td>
<td>bal.</td>
</tr>
<tr>
<td>HF</td>
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<td>21</td>
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<td>bal.</td>
</tr>
<tr>
<td>HH</td>
<td></td>
<td>26</td>
<td>12</td>
<td>bal.</td>
</tr>
<tr>
<td>HK</td>
<td></td>
<td>26</td>
<td>20</td>
<td>bal.</td>
</tr>
<tr>
<td>HT</td>
<td></td>
<td>15</td>
<td>35</td>
<td>bal.</td>
</tr>
<tr>
<td>HW</td>
<td></td>
<td>12</td>
<td>bal.</td>
<td>28</td>
</tr>
<tr>
<td>Super alloys:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inconel X</td>
<td></td>
<td>15</td>
<td>bal.</td>
<td>7</td>
</tr>
<tr>
<td>A 286</td>
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<td>25</td>
<td>bal.</td>
</tr>
<tr>
<td>Stellite 25</td>
<td></td>
<td>20</td>
<td>10</td>
<td>Co-base</td>
</tr>
<tr>
<td>Stellite 21 (cast)</td>
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<td>27</td>
<td>3</td>
<td>Co-base</td>
</tr>
<tr>
<td>Stellite 31 (cast)</td>
<td></td>
<td>25</td>
<td>2</td>
<td>Co-base</td>
</tr>
</tbody>
</table>

choose materials resistant to shock. Usually a minimum Charpy value of 15 ft . lbf (keyhole notch) is specified at the operating temperature. For severe loading, a value of 20 ft . lbf is recommended. Ductility tests are performed on notched specimens since smooth specimens usually show amazing ductility. Table 7 provides a brief summary of metals and alloys recommended for low-temperature use.

Among the most important properties of materials at the other end of the temperature spectrum are creep, rupture, and short-time strengths. Stress rupture is another important consideration at high temperatures since it relates stress and time to produce rupture. Ferritic alloys are weaker than austenitic compositions, and in both groups molybdenum increases strength. Higher strengths are available in Inconel, cobalt-based Stellite 25, and iron-base A286. Other properties which become important at high temperatures include thermal conductivity, thermal expansion, ductility, alloy composition, and stability.

Actually, in many cases strength and mechanical properties become of secondary importance in process applications, compared with resistance to the corrosive surroundings. All common heat-resistant alloys form oxides when exposed to hot oxidizing environments. Whether the alloy is resistant depends upon whether the oxide is stable and forms a protective film. Thus, mild steel is seldom used above 500°C because of excessive scaling rates. Higher temperatures require chromium. This is evident, not only from Table 5, but also from Table 8 which lists the important commercial alloys for high-temperature use.

GASKET MATERIALS

Metallic and nonmetallic gaskets of many different forms and compositions are used in industrial equipment. The choice of a gasket material depends on the corrosive action of the chemicals that may contact the gasket, the location of the gasket, and the type of gasket construction. Other factors of importance are the cost of the materials, pressure and temperature involved, and frequency of opening the joint.

TABULATED DATA FOR SELECTING MATERIALS OF CONSTRUCTION

Table 9 presents information on the corrosion resistance of some common metals, nonmetals, and gasket materials. Table 10 presents similar information for various types of plastics. These tables can be used as an aid in choosing materials of construction, but no single table can take into account all the factors that can affect corrosion. Temperature level, concentration of the corrosive agent, presence of impurities, physical methods of operation, and slight alterations in the composition of the constructional material can affect the degree of corrosion resistance. The final selection of a material of construction,
**TABLE 9**
Corrosion resistance of constructional materials?

<table>
<thead>
<tr>
<th>Chemical</th>
<th>Iron and steel</th>
<th>Stainless steel</th>
<th>Metals</th>
<th>Nonmetals</th>
<th>Acceptable nonmetallic gasket materials</th>
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<tr>
<td></td>
<td></td>
<td>18-8</td>
<td>18-8 Mo</td>
<td>Nickel</td>
<td>Monel</td>
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<tr>
<td>Acetic acid, crude</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Acetic acid, pure</td>
<td>X</td>
<td>X</td>
<td>C</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Acetic anhydride</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Acetone</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
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<tr>
<td>Aluminum chloride</td>
<td>X</td>
<td>C</td>
<td>X</td>
<td>X</td>
<td>C</td>
</tr>
<tr>
<td>Aluminum sulfate</td>
<td>X</td>
<td>C</td>
<td>C</td>
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</tr>
<tr>
<td>Alums</td>
<td>X</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>C</td>
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<tr>
<td>Ammonia (gaseous)</td>
<td>A</td>
<td>A</td>
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<tr>
<td>Ammonium chloride</td>
<td>C</td>
<td>A</td>
<td>C</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Ammonium hydroxide</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>Ammonium phosphate (monobasic)</td>
<td>C</td>
<td>A</td>
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<td>A</td>
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<tr>
<td>Ammonium phosphate (dibasic)</td>
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<td>A</td>
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<tr>
<td>Ammonium phosphate (tribasic)</td>
<td>A</td>
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<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Ammonium sulfate</td>
<td>C</td>
<td>A</td>
<td>C</td>
<td>C</td>
<td>A</td>
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<tr>
<td>Aniline</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

**Code designation for corrosion resistance**
- A = acceptable, can be used successfully
- C = caution, resistance varies widely depending on conditions; used when some corrosion is permissible
- X = unsuitable
- Blank = information lacking

**Code designation for gasket materials**
- a = asbestos, white (compressed or woven)
- b = asbestos, blue (compressed or woven)
- c = asbestos (compressed and rubber-bonded)
- d = asbestos (woven and rubber-frictioned)
- e = CR-S or natural rubber
- f = Teflon
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<tr>
<th>Compound</th>
<th>A</th>
<th>A</th>
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<td>Boric acid</td>
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### TABLE 10

**Chemical resistance of plastics in various solvents**

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<td>U</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Triethanolamine</td>
<td>S₁</td>
<td>S₁</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>U</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Xylene</td>
<td>U</td>
<td>U</td>
<td>U</td>
<td>U</td>
<td>U</td>
<td>S₂</td>
<td>S</td>
<td>S</td>
<td>U</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
</tbody>
</table>

†From Biennial Materials of Construction Reports published by *Chemical Engineering.*
therefore, may require reference to manufacturers’ bulletins and consultation with persons who are experts in the particular field of application.

**SELECTION OF MATERIALS**

The chemical engineer responsible for the selection of materials of construction must have a thorough understanding of all the basic process information available. This knowledge of the process can then be used to select materials of construction in a logical manner. A brief plan for studying materials of construction is as follows:

1. **Preliminary selection**
   - Experience, manufacturer’s data, special literature, general literature, availability, safety aspects, preliminary laboratory tests
2. **Laboratory testing**
   - Reevaluation of apparently suitable materials under process conditions
3. **Interpretation of laboratory results and other data**
   - Effect of possible impurities, excess temperature, excess pressure, agitation, and presence of air in equipment
   - Avoidance of electrolysis
   - Fabrication method
4. **Economic comparison of apparently suitable materials**
   - Material and maintenance cost, probable life, cost of product degradation, liability to special hazards
5. **Final selection**

In making an economic comparison, the engineer is often faced with the question of where to use high-cost claddings or coatings over relatively cheap base materials such as steel or wood. For example, a column requiring an expensive alloy-steel surface in contact with the process fluid may be constructed of the alloy itself or with a cladding of the alloy on the inside of carbon-steel structural material. Other examples of commercial coatings for chemical process equipment include baked ceramic or glass coatings, flame-sprayed metal, hard rubber, and many organic plastics. The durability of coatings is sometimes questionable, particularly where abrasion and mechanical-wear conditions exist. As a general rule, if there is little economic incentive between a coated type versus a completely homogeneous material, a

selection should favor the latter material, mainly on the basis of better mechanical stability.

ECONOMICS IN SELECTION OF MATERIALS

First cost of equipment or material often is not a good economic criterion when comparing alternate materials of construction for chemical process equipment. Any cost estimation should include the following items:

1. Total equipment or materials costs
2. Installation costs
3. Maintenance costs
4. Estimated life
5. Replacement costs

When these factors are considered, cost comparisons bear little resemblance to first costs. Table 11 presents a typical analysis of comparative costs for alternative materials when based on return on investment. One difficulty with such a comparison is the uncertainty associated with “estimated life.” Well-designed laboratory and plant tests can at least give order-of-magnitude estimates. Another difficulty arises in estimating the annual maintenance cost. This can only be predicted from previous experience with the specific materials.

Table 11 could be extended by the use of continuous compounding interest methods as outlined in Chaps. 7 and 10 to show the value of money to a company above which (or below which) material A would be selected over B, B

<table>
<thead>
<tr>
<th>TABLE 11</th>
<th>Alternative investment comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Material</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Purchased cost</td>
<td>$25,000</td>
</tr>
<tr>
<td>Installation cost</td>
<td>15,000</td>
</tr>
<tr>
<td>Total installed cost</td>
<td>40,000</td>
</tr>
<tr>
<td>Additional cost over A</td>
<td>50,000</td>
</tr>
<tr>
<td>Estimated life, years</td>
<td>4</td>
</tr>
<tr>
<td>Estimated maintenance cost/year</td>
<td>5,000</td>
</tr>
<tr>
<td>Annual replacement cost</td>
<td>10,000</td>
</tr>
<tr>
<td>(installed cost/estimated life)</td>
<td></td>
</tr>
<tr>
<td>Total annual cost</td>
<td>15,000</td>
</tr>
<tr>
<td>Annual savings vs. cost for A</td>
<td>5,500</td>
</tr>
<tr>
<td>Tax on savings, 34%</td>
<td>1,870</td>
</tr>
<tr>
<td>Net annual savings</td>
<td>3,630</td>
</tr>
<tr>
<td>Return on investment over A (net savings/additional cost over A)</td>
<td>100</td>
</tr>
</tbody>
</table>
over C, etc. Table 11 indicates that material $B$ is always better than material A (this, of course, is inherent in the yearly return on investment method used). However, depending on the value of money to a company, this may not always be true.

**FABRICATION OF EQUIPMENT**

Fabrication expenses account for a large fraction of the purchased cost for equipment. A chemical engineer, therefore, should be acquainted with the methods for fabricating equipment, and the problems involved in the fabrication should be considered when equipment specifications are prepared.

Many of the design and fabrication details for equipment are governed by various codes, such as the **ASME** Codes. These codes can be used to indicate definite specifications or tolerance limits without including a large amount of descriptive restrictions. For example, fastening requirements can often be indicated satisfactorily by merely stating that all welding should be in accordance with the **ASME** Code.

The exact methods used for fabrication depend on the complexity and type of equipment being prepared. In general, however, the following steps are involved in the complete fabrication of major pieces of chemical equipment, such as tanks, autoclaves, reactors, towers, and heat exchangers:

1. Layout of materials
2. Cutting to correct dimensions
3. Forming into desired shape
4. Fastening
5. Testing
6. Heat-treating
7. Finishing

**Layout**

The first step in the fabrication is to establish the layout of the various components on the basis of detailed instructions prepared by the fabricator. Flat pieces of the metal or other constructional material involved are marked to indicate where cutting and forming are required. Allowances must be made for losses caused by cutting, shrinkage due to welding, or deformation caused by the various forming operations.

After the equipment starts to take shape, the location of various outlets and attachments will become necessary. Thus, the layout operation can continue throughout the entire fabrication. If tolerances are critical, an exact layout, with adequate allowances for deformation, shrinkage, and losses, is absolutely essential.
Cutting

Several methods can be used for cutting the laid-out materials to the correct size. Shearing is the cheapest method and is satisfactory for relatively thin sheets. The edge resulting from a shearing operation may not be usable for welding, and the sheared edges may require an additional grinding or machining treatment.

Burning is often used for cutting metals. This method can be employed to cut and, simultaneously, prepare a beveled edge suitable for welding. Carbon steel is easily cut by an oxyacetylene flame. The heat effects on the metal are less than those involved in welding. Stainless steels and nonferrous metals that do not oxidize readily can be cut by a method known as powder or flux burning. An oxyacetylene flame is used, and powdered iron is introduced into the cut to increase the amount of heat and improve the cutting characteristics. The high temperatures involved may affect the materials, resulting in the need for a final heat-treatment to restore corrosion resistance or removal of the heat-affected edges.

Sawing can be used to cut metals that are in the form of flat sheets. However, sawing is expensive, and it is used only when the heat effects from burning would be detrimental.

Forming

After the constructional materials have been cut, the next step is to form them into the desired shape. This can be accomplished by various methods, such as by rolling, bending, pressing, bumping (i.e., pounding), or spinning on a die. In some cases, heating may be necessary in order to carry out the forming operation. Because of work hardening of the material, annealing may be required before forming and between stages during the forming.

When the shaping operations are finished, the different parts are assembled and fitted for fastening. The fitting is accomplished by use of jacks, hoists, wedges, and other means. When the fitting is complete and all edges are correctly aligned, the main seams can be tack-welded in preparation for the final fastening.

Fastening

Riveting can be used for fastening operations, but electric welding is far more common and gives superior results. The quality of a weld is very important, because the ability of equipment to withstand pressure or corrosive conditions is often limited by the conditions along the welds. Although good welds may be stronger than the metal that is fastened together, design engineers usually assume a weld is not perfect and employ weld efficiencies of 80 to 95 percent in the design of pressure vessels.
The most common type of welding is the **manual shielded-arc** process in which an electrode approximately 14 to 16 in. long is used and an electric arc is maintained manually between the electrode and the material being welded. The electrode melts and forms a filler metal, while, at the same time, the work material fuses together. A special coating is provided on the electrode. This coating supplies a flux to float out impurities from the molten metal and also serves to protect the metal from surrounding air until the metal has solidified and cooled below red heat. The type of electrode and coating is determined by the particular materials and conditions that are involved in the welding operation.

A **submerged-arc** process is commonly used for welding stainless steels and carbon steels when an automatic operation is acceptable. The electrode is a continuous roll of wire fed at an automatically controlled rate. The arc is submerged in a granulated flux which serves the same purpose as the coating on the rods in the shielded-arc process. The appearance and quality of the submerged-arc weld is better than that obtained by an ordinary shielded-arc manual process; however, the automatic process is limited in its applications to main seams or similar long-run operations.

**Heliarc welding** is used for stainless steels and most of the nonferrous materials. This process can be carried out manually, automatically, or semiautomatically. A stream of helium or argon gas is passed from a nozzle in the electrode holder onto the weld, where the inert gas acts as a shielding blanket to protect the molten metal. As in the shielded-arc and submerged-arc processes, a filler rod is fed into the weld, but the arc in the heliarc process is formed between a tungsten electrode and the base metal.

In some cases, fastening can be accomplished by use of various solders, such as brazing solder (mp, 840 to 905°C) containing about 50 percent each of copper and zinc; silver solders (mp, 650 to 870°C) containing silver, copper, and zinc; or ordinary solder (mp, 220°C) containing 50 percent each of tin and lead. Screw threads, packings, gaskets, and other mechanical methods are also used for fastening various parts of equipment.

**Testing**

All welded joints can be tested for concealed imperfections by X rays, and code specifications usually require X-ray examination of main seams. Hydrostatic tests can be conducted to locate leaks. Sometimes, delicate tests, such as a helium probe test, are used to check for very small leaks.

**Heat-treating**

After the preliminary testing and necessary repairs are completed, it may be necessary to heat-treat the equipment to remove forming and welding stresses, restore corrosion-resistance properties to heat-affected materials, and prevent
stress-corrosion conditions. A low-temperature treatment may be adequate, or the particular conditions may require a full anneal followed by a rapid quench.

**Finishing**

The finishing operation involves preparing the equipment for final shipment. Sandblasting, polishing, and painting may be necessary. Final pressure tests at 1 \(\frac{1}{2}\) to 2 or more times the design pressure are conducted together with other tests as demanded by the specified code or requested by the inspector.

**PROBLEMS**

1. A new plant requires a large rotary vacuum filter for the filtration of zinc sulfite from a slurry containing 1 kg of zinc sulfite solid per 20 kg of liquid. The liquid contains water, sodium sulfite, and sodium bisulfite. The filter must handle 8000 kg of slurry per hour. What additional information is necessary to design the rotary vacuum filter? How much of this information could be obtained from laboratory or pilot-plant tests? Outline the method for converting the test results to the conditions applicable in the final design.

2. For each of the following materials of construction, prepare an approximate plot of temperature versus concentration in water for sulfuric acid and for nitric acid, showing conditions of generally acceptable corrosion resistance:
   
   (a) Stainless steel type 302.
   (b) Stainless steel type 316.
   (c) Karbate
   (d) Haveg


3. A process for sulfonation of phenol requires the use of a 3000-gal storage vessel. It is desired to determine the most suitable material of construction for this vessel. The time value of money is to be taken into account by use of an interest rate of 10 percent.

   The life of the storage vessel is calculated by dividing the corrosion allowance of \(\frac{1}{8}\) in. by the estimated corrosion rate. The equipment is assumed to have a salvage value of 10 percent of its original cost at the end of its useful life.

   For the case in question, corrosion data indicate that only a few corrosion-resistant alloys will be suitable:

<table>
<thead>
<tr>
<th>Vessel Type</th>
<th>Installed cost</th>
<th>Average corrosion rate, in./yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel clad</td>
<td>$80,000</td>
<td>0.020</td>
</tr>
<tr>
<td>Monel clad</td>
<td>$95,000</td>
<td>0.010</td>
</tr>
<tr>
<td>Hastelloy B</td>
<td>$180,000</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

   Determine which material of construction would be used with appropriate justification for the selection.
4. What materials of construction should be specified for the thiophane process described in Prob. 20 of Chap. 22? Note the extremes of temperatures and corrosion which are encountered in this process because of the regeneration step and the presence of $\text{H}_2\text{S}$ and caustic.

5. A manhole plate for a reactor is to be 2 in. thick and 18 in. in diameter. It has been proposed that the entire plate be made of stainless steel type 316. The plate will have 18 bolt holes, and part of the face will need to be machined for close gasket contact. If the base price for stainless steel type 316 in the form of industrial plates is $4.00 per pound, estimate the purchased cost for the manhole plate.

6. Six tanks of different constructional materials and six different materials to be stored in these tanks are listed in the following columns:

<table>
<thead>
<tr>
<th>Tanks</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass-lined</td>
<td>20% hydrochloric acid</td>
</tr>
<tr>
<td>Carbon steel</td>
<td>10% caustic soda</td>
</tr>
<tr>
<td>Concrete</td>
<td>75% phosphoric acid for food products</td>
</tr>
<tr>
<td>Nickel-lined</td>
<td>98% sulfuric acid</td>
</tr>
<tr>
<td>Stainless steel type 316</td>
<td>Vinegar</td>
</tr>
<tr>
<td>Wood</td>
<td>Water</td>
</tr>
</tbody>
</table>

All tanks must be used, and all materials must be stored without using more than one tank for any one material. Indicate the material that should be stored in each tank.

7. For the design of internal-pressure cylindrical vessels, the API-ASME Code for Unified Pressure Vessels recommends the following equations for determining the minimum wall thickness when extreme operating pressures are not involved:

$$t = \frac{PD_m}{2SE} + C \quad \text{applies when} \quad \frac{D_o}{D_i} < 1.2$$

or

$$t = \frac{D_i}{2}\left(\sqrt{\frac{SE + P}{SE - P}} - 1\right) + C \quad \text{applies when} \quad \frac{D_o}{D_i} > 1.2$$

where $t =$ wall thickness, in.
$P =$ internal pressure, psig (this assumes atmospheric pressure surrounding the vessel)
$D_m =$ mean diameter, in.
$D_i =$ ID, in.
$D_o =$ OD, in.
$E =$ fractional efficiency of welded or other joints
$C =$ allowances, for corrosion, threading, and machining, in.
$S =$ design stress, lb/in.* (for the purpose of this problem, $S$ may be taken as one-fourth of the ultimate tensile strength)

A cylindrical storage tank is to have an ID of 12 ft and a length of 36 ft. The seams will be welded, and the material of construction will be plain carbon steel (0.15 percent C). The maximum working pressure in the tank will be 100 psig, and the